Lecture 05: Circuit Solution by Circuit Reduction

Sources Connected in Series and in Parallel :

Both circuits are invalid. Why?

Circuit (a) violates KVL $\Rightarrow$ *ideal* voltage sources cannot be combined in *parallel*

(*unless they have the same voltage*)

Circuit (b) violates KCL $\Rightarrow$ *ideal* current sources cannot be combined in *series*

(*unless they have the same current*)

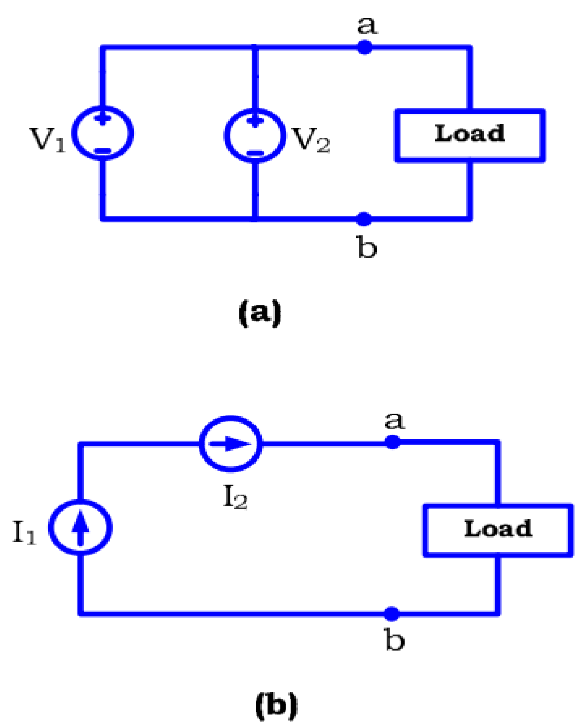


Figure 1

We can connect ideal voltage sources in *series*.

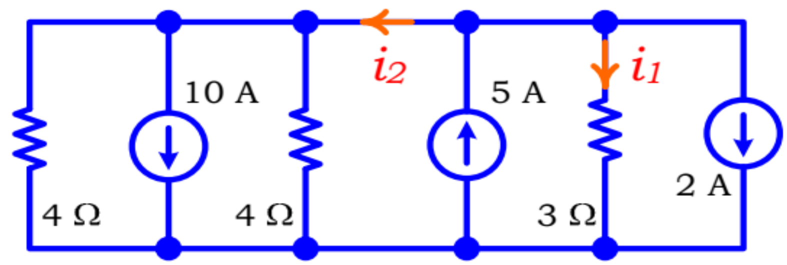
Voltage sources in *series* can be reduced to a single voltage source

$$V_{eq} = V_1 - V_2 + V_3 + V_4$$

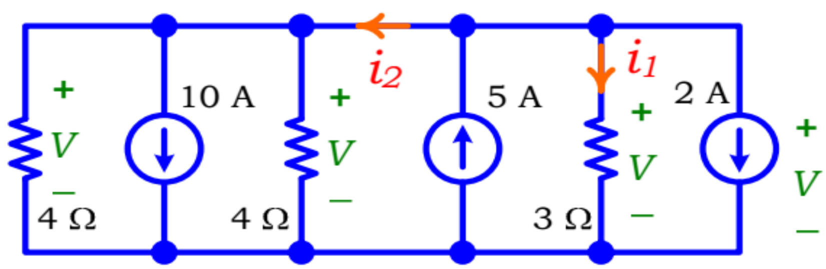
Lecture 05: Circuit Solution by Circuit Reduction

Example 1

Determine the currents i1 and i2 in the circuit of Fig. …



(a)



(b)

Solution:

$$V = \frac{-10A + 5A - 2A}{\frac{1}{4}S + \frac{1}{4}S + \frac{1}{3}S}$$

$$= \frac{-7A}{\frac{5}{6}S} = -8.4V$$

Ohm's law gives the currents through the resistors. Current i1 is labeled with the passive sign convention with respect to voltage V. Hence

$$i_1 = \frac{V}{3\Omega}$$

$$= \frac{-8.4A}{3\Omega} = -2.8A$$

Circuit i2 is the sum of the currents through the 3 ohm resistor , the 5-A current source, and the 2-A current source . Applying KCL yields:

