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International Arbitrage Pricing Theory: An Empirical Investigation

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ABSTRACT

In this paper, we test the arbitrage pricing theory (APT) in an international setting. Inter-battery factor analysis is used to estimate the international common factors and the Chow test is used in testing the validity of the APT. Our inter-battery factor analysis results show that the number of common factors between a pair of countries ranges from one to five, and our cross-sectional test results lead us to reject the joint hypothesis that the international capital market is integrated and that the APT is internationally valid. Our results, however, do not rule out the possibility that the APT holds locally or regionally in segmented capital markets. Finally, the basic results of both the inter-battery factor analysis and the cross-sectional tests are largely invariant to the numeraire currency chosen.

NUMEROUS AUTHORS, NOTABLY SOLNIK [21], Grauer, Litzenberger, and Stehle [11], and Stulz [24] have derived various versions of the international asset pricing model (IAPM) under alternative views of the structure of international capital markets. However, only a few serious attempts have been made to test various versions of the IAPM. These tests are largely inconclusive (see Solnik [20] and Stehle [23]). Apart from the problem stressed by Roll [16] of identifying the world market portfolio, previous tests of the IAPMs suffer from the technical problem of aggregating assets of national investors using different numeraire currencies. Differences in the numeraire arise from differences in consumption baskets in an environment characterized by exchange rate uncertainty.

In a fruitful attempt to extend the arbitrage pricing theory (APT) of Ross [18] to an international setting, Solnik [22] derives an international arbitrage pricing theory which is largely devoid of the aforementioned difficulties and thus more amenable to empirical testing.[1] As shown by Solnik, testability of the APT in an

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[1] The consumption-based IAPM of Stulz [24] is another model which seems to be more amenable to empirical testing. In the Stulz IAPM, the (world) market portfolio does not play an essential role. Empirical tests of the model, however, could be hampered by at least two difficulties. First, as pointed out by Cornell [5], the effects of the state variables are impounded in the consumption betas, implying that the consumption betas will be nonstationary if the state variables are random. Second, given that the national income accounts, the main source of aggregate consumption data, are subject to

international setting stems from the fact that, unlike asset returns, factors do not have to be translated from one currency to another. Furthermore, since the APT addresses relative pricing on any set of n assets following a particular return-generating process, it can be tested by examining only subsets of the universe of assets. Neither the international market portfolio nor a set of mean-variance efficient portfolios of the primary assets implied by the existing IAPMs play an essential role.

In fact, derivation of the asset pricing relationship via arbitrage consideration is not new in international finance. Let us consider an "arbitrage" portfolio which consists of: (i) borrowing a certain amount in U.S. dollars; (ii) lending the equivalent pound amount in the U.K.; and (iii) selling the proceeds of the pound investment forward. Clearly, this portfolio entails neither net investment nor (exchange) risk. To preclude arbitrage opportunities, such a portfolio should yield zero profit. From this arbitrage condition follows an international parity relationship stating that the interest rate differential should be equal to the forward exchange premium or discount. This, of course, is the well-known interest rate parity relationship (IRP). Thus, the IRP is akin in spirit to the APT. Unlike the APT, however, the IRP is incapable of pricing equities, the future payoffs of which are not fixed in any particular currency.

The purpose of this paper is to test the APT in an international setting (IAPT). Specifically, we address various issues as outlined in the following procedure:

(i) Extracting the number of international factors common to the universe of assets across national boundaries;
(ii) Testing the asset pricing relationship implied by the IAPT; and
(iii) Examining whether the factor structure and the asset pricing relationship are invariant to the numeraire chosen by using two major currencies, i.e., the U.S. dollar and the Japanese yen.

As will be discussed in detail, our test involves the joint hypothesis of the international capital market being integrated and the APT being valid internationally.

The rest of the paper is organized as follows. Section I briefly reviews the international arbitrage pricing theory. Section II discusses the test methodology and the hypotheses to be tested. Section III presents the empirical results. Section IV concludes the paper.

I. International Arbitrage Pricing Theory: A Review

Suppose there exist k factors in the world economy which generate the random returns on a set of n international assets in terms of a given numeraire currency,

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errors and omissions, it would be a formidable task to measure the aggregate world real consumption rate without error. Recently, Shanken [19] has questioned the testability of APT itself. In response to this, Dybvig and Ross [7] specify certain testability restrictions and argue that these restrictions are reasonably satisfied by the real world economy.

say, the U.S. dollar:

$$\tilde{r}_i = E_i + b_{i1}\tilde{\delta}_1 + b_{i2}\tilde{\delta}_2 + \cdots + b_{ik}\tilde{\delta}_k + \varepsilon_i, \qquad i = 1, \cdots, n \tag{1}$$

where E_i is the expected return on the ith asset, $\tilde{\delta}_j$'s are zero mean international common factors, b_{ij} is the sensitivity of the ith asset to the jth factor, and $\tilde{\varepsilon}_i$'s are the residual terms of the assets. As usual, it is assumed that $E(\tilde{\varepsilon}_i \mid \tilde{\delta}_j) = 0$ for $i = 1, \cdots, n, j = 1, \cdots, k, n > k$ and $E(\tilde{\varepsilon}_i^2) = \sigma_i^2 < \infty$.