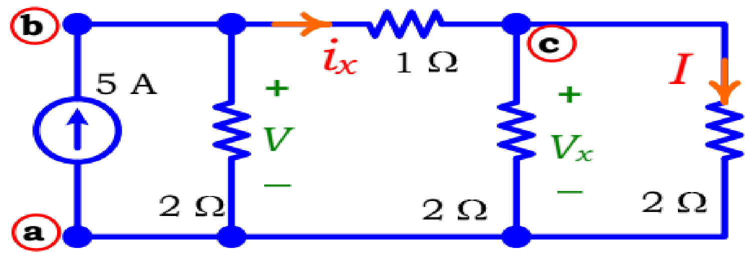
$$i_2 = 5A - i_1 - 2A = 5.8A$$

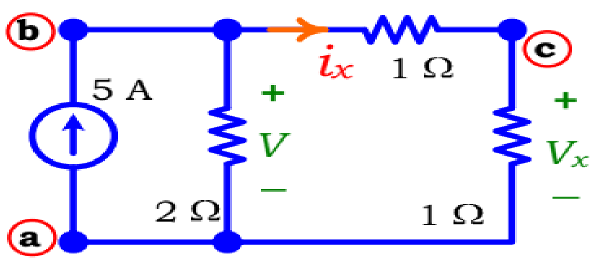
This can also be calculated as the sum of the currents through the 4-ohm resistors and the 10-A source:

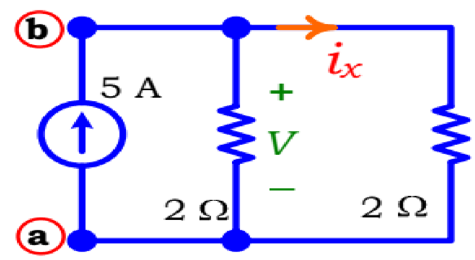
$$i_2 = \frac{V}{4\Omega} + \frac{V}{4\Omega} + 10 = 5.8A$$

Example 2

Determine voltages V and vx and currents I and ix in the circuit of Fig. …







Solution:

Combine the 2 resistors in // (2 ohm resistors) to get a resistor of 1 Ohm. Circuit in step 1. Note that to find the current I which has been lost, we have to come back to the original circuit once we find vx which remains after circuit reduction. Similarly ix remains.

Finally add the equivalent resistor to the 1 Ohm series resistor to get 1 +1 or 2 Ohm resistor, circuit in step2. Note that node c and voltage vx have disappeared in this reduction, but voltage V remains since it is across the parallel combination. Also current ix remains. No more reduction is required since we have a single node now and we can therefore determine V as:

$$V = \frac{5A}{\frac{1}{2}\Omega + \frac{1}{2}\Omega} = 5V$$

The current ix can also be determined using Ohm's law as:

$$i_x = \frac{V}{2\Omega} = \frac{5}{2}A$$

Moving back to Step 1, we determine the voltage vx using Ohm's law as:

$$v_x = i_x \times 1\Omega = \frac{5}{2}V$$

The current I can now be determined from the original circuit as

$$I = \frac{v_x}{2\Omega} = \frac{5}{4}A$$

Lecture 05: Circuit Solution by Circuit Reduction

Equivalent Resistance of N Resistors in Series:

$$R_{eq} = R_1 + R_2 + \ldots + R_N = \sum_{i=1}^{N} R_i$$

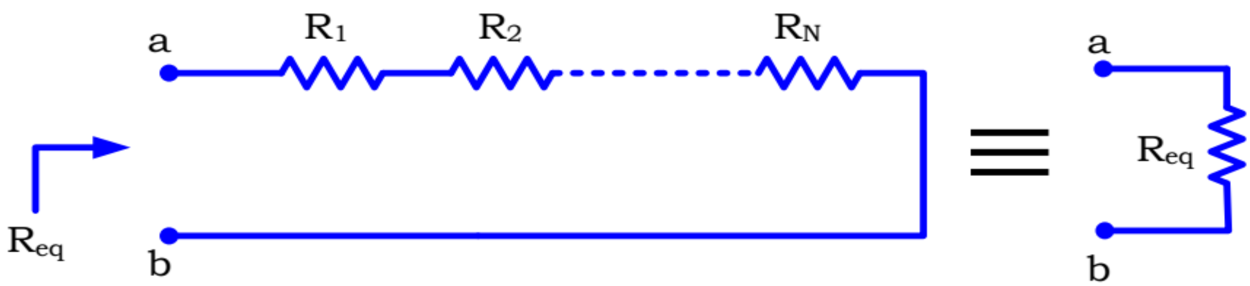


Figure 3

Equivalent Resistance of N Resistors in Parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N} = \sum_{i=1}^{N} \frac{1}{R_i}$$