Assuming that investors have homogeneous expectations concerning the k-factor generating process of Equation (1), we can derive the IAPT in terms of the U.S. dollar in the usual manner. Suppose that there is a sufficient number of assets so that a portfolio with the following characteristics can be formed:

$$\underline{x}^t\underline{1} = 0, \tag{2a}$$

$$\underline{x}^t\underline{b}_j = 0, \qquad j = 1, \cdots, k \tag{2b}$$

where $\underline{x}^t$ is an n-dimensional (row) vector of portfolio weights; $\underline{1}$ is an n-dimensional vector of ones; $\underline{b}_j$ is an n-dimensional vector of factor loadings b_{ij}'s. These portfolios entail neither net investment nor systematic risk. Further, the idiosyncratic risk of these portfolios should become negligible as the number of securities grows large. Consequently, in order to preclude arbitrage opportunities, these portfolios must earn zero profits, which in return implies the following relationship.

$$\underline{E} \simeq \lambda_0 + \lambda_1\underline{b}_1 + \cdots + \lambda_k\underline{b}_k \tag{3}$$

where $\underline{E}$ is an n-dimensional vector of E_i's.

The k weights, $\lambda_1, \cdots, \lambda_k$, can be viewed as risk premia. If a riskless asset exists with return, E_0, then $\lambda_0 = E_0$ and $\lambda_j = E^j - E_0$ where E^j is the expected return on portfolios with only systematic factor j risk. It is well known in the APT literature that the IAPT of Equation (3) holds only as an approximation, particularly in a finite economy, as shown by Ross [18] and others. In a large economy with infinitely many assets, the model holds as an exact equality under certain conditions (see Dybvig and Ross [7], for instance). However, the magnitude of mispricing due to the approximation should be mitigated in the international context by the fact that there are more assets in the world economy than in any particular national economy.

Although Equation (3) applies to a set of international assets, rather than a set of local assets as in the domestic APT, its structure is identical to the standard APT of Ross [18]. However, Solnik [22] demonstrates that the APT structure in Equation (3) is invariant to the currency chosen. Solnik shows that this invariance result is dependent upon two other invariance propositions he demonstrates, namely, (i) an arbitrage portfolio that is riskless in a given currency is also riskless in any other currency, and (ii) the factor structure in Equation (1) is also invariant to the choice of a currency in terms of decomposition into k factors and a residual. To derive the "invariance" propositions, Solnik requires that the exchange rates (like security returns) follow the k-factor model of Equation (1), with the assumption of mutually independent idiosyncratic terms.[2]

[2] For a detailed derivation and discussion of the invariance propositions, see Solnik [22].

II. Test Methodology and Hypotheses

In this section, we discuss the testing procedures, the hypotheses to be tested, and the data used in this study. We shall begin with a discussion of the joint nature of the hypothesis.

A. International Market Integration and IAPT: Joint Hypothesis

Our test involves a joint hypothesis like any other test of the asset pricing models. In the domestic setting, for instance, most of the studies test the joint hypothesis of the market being efficient and the underlying asset pricing model being valid. In an international setting, there is one additional hypothesis, i.e., the markets being integrated. International capital markets can be viewed as integrated if assets in various national markets are traded as though their prices are determined in a unified market so as to yield the same price in a given currency across countries.

Capital markets can be segmented along regional lines due to severe imperfections resulting from discriminatory border taxes, possibilities of expropriation, exchange controls, information gaps, etc. The existence of exchange rate uncertainty per se does not cause segmentation. Indeed, as we saw earlier, the IAPT was developed in an environment characterized by exchange rate fluctuations. We should also point out that, even if the IAPT fails to hold internationally, it can characterize a subset or a segment of international capital markets. This paper does not provide such regional tests of APT.

We cannot evaluate the extent of capital market integration between two countries by looking at the number of common factors. A strong single common factor may depict more integration than several weak factors. By the same token, we cannot judge capital market integration on the basis of economic integration. Two economies, the industrial bases of which are quite similar, could well have their capital markets segmented by virtue of the frictions that limit accessibility by foreign residents to domestic capital markets. The frictions that potentially segment capital markets are apparent, but their actual significance is an empirical matter. As mentioned earlier, we cannot infer capital market integration from factor structure or correlation structure. We must test if the factors are priced identically across markets, which should be the case if the IAPT is valid and capital markets are integrated. In this sense we seek to test a joint hypothesis.

B. Estimation of Factor Loadings: Inter-Battery Factor Analysis

Testing of the IAPT will be carried out in two parts. The first part involves estimation of the systematic risks, i.e., factor loadings for each asset, while the second part involves testing the pricing implications of the IAPT using cross-sectional regression analysis. Due to the well-known technological constraint, we adopted the group approach first used by Roll and Ross [17]. Unlike Roll and Ross, we grouped stocks according to their country membership, rather than, say, alphabetically. In view of the existing empirical findings indicating that there is a strong country factor influencing the return-generating process (see Eun and Resnick [10]), grouping stocks by their country membership is appropriate.

One of the major difficulties encountered in the group approach is the problem of comparing the factor structure across different asset groups. Granting that the factor structure is more likely to vary across countries than within a country, the problem of factor comparability becomes even more acute in an international setting. However, as Cho [4] shows, this problem can be substantially alleviated by using inter-battery factor analysis. Unlike traditional factor analysis, inter-battery factor analysis estimates the common factor loadings of two different groups of assets by examining only the inter-group sample covariance matrix rather than the entire sample covariance matrix. If two groups have the same set of factors, then it should be reflected in the inter-group covariance matrix. Conversely, the inter-group covariance matrix should reflect only those factors that are common between two groups and not those factors that are common for only one group.