Special Case: If *two* resistors R_1 & R_2 are in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2} \quad \Rightarrow \quad R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \quad \Rightarrow \quad R_{eq} = \frac{\text{Product}}{\text{Sum}}$$

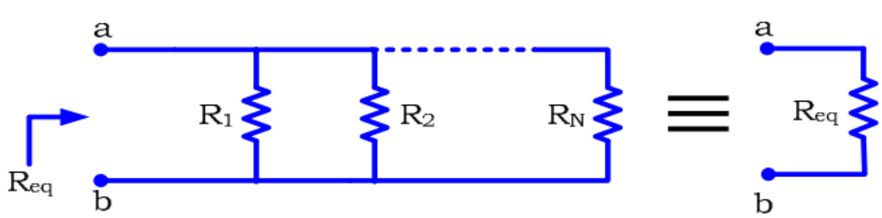


Figure 4

Example 1:

Calculate the equivalent resistance seen to the right of $a-b$.

Lecture 05: Circuit Solution by Circuit Reduction

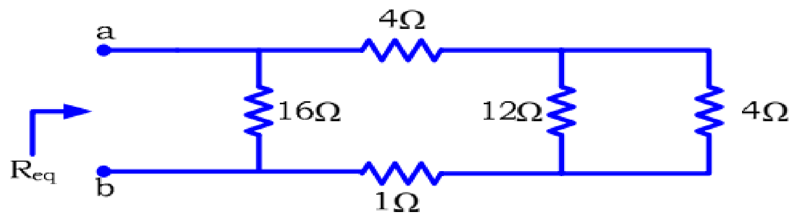


Figure 5

Solution:

12Ω & 4Ω in parallel $\Rightarrow$ $\dfrac{12\times 4}{12+4}=\dfrac{48}{16}=3\Omega$

4Ω & 3Ω & 1Ω in series $\Rightarrow$ $4+3+1=8\Omega$

16Ω & 8Ω in parallel $\Rightarrow$ $\dfrac{16\times 8}{16+8}=\dfrac{16\times 8}{24}=\dfrac{16}{3}=5.33\Omega$

$\therefore R_{eq}=5.33\Omega$

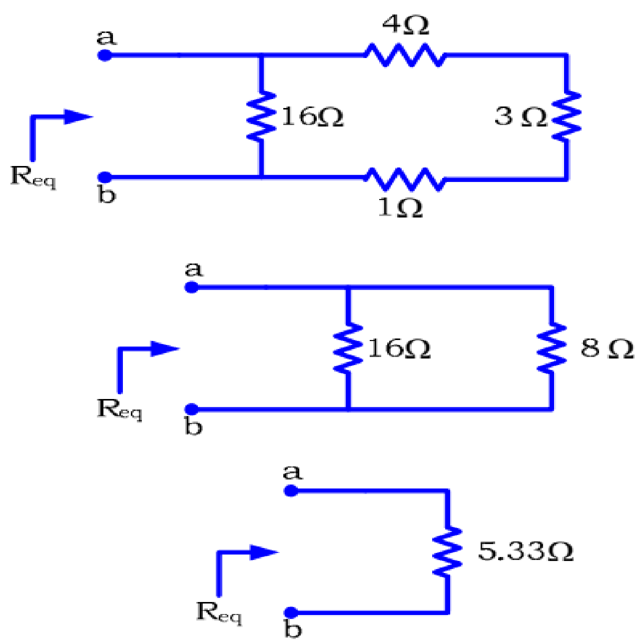


Figure 6

Conductance

The conductance G of a resistor is the reciprocal of the resistance R

$$G=\frac{1}{R}$$

Unit of G is $\frac{1}{\Omega}$ or Semen $[S]$ $\Rightarrow$ $\frac{1}{\Omega} \equiv S$