Consider a pair of groups of assets whose returns are generated by the k-factor model:

$$\begin{bmatrix} \tilde{\underline{r}}_J \\ \tilde{\underline{r}}_K \end{bmatrix} = \begin{bmatrix} \underline{E}_J \\ \underline{E}_K \end{bmatrix} + \begin{bmatrix} \underline{b}_J \\ \underline{b}_K \end{bmatrix} \tilde{\underline{\delta}} + \begin{bmatrix} \tilde{\underline{\varepsilon}}_J \\ \tilde{\underline{\varepsilon}}_K \end{bmatrix}, \tag{4}$$

where J and K represent two different country groups of assets; $\underline{b}_H$, $H = J$ and K, are the $n_H \times k$ factor loading matrices; n_H is the number of securities in a group, H; $\tilde{\underline{\delta}}$ is the $k \times 1$ column vector of common factors; and $\tilde{\underline{\varepsilon}}_H$ are the $n_H \times 1$ vectors of residual terms for $H = J$ and K. We assume that $\tilde{\underline{\delta}}$ and $\tilde{\underline{\varepsilon}}_H$, $H = J$ and K, have zero means and $\tilde{\underline{\varepsilon}}_H$ are orthogonal to $\tilde{\underline{\delta}}$. Furthermore, we assume that $\tilde{\underline{\varepsilon}}_J$ and $\tilde{\underline{\varepsilon}}_K$ are orthogonal to each other so that the covariance matrix of $[\tilde{\underline{\varepsilon}}_J^t, \tilde{\underline{\varepsilon}}_K^t]^t$ is block diagonal with covariance matrices of $\tilde{\underline{\varepsilon}}_H$ being ψ_H for $H = J$ and K. Note that we allow more than k common factors within each group by not assuming ψ_H to be a diagonal matrix. For the sake of convenience, it is also assumed that the covariance matrix of $\tilde{\underline{\delta}}$ is an identity matrix. Then, the traditional factor analysis would find estimates $\hat{\underline{b}}_H$ and $\hat{\psi}_H$ for $H = J$ and K in such a way that the estimated matrix, $\hat{V}$, closely replicates the sample covariance matrix, V, where

$$\hat{V} = \begin{bmatrix} \hat{\underline{b}}_J \hat{\underline{b}}_J^t + \hat{\psi}_J & \hat{\underline{b}}_J \hat{\underline{b}}_K^t \\ \hat{\underline{b}}_K \hat{\underline{b}}_J^t & \hat{\underline{b}}_K \hat{\underline{b}}_K^t + \hat{\psi}_K \end{bmatrix}, \tag{5}$$

$$V = \begin{bmatrix} V_{JJ} & V_{JK} \\ V_{KJ} & V_{KK} \end{bmatrix}. \tag{6}$$

On the other hand, the inter-battery factor analysis estimates $\hat{\underline{b}}_J$ and $\hat{\underline{b}}_K$ by relating their product $\hat{\underline{b}}_J \hat{\underline{b}}_K^t$ to V_{JK}.

There are several advantages to using inter-battery factor analysis rather than standard factor analysis. First, one can factor analyze a larger dimensional problem than is possible by traditional factor analysis. This is true because inter-battery factor analysis focuses only on the submatrix V_{JK} rather than the entire matrix V. Second, since the solution of the inter-battery factor analysis is in a closed form, a global optimal solution can be determined without going through an iteration procedure. Note that standard factor analysis does not guarantee a global optimal solution. Third, as we show later, inter-battery factor analysis does not appear as sensitive to the number of variables included in a sample as the standard factor analysis. Finally, inter-battery factor analysis estimates the factor loadings by constraining the factor structures between the two groups to be the same. This enables us to carry out the cross-sectional analysis in a more efficient manner using the Chow test.

C. Hypotheses Testing

Once we obtain estimates of international factor loadings, we can test the basic cross-sectional pricing relationship of the IAPT in Equation (3). Cross-sectional tests are performed by comparing the risk-free rate and the risk premia between two different country groups using the Chow test as was done in Brown and Weinstein [3]. Specifically, we test the following null hypotheses:[3]

(H1) the risk-free rate is the same between two country groups;
(H2) the risk premia are the same between two country groups;
(H3) both the risk-free rate and the risk premia are the same between two country groups.

Each of the above hypotheses will be tested using the U.S. dollar and the Japanese yen as the numeraire currency.

As previously mentioned, our test involves a joint hypothesis that the international capital market is integrated and that the APT is valid internationally. If the APT holds internationally, then none of the above hypotheses, H1–H3, should be rejected. If any of the hypotheses is rejected, then the APT does not hold internationally. It should be stressed, however, that, even if the APT does not hold internationally, it may hold locally in segmented capital markets.