For N conductances in series

$$\frac{1}{G_{eq}} = \sum_{i=1}^{N} \frac{1}{G_i}$$

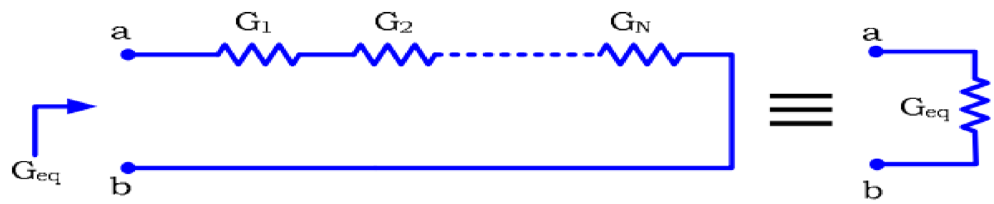


Figure 7

For N conductances in parallel

$$G_{eq} = \sum_{i=1}^{N} G_i$$

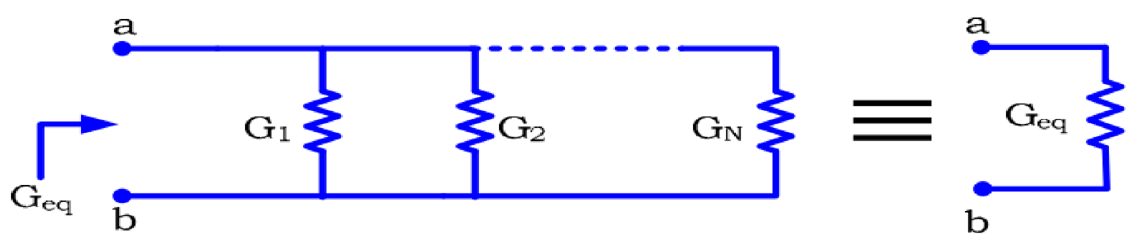


Figure 8

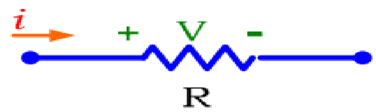
Power absorbed by a resistor:

Using circuit a) $p_R = +iv = +i(iR) = i^2 R = \frac{v^2}{R}$

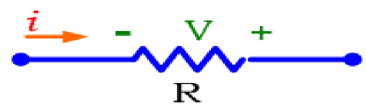
Using circuit b) $p_R = -iv = -i(-iR) = i^2 R = \frac{v^2}{R}$

$\therefore\ p_R = \frac{v^2}{R} = i^2 R$ (*regardless* of the direction of i and the polarity of v)

$\therefore\ p_R \geq 0$ $\Rightarrow$ a resistor *does not generate* electric power, it *always absorbs* it.



(a)



(b)

Figure 9

Example 2:

In the given circuit, calculate:

a) G_{eq} seen by the voltage source
b) R_{eq}
c) the power absorbed by the load

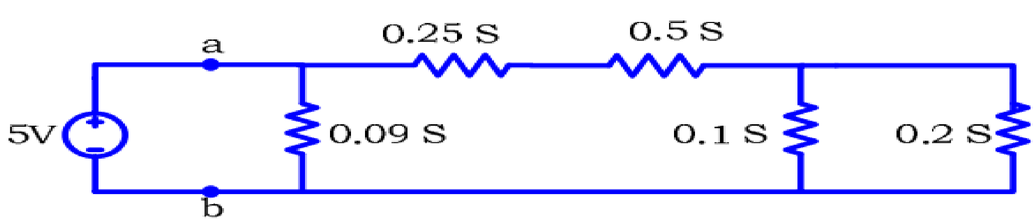


Figure 10

Solution

a)

$0.1S$ & $0.2S$ (parallel) $\Rightarrow$ $0.1+0.2=0.3S$

$0.25S$ & $0.5S$ & $0.3S$ (series) $\Rightarrow$ $\frac{1}{0.25}+\frac{1}{0.5}+\frac{1}{0.3}=4+2+3.33=9.33 \Rightarrow$