Let us briefly provide the hypotheses testing procedure. If the APT holds, then the expected returns on assets must be linear combinations of the factor loadings:

$$\underline{E}_H = [\underline{1} : \underline{b}_H] \begin{bmatrix} \lambda_{0H} \\ \underline{\lambda}_H \end{bmatrix}, \qquad \text{for } H = J \text{ and } K \tag{7}$$

where λ_{0H} is the risk-free rate; and $\underline{\lambda}_H$ represents a column vector of the k risk premia for group H. Substituting Equation (7) into Equation (4), we obtain

$$\begin{bmatrix} \tilde{\underline{r}}_J \\ \tilde{\underline{r}}_K \end{bmatrix} = \begin{bmatrix} \underline{1} : \underline{b}_J : 0 : 0 \\ 0 : 0 : \underline{1} : \underline{b}_K \end{bmatrix} \begin{bmatrix} \lambda_{0J} \\ \underline{\lambda}_J \\ \lambda_{0K} \\ \underline{\lambda}_K \end{bmatrix} + \begin{bmatrix} \tilde{\underline{e}}_J \\ \tilde{\underline{e}}_K \end{bmatrix}, \tag{8}$$

where $\tilde{\underline{e}}_H = \underline{b}_H \tilde{\underline{\delta}} + \tilde{\underline{\varepsilon}}_H$ for $H = J$ and K.

[3] We also attempted to test for the effect of residual risk on international asset pricing, although this is not of direct interest to the paper. We do not report the results, because our tests are based on residuals extracted from the same sample, and hence there is bias toward rejecting the hypothesis that residual risk does not affect pricing. The bias is due to spurious correlation between sample mean and residual variation. Despite this bias, however, we do not reject the hypothesis on a data that excludes Japan and Australia. Consequently, we think our residual variance has negligible impact on international asset pricing if the bias were corrected for.

Noting that Equation (8) should hold for each time period and assuming the stationarity of factor loadings, we obtain the mean returns as follows:

$$\begin{bmatrix} \bar{\underline{r}}_J \\ \bar{\underline{r}}_K \end{bmatrix} = \begin{bmatrix} \underline{1}: \underline{b}_J: 0: 0 \\ 0: 0: \underline{1}: \underline{b}_K \end{bmatrix} \begin{bmatrix} \lambda_{0J} \\ \lambda_J \\ \lambda_{0K} \\ \lambda_K \end{bmatrix} + \begin{bmatrix} \bar{\underline{e}}_J \\ \bar{\underline{e}}_K \end{bmatrix}, \tag{9}$$

where $\bar{\underline{r}}_H = \sum_{t=1}^{T} \tilde{r}_{Ht}/T$; $\bar{\underline{e}}_H = \sum_{t=1}^{T} \tilde{\underline{e}}_{Ht}/T$ for $H = J$ and K; and T is the total number of time periods.

Equation (9) is the "unconstrained" regression equation. Now, by imposing each of the hypotheses on Equation (9), we can derive the "constrained" regression equations. The Chow test entails comparison of the constrained residual sum of squares (SSE_c) with the unconstrained residual sum of squares (SSE_u). A given hypothesis is not rejected when the two residual sum of squares are close in value. Furthermore, if df_c and df_u denote the degrees of freedom for the constrained and unconstrained regressions, respectively, then

$$F = \frac{(SSE_c - SSE_u)/(df_c - df_u)}{SSE_u/df_u} \tag{10}$$

has an F-distribution with $(df_c - df_u)$ and df_u degrees of freedom. Note that one has to carry out the regressions using the Generalized Least Square methodology due to the correlations among residuals.[4]

D. Data

Our sample consists of 349 stocks representing 11 different countries, the monthly returns of which are available for the entire period of January 1973 through December 1983.[5] This sample period is roughly characterized by flexible exchange rates. Monthly return data for the U.S. and the foreign stocks were obtained, respectively, from the monthly version of the tape furnished by Center for Research in Security Prices (CRSP) of the University of Chicago and various monthly issues of *Capital International Perspective* (*CIP*). The returns were adjusted for dividends. The exchange rates, which were used in converting stock returns from one currency into another, were obtained from *CIP*.

The sample firms were divided into 11 groups according to their country membership:

1. United States (US: 60 stocks)
2. Canada (CA: 28 stocks)
3. France (FR: 24 stocks)
4. Germany (GE: 22 stocks)
5. Netherlands (NE: 26 stocks)
6. Switzerland (SW: 22 stocks)

[4] Readers are referred to Cho [4] for a detailed discussion of the inter-battery factor analysis and the test statistics.

[5] The period for the Canadian stocks was January 1975 through December 1983.

7. United Kingdom (UK: 48 stocks)
8. Australia (AU: 26 stocks)
9. Hong Kong (HK: 14 stocks)
10. Singapore (SI: 24 stocks)
11. Japan (JA: 55 stocks)

Thus, 4 Asia-Pacific, 5 European and 2 North American countries were represented in the sample. With the exception of Hong Kong, each country group contained at least 20 stocks and not more than 60 stocks. This was necessary to have large enough sample sizes in the second-stage cross-sectional regressions and to ease the calculation of the correlation matrices of the combined groups.

Table I presents the average correlation coefficients among country groups in terms of both the U.S. dollar and the Japanese yen. Since factors are derived from correlation matrices, it is worthwhile to briefly examine them. As expected in accordance with the existing literature, securities are found to be, on average, less positively correlated across countries than within a country. In fact, for our sample, the mean of the average inter-country correlations is 0.2336 (0.2029) in the U.S dollar (Japanese yen). This is compared with the mean of the average intra-country correlations, which is 0.5175 (0.4995) in the U.S. dollar (Japanese yen). It is also noted from comparing the two panels of Table I that the

Table I

Average Correlation Coefficients

	US	CA	FR	GE	NE	SW	UK	AU	HK	SI	JA
					Panel A: U.S. Dollar						
US	0.379										
CA	0.263	0.427									
FR	0.196	0.224	0.546								
GE	0.141*	0.150	0.290	0.539							
NE	0.189*	0.202	0.293	0.389	0.484						
SW	0.201	0.244	0.339	0.402	0.403	0.579					
UK	0.225	0.260	0.315	0.233	0.261	0.323	0.617				
AU	0.225	0.258	0.239	0.154	0.155	0.284	0.282	0.488			
HK	0.111*	0.139*	0.185	0.204	0.198	0.260	0.213	0.192	0.672		
SI	0.232	0.221	0.208	0.211	0.234	0.306	0.324	0.268	0.294	0.562	
JA	0.107*	0.134*	0.213	0.242	0.222	0.254	0.179	0.148	0.226	0.191	0.401
					Panel B: Japanese Yen						
US	0.463										
CA	0.332	0.451									
FR	0.206	0.205	0.514								
GE	0.192	0.146*	0.245	0.515							
NE	0.234	0.213	0.254	0.360	0.466						
SW	0.243	0.236	0.289	0.357	0.368	0.541					
UK	0.254	0.263	0.291	0.213	0.247	0.301	0.609				
AU	0.267	0.268	0.208	0.136	0.138	0.263	0.272	0.489			
HK	0.074*	0.103*	0.102*	0.108*	0.112*	0.161	0.160	0.132*	0.635		
SI	0.255	0.211	0.109	0.189	0.219	0.286	0.313	0.262	0.261	0.555	
JA	0.080*	0.062*	0.108*	0.125*	0.118*	0.120*	0.111*	0.070*	0.110*	0.131*	0.256

* Not significantly different from zero at 0.05 significance level.

international correlation structure is similar and varies only a little between the two currencies used to measure security returns.

III. Empirical Results

In this section, the inter-battery factor analysis is conducted to estimate the international common factors. Then, cross-sectional tests are performed in order to investigate the validity of the IAPT.

A. International Common Factors

All of the test samples were constructed by combining two different country groups. To be specific, each of the 11 country groups was combined with each of the remaining 10 country groups to generate 55 distinct samples. This procedure allows us to generate the greatest number of samples and test the validity of the IAPT across all groups of securities. We used the inter-battery factor analysis technique, which allowed us to constrain the factor structure between a pair of countries to be the same or common. Recall that this commonality is not ensured by the traditional factor analysis technique. On the other hand, our approach focuses only on common factors across countries and hence does not consider purely national factors. Nonetheless, it still allows us to test the pricing of common factors as predicted by the IAPT. This is, of course, the primary goal of the paper.

Results of the inter-battery factor analyses on the 55 samples, which are obtained using the significance level of 0.1, are summarized in Table II. Panel A and Panel B, respectively, present the number of factors to be used in subsequent analyses and their corresponding p-levels in terms of the U.S. dollar. Panel C and Panel D present the same in terms of the Japanese yen. It is interesting to note that the numbers in Panel A and Panel C are about the same. There are five cases in which Panel A has more factors, while there are six cases in which Panel C has more factors. In all of the 11 cases, however, the difference is only one factor. Thus, the factor structure is largely invariant to the numeraire currency chosen between the dollar and the yen. Hence, our discussion will be mainly focused on Panel A.

First, casual observation reveals that the samples that are paired with the U.S. group seem to have more factors than other samples. One apparent explanation might be that the U.S. group had more stocks than other groups. In traditional factor analysis, as was documented by Kryzanowski and To [13] and Dhrymes, Friend, and Gultekin [6], one should expect to find more factors as the number of variables in a sample increases.[6] Their studies show that the correlation coefficient between the number of factors and the size of samples is about 0.98. Our results, however, seem to indicate that we do not have as high a correlation.

[6] From a statistical point of view, we should note that χ^2 is positively related to the number of variables. Thus, adding a variable would, in general, increase χ^2 and, as long as this increase is more than that compensated by the increase in the degrees of freedom, one would tend to reject the hypothesis more often than not.

Table II

International Common Factors

	US	CA	FR	GE	NE	SW	UK	AU	HK	SI
Panel A: Number of Factors (U.S. Dollar)										
CA	5									
FR	2	1								
GE	3	1	3							
NE	3	2	2	4						
SW	4	2	2	2	2					
UK	5	2	1	3	3	2				
AU	3	1	2	1	1	1	1			
HK	3	1	1	2	1	2	1	1		
SI	3	1	3	2	2	4	2	1	4	
JA	5	1	2	1	2	3	3	1	3	2
Panel B: *p*-level (U.S. Dollar)										
CA	0.250									
FR	0.511	0.314								
GE	0.159	0.460	0.405							
NE	0.190	0.263	0.329	0.291						
SW	0.138	0.127	0.286	0.217	0.273					
UK	0.104	0.279	0.226	0.221	0.456	0.168				
AU	0.101	0.413	0.272	0.845	0.583	0.187	0.124			
HK	0.292	0.244	0.539	0.269	0.101	0.163	0.206	0.282		
SI	0.340	0.576	0.164	0.180	0.309	0.262	0.230	0.403	0.244	
JA	0.294	0.115	0.364	0.168	0.471	0.253	0.114	0.237	0.245	0.262
Panel C: Number of Factors (Japanese Yen)										
CA	5									
FR	2	1								
GE	3	1	2							
NE	4	2	2	4						
SW	5	2	2	2	2					
UK	6	2	1	3	3	2				
AU	3	1	2	1	1	1	2			
HK	3	1	1	2	1	2	1	1		
SI	3	1	3	2	1	3	2	1	4	
JA	5	2	1	1	2	3	4	1	2	2
Panel D: *p*-level (Japanese Yen)										
CA	0.200									
FR	0.407	0.445								
GE	0.228	0.467	0.114							
NE	0.436	0.444	0.335	0.311						
SW	0.278	0.135	0.346	0.238	0.288					
UK	0.487	0.322	0.165	0.202	0.370	0.173				
AU	0.144	0.369	0.308	0.804	0.577	0.231	0.426			
HK	0.225	0.308	0.552	0.331	0.231	0.247	0.313	0.263		
SI	0.285	0.592	0.168	0.366	0.105	0.151	0.210	0.285	0.354	
JA	0.224	0.512	0.128	0.165	0.488	0.440	0.465	0.256	0.152	0.224