$\frac{1}{9.33}=0.107S$

$0.107S$ & $0.09S$ (parallel) $\Rightarrow$ $0.107+0.09=0.197S$

$\therefore G_{eq}=0.197S$

b) $R_{eq} = \frac{1}{Geq} = \frac{1}{0.197} = 5.08\Omega$

c) $p_{5.08\Omega} = \frac{v^2}{R} = \frac{(5)^2}{5.08} = 4.97W$

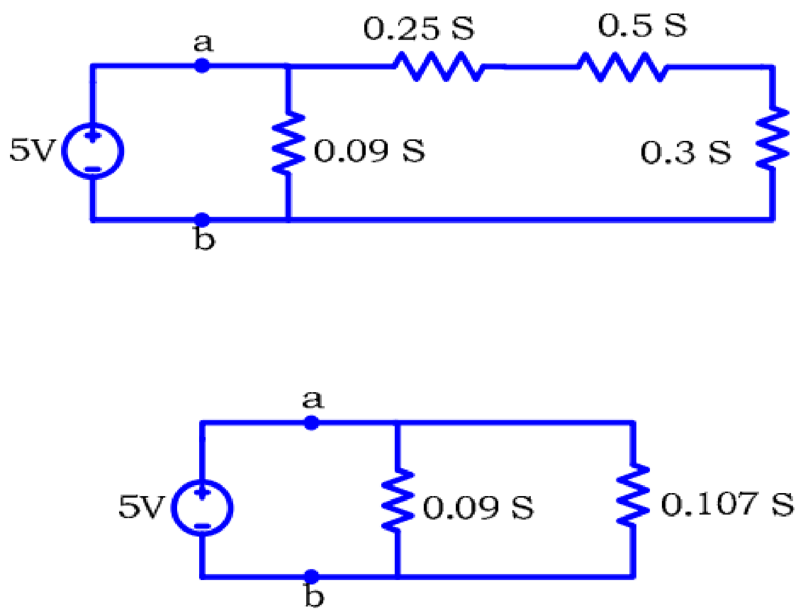


Figure 11

The meaning of the series connection

3Ω & $6V$ are in series.

$10V$ & $5A$ are in series.

4Ω & $20V$ & 5Ω are in series.

Why?

$6V$ & 2Ω are *not* in series.

2Ω & $11V$ are *not* in series.

Why?

Lecture 05: Circuit Solution by Circuit Reduction

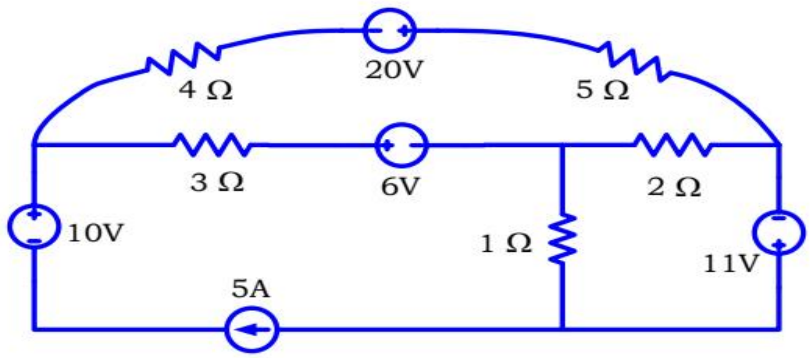


Figure 12

3Ω & $6V$ are in series $\Rightarrow$ the *same current* I_1 passes through them.

$10V$ & $5A$ are in series $\Rightarrow$ the *same current* $5A$ passes through them.

4Ω & $20V$ & 5Ω are in series $\Rightarrow$ the *same current* I_4 passes through them.

$6V$ & 2Ω are *not* in series $\Rightarrow$ *different currents* I_1 & I_3 pass through them.

2Ω & $11V$ are *not* in series. $\Rightarrow$ *different currents* I_3 & I_5 pass through them.

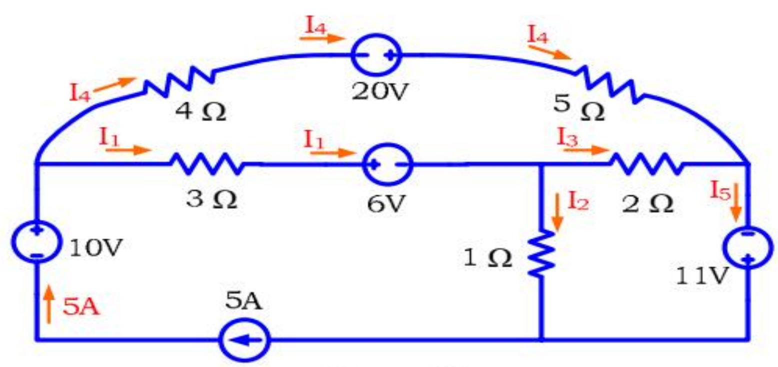


Figure 13

The meaning of the parallel connection

$3A$ & 4Ω are in parallel

6Ω & 8Ω are in parallel

$2V$ & 8Ω *are not* in parallel

Why?