For example, (HK, UK) has 62 stocks resulting in one factor, whereas (HK, SI) has 38 stocks resulting in four factors. In fact, the correlation coefficient between the number of factors and the size of samples is found to be about 0.45 in our tests. This clearly shows one of the advantages that inter-battery factor analysis has over standard factor analysis. It should be noted, however, that our results may still be biased due to the differences in sizes across different samples.

Next, we observe that the number of factors ranges from one to five. In the domestic setting, Cho [4] finds that the number of factors ranges from two to nine. Thus, our results show that the number of factors fluctuates less in an international setting than in a domestic setting. However, we still observe a rather strong sampling fluctuation in the number of factors. It is not clear why we observe this fluctuation. However, as suggested by Cho [4], the observed fluctuations in the number of factors may reflect the homogeneity of the groups involved. For example, if two groups represent the same industry, then one should expect to find only those factors that are relevant to that industry. On the other hand, if two groups represent several different industries, then one should expect to find a wide range of factors that are relevant for the combined industries.

Against this backdrop, the number of factors reported in Table II may be interpreted as reflecting the complexity of the economic relationship between two countries. If two countries are integrated through many levels of economic activity (i.e., high "economic" integration), then we should expect to find more factors. Conversely, if two countries are integrated only through limited levels of economic activity (i.e., low "economic" integration), then we should expect to find a smaller number of factors.[7] This may also explain why fewer common factors were found in the international capital market than in the domestic capital market. In other words, economies are less integrated internationally than domestically. To the extent that the number of international common factors reflects the degree of economic integration, the United States can be said to be highly economically integrated with Canada, the United Kingdom and Japan, and least integrated with France.

Finally, Table III reports the average number of factors that each country has in common with the other countries. Suppose we want to estimate the average number of factors at an α significance level. Considering that the number of factors in Table II represents various p-levels, one cannot take a simple average of the number of factors in the table.[8] However, we can use a procedure similar to Roll and Ross [17]. By choosing an α significance level, we implicitly allow $100\alpha\%$ of our samples to reject the null hypothesis. Hence, we can estimate the average number of factors by identifying the smallest number of factors, say k, at which less than $100\alpha\%$ of our samples could reject the hypothesis that k factors are sufficient. For this procedure, we need p-level distributions for each factor, and an illustrative case is provided in Panel C of Table III for the United States.

Let us, for example, determine the average number of factors for the United States at the 0.1 significance level. As shown in Panel C of Table III, with three factors, there are four samples, namely, those paired with Canada, Switzerland,

[7] Economic integration discussed here in association with the number of factors should not be confused with the issue of capital market integration to be investigated later in this paper. Capital markets of two countries, which are characterized by high economic integration, may well be segmented from each other.

[8] For example, (US, FR) has two factors with a p-level of 0.5112 and (US, AU) has three factors with a p-level of 0.1009. Thus, if one takes the simple average of these two numbers of factors, one may not be able to calculate the p-level for the average number of factors.

Table III

Average Number of International Common Factors and p-Level Distribution of U.S. Common Factors

Panel A: U.S. Dollar

p-level	US	CA	FR	GE	NE	SW	UK	AU	HK	SI	JA	World
0.1	5	2	3	3	3	4	3	2	3	4	3	4
0.2	5	2	2	3	3	3	3	2	3	4	3	4
0.3	5	3	3	3	3	3	3	2	3	3	4	3
0.4	4	2	3	3	3	3	3	2	3	3	3	3
0.5	5	2	3	3	3	4	3	2	3	3	3	3

Panel B: Japanese Yen

p-level	US	CA	FR	GE	NE	SW	UK	AU	HK	SI	JA	World
0.1	5	2	2	3	4	3	4	2	3	3	4	4
0.2	5	2	2	3	3	3	3	2	3	4	3	3
0.3	6	2	2	3	3	3	3	2	3	4	3	3
0.4	4	2	3	3	3	3	3	2	3	3	3	3
0.5	5	2	3	3	3	3	3	2	2	3	3	3

Panel C: p-level Distributions of U.S. Common Factors (U.S. Dollar)

Factors	CA	FR	GE	NE	SW	UK	AU	HK	SI	JA
2	0.000	0.511	0.023	0.009	0.001	0.000	0.009	0.064	0.028	0.000
3	0.004	0.942	0.159	0.190	0.029	0.000	0.101	0.292	0.340	0.001
4	0.022	0.997	0.455	0.630	0.138	0.005	0.418	0.650	0.774	0.035
5	0.250	0.999	0.747	0.888	0.420	0.104	0.782	0.835	0.960	0.294

the United Kingdom, and Japan, that reject the hypothesis. With four factors, there are three samples, namely, those paired with Canada, the United Kingdom, and Japan, that reject the hypothesis. With five factors, there are no samples that reject the hypothesis. Since there are ten samples that contain the U.S. group, at most one sample (i.e., 10% of our samples) should be allowed to reject the hypothesis at the 0.1 significance level. Thus, we conclude that on the average, the United States has five factors in common with the other countries.

The results of Panel A and Panel B in Table III were obtained from the procedures illustrated above. Panel A and Panel B present the average number of factors in terms of the U.S. dollar and the Japanese yen, respectively. Notice that the results reported in the two panels are quite similar. Again, it seems that the international factor structure is largely invariant to the numeraire currency chosen between the two currencies. Also, the average number of international common factors is fairly consistent across different significance levels. If our results were biased due to the difference in sample sizes, then we should have observed marked differences in the number of factors across different p-levels.[9] Thus, the observed consistency in the number of factors seems to indicate that the possible bias due to the difference in sample sizes is not serious in our study. In summary, Table III indicates that, on average, there are about three or four worldwide common factors.[10] It also indicates that the United States and Singapore seem to be most highly economically integrated with the other countries, whereas Australia and Canada seem to be least integrated. A possible explanation for these results on this particular economic integration is the heavy emphasis on manufacturing industries by the U.S. and Singapore and on raw materials by Australia and Canada.

B. *Tests of the International Arbitrage Pricing Theory*

We investigate the validity of the IAPT by testing the three hypotheses enumerated in Section II.C, both in the U.S. dollar and the Japanese yen. In order to save space, we briefly summarize the results in the following paragraphs and report the test results only for the hypothesis (H3) in Table IV.[11] The hypothesis (H3) implies that both the intercept and risk premia are all equal between two country groups. In conducting these tests, the number of factors was not constrained to be the same across the entire sample; rather, it was allowed to vary as long as the significance level was similar across the entire sample.[12]

At the 0.05 significance level, the hypothesis (H1) of equal intercept (or the risk-free rate) between two country groups is rejected in three out of 55 total cases in terms of the U.S. dollar, which is 5.45% of the overall sample. In terms

[9] As mentioned in footnote 6, the p-levels at which a given number of factors is accepted depend on the sample sizes. We would not expect this kind of consistency in the number of factors across the different p-levels if the bias were serious.

[10] Worldwide common factors are estimated as above by examining all of the 55 samples.

[11] The test results for the hypotheses (H1) and (H2) are available upon request.

[12] Since we do not know the number of factors in the population, we allow our sampling fluctuation in the number of factors. This was done out of our desire not to overfit the model by considering more factors than necessary.

Table IV

Cross-Sectional Test of Equal Intercepts and Risk Premia

	US	CA	FR	GE	NE	SW	UK	AU	HK	SI
					Panel A: F-Statistics (U.S. Dollar)					
CA	3.804									
FR	18.422	18.331								
GE	1.662	0.075	4.603							
NE	3.611	7.149	9.495	1.587						
SW	8.915	2.887	0.370	3.056	5.357					
UK	15.600	4.697	1.942	7.448	5.817	4.068				
AU	2.386	8.388	3.330	0.327	0.329	0.205	4.569			
HK	4.441	4.424	0.110	1.626	5.223	1.035	3.631	0.449		
SI	1.516	2.910	0.767	1.593	0.675	3.735	2.658	3.406	1.218	
JA	13.198	2.891	3.752	5.973	6.148	5.295	25.582	0.444	4.014	0.752
					Panel B: p-levels (U.S. Dollar)					
CA	0.002*									
FR	0.000*	0.000*								
GE	0.168	0.928	0.004*							
NE	0.009*	0.001*	0.000*	0.187						
SW	0.000*	0.046*	0.775	0.040*	0.003*					
UK	0.000*	0.005*	0.151	0.000*	0.001*	0.010*				
AU	0.058	0.001*	0.028*	0.723	0.721	0.816	0.014*			
HK	0.003*	0.019*	0.896	0.204	0.010*	0.391	0.033*	0.642		
SI	0.206	0.064	0.553	0.206	0.572	0.008*	0.056	0.042*	0.327	
JA	0.000*	0.061	0.015*	0.004*	0.001*	0.001*	0.000*	0.643	0.006*	0.525
					Panel C: F-Statistics (Japanese Yen)					
CA	3.280									
FR	18.089	18.467								
GE	2.062	0.271	5.663							
NE	7.270	7.591	10.467	1.886						
SW	8.957	1.149	0.465	2.991	6.046					
UK	17.574	6.902	1.968	11.228	6.046	3.186				
AU	2.625	6.431	3.106	0.462	0.195	0.262	3.009			
HK	4.553	2.986	1.185	2.201	7.373	1.094	4.458	0.577		
SI	1.885	2.409	0.406	5.868	0.790	4.847	2.543	2.342	1.368	
JA	9.980	5.398	1.163	5.538	7.377	5.869	20.762	0.550	0.916	1.138
					Panel D: p-levels (Japanese Yen)					
CA	0.006*									
FR	0.000*	0.000*								
GE	0.094	0.764	0.003*							
NE	0.000*	0.000*	0.000*	0.120						
SW	0.000*	0.340	0.709	0.043*	0.002*					
UK	0.000*	0.000*	0.148	0.000*	0.000*	0.030*				
AU	0.041*	0.003*	0.036*	0.633	0.824	0.771	0.036*			
HK	0.003*	0.063	0.318	0.109	0.002*	0.367	0.016*	0.567		
SI	0.122	0.101	0.803	0.002*	0.460	0.003*	0.006*	0.044*	0.266	
JA	0.000*	0.002*	0.318	0.006*	0.000*	0.000*	0.000*	0.580	0.438	0.340