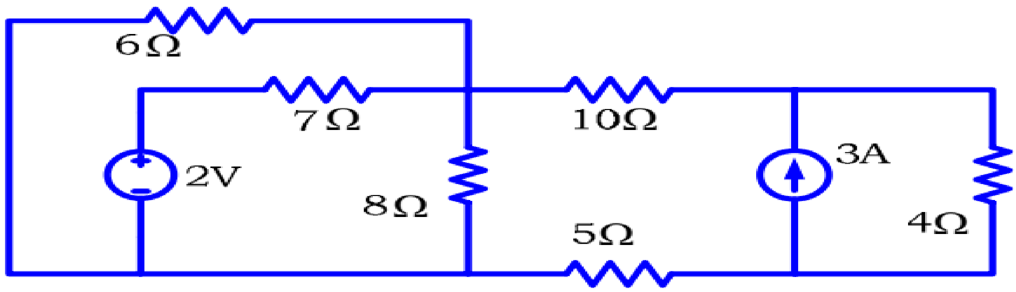


Figure 14

The *same voltage* appears across $3A$ & 4Ω $\Rightarrow$ they are in parallel

The *same voltage* appears across 6Ω & 8Ω $\Rightarrow$ they are in parallel

Different voltages appear across $2V$ & 8Ω $\Rightarrow$ they are *not* in parallel

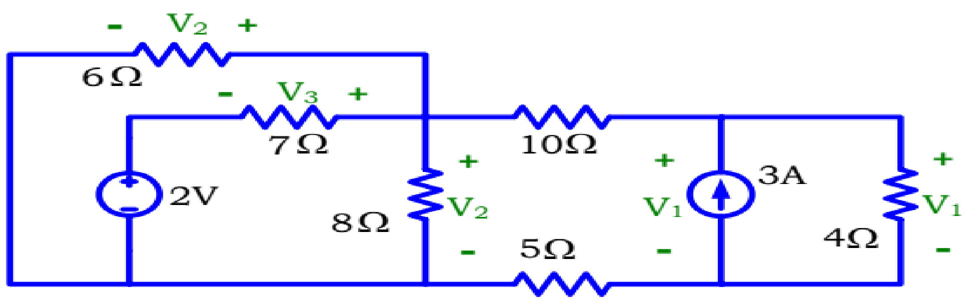


Figure 15

Example 3:

Calculate:

a) the power absorbed by the 3Ω resistor

b) the *equivalent resistance* seen by the $10V$ voltage source

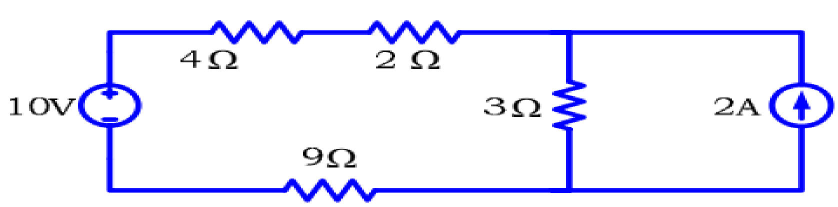


Figure 16

Solution:

a)

2Ω & 4Ω & 9Ω are in series $\Rightarrow$ $2+4+9=15\Omega$

Define v_1 & v_2 & i_1 & i_2 (*arbitrary* choice of *voltage polarity* and *current direction*)

KVL $\Rightarrow$ $-10+v_1+v_2=0$

Ohm's Law $\Rightarrow$ $-10+15i_1+3i_2=0$ (1)

KCL $\Rightarrow$ $i_1+3=i_2$ (2)

Solving (1) & (2) $\Rightarrow$ $-10+15(i_2-3)+3i_2=0$ $\Rightarrow$ $18i_2=55$ $\Rightarrow$
$i_2=\frac{55}{18}=3.056A$

$\therefore p_{3\Omega}=3i_2^2=3(3.056)^2=28.02W$

b)

Using (3) $\Rightarrow$ $i_1=i_2-3=3.056-3=0.056A$

$\therefore R_{eq}=+\frac{v}{i_1}=+\frac{10}{0.056}=178.57\Omega$