* p-levels that are smaller than the significance level of 0.05.

of the Japanese yen, the hypothesis is rejected in two out of 55 cases, which is 3.64% of the entire sample. For the null hypothesis to be true at the 0.05 significance level, there should be at most two cases (5% of the entire sample) in which the hypothesis is rejected, if the samples are independent. Thus, our results indicate that the hypothesis should be rejected in terms of the dollar but not the yen. Considering, however, that our samples are not independent, and also that the normality assumption might have been violated, we are inclined not to reject H1 in terms of the dollar as well as the yen.

The hypothesis (H2) of the equal risk premia between two country groups is rejected in 30 out of 55 total cases in terms of both the dollar and the yen, which is about 55% of the entire sample. Table IV reports the F-statistics and p-levels for the hypothesis (H3). Again, at the 0.05 significance level, the hypothesis is rejected in 32 cases out of 55 cases in terms of both the dollar and the yen, which is about 58% of the entire sample. These empirical results lead us to reject both H2 and H3, irrespective of the numeraire currency chosen.

The empirical results presented above can be summarized as follows:

(H1) equal intercepts: not rejected.
(H2) equal risk premia: rejected.
(H3) equal intercepts and risk premia: rejected.

It should be pointed out that these results are invariant to the numeraire currency chosen. As previously mentioned, if the APT holds in an integrated international capital market, then all of the three hypotheses must not be rejected. Given that two of the hypotheses are rejected, the empirical results lead us to reject the joint hypothesis that capital markets are integrated and that the APT holds internationally.[13]

One caveat is that we have used the same data for factor estimation and testing the equality of the risk premia. This caveat is endemic to the tests conducted by Roll and Ross [17] as well. However, the bias would have worked in favor of the IAPT by increasing the test statistics associated with risk premia. Despite this, our tests reject the IAPT.[14] Still another caveat is that, due to the nature of testing the joint hypothesis, it is impossible to determine whether rejection of the joint hypothesis reflects the failure of the IAPT or segmentation of capital markets. If capital markets are segmented, then the APT cannot be valid internationally by definition, but can be valid locally or regionally. Thus, our results do not rule out the possibility of the APT being valid locally or regionally in segmented capital markets. For example, there are 20 (22) cases in terms of the dollar (the yen) out of 55 total cases, about 36% (40%) of the entire sample, in which none of the three hypotheses is rejected. This suggests the possibility that the APT holds locally or regionally in segmented capital markets. One

[13] The test results of H1, however, indicate that the risk-free rates "implicit" in the IAPT are equal in a given currency across countries. Our results thus seem to be consistent with the notion of an integrated international capital market for the risk-free assets (or the zero-beta assets) in which there exist no arbitrage opportunities across countries.

[14] We have also conducted a test of the effect of residual risk on international asset pricing. Our results are inconclusive and can be obtained from the authors upon request. See footnote 3 for further explanation.

possible cause of market segmentation is an investment barrier in the form of differential tax regimes and border (or withholding) taxes across countries (see Black [2]). Of course, local tax asymmetry between capital gains and dividends may impact the structural form of APT even in the domestic setting, since the model is originally derived under competitive and frictionless capital markets. The analogy to this is the effect of taxes on capital asset pricing as in Litzenberger and Ramaswamy [15]. Obviously, these tax effects are more pronounced in an international setting so as to segment national or regional markets and hence possibly explain some of our results.

Thus, an interesting topic for future research would be to determine different regions or segments of the world in which security prices behave as if they are determined regionally. For example, the existence of a regional APT would be important in identifying those areas of the world that create an incentive for multinational firms to play a role in integrating the international capital market as argued by Errunza and Senbet [8, 9]. With more refined data on the degree of international involvement spanning geographic diversification and the corresponding "regionalization" of APT, one can study further the extent to which multinational firms provide valuable financial intermediation services through direct foreign investment.

IV. Conclusions

In this paper, we have provided an empirical investigation of the arbitrage pricing theory in an international setting. Inter-battery factor analysis was used to estimate the common factors between two country groups, and the Chow test was used in testing the validity of the APT.

Our inter-battery factor analysis results have shown that there are about three or four worldwide common factors and that the number of common factors between two countries ranges from one to five depending on the degree of their economic integration. These results are rather similar in terms of the U.S. dollar and the Japanese yen. We have also observed that the inter-battery factor analysis produces less bias concerning the effect of sample size on the number of factors extracted than the standard factor analysis.

Our cross-sectional test results led us to reject the joint hypothesis that the international capital market is integrated and that the APT is valid internationally. At present, we are unable to determine whether rejection of the joint hypothesis reflects segmentation of capital markets or the failure of the international APT. Resolution of this issue is left for future research. Our empirical results do not rule out the possibility of the APT being valid locally or regionally in segmented capital markets.

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