Lecture 09: Thevenin Equivalent Circuits

Thevenin Equivalent Circuit:

Given an electrical circuit ⇒ split it into circuits A & B

Call circuit B "*the load*"

Notice that circuits A & B are connected by the *two* terminals *a* & *b*

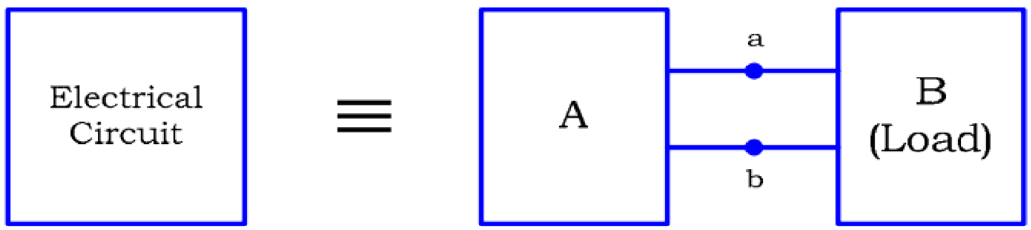

Figure 1

Thevenin theorem:

In general, it is possible to replace circuit A with a voltage source in *series* with a resistor.

The voltage source is labeled V_{th} (Thevenin voltage)

The resistance is labeled R_{th} (Thevenin resistance)

The new circuit is called the *Thevenin Equivalent Circuit* (TEC) of circuit A

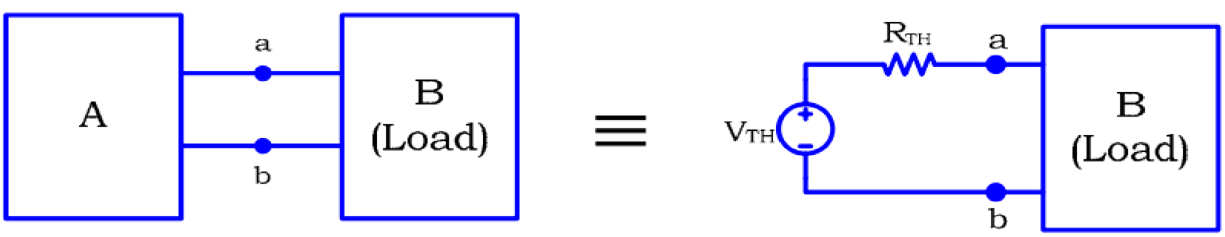

Figure 2

We will consider *four* methods for finding the TEC.

Only two methods will be presented in this class.

Finding the TEC (Method 1):

Lecture 09: Thevenin Equivalent Circuits

Calculate 1) $V_{th} = V_{oc}$ & 2) $R_{th} = \frac{V_{oc}}{i_{sc}}$

1) Calculate V_{th}:

First we remove the load (circuit B)

The terminals a & b become *open-circuited* (no load)

The resulting voltage across the terminals a & b is labeled V_{oc} (Open-circuit voltage)

Current through the *open circuit* is *zero* $\Rightarrow$ no current flows through R_{th}

KVL $\Rightarrow$ $-V_{th} + 0R_{th} + V_{oc} = 0$ $\Rightarrow$ $V_{th} = V_{oc}$

We calculate the *open circuit voltage* of circuit A and equate it to V_{th}.

Figure 3

2) Calculate R_{th} using $R_{th} = \frac{V_{oc}}{i_{sc}}$

Remove the load and place a *short circuit* across "$a - b$"

The current that flows in the short circuit is labeled i_{sc}

The voltage across the *short* circuit is *zero*

KVL $\Rightarrow$ $-V_{th} + i_{sc}R_{th} + 0 = 0$ $\Rightarrow$ $V_{th} = i_{sc}R_{th}$

$\therefore R_{th} = \frac{V_{th}}{i_{sc}}$ [this can be used to calculate R_{th}]

Figure 4

Summary:

Lecture 09: Thevenin Equivalent Circuits

1) Calculate $V_{oc} = V_{th}$

2) Calculate i_{sc} $\Rightarrow$ use $R_{th} = \dfrac{V_{oc}}{i_{sc}} = \dfrac{V_{th}}{i_{sc}}$

Example 1:

Calculate the TEC *seen* by the 3Ω resistor.

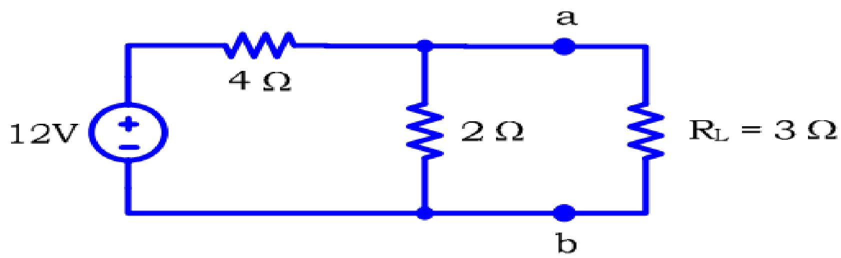

Figure 5

Solution:

Remove the 3Ω load

The voltage V_{oc} is also across the 2Ω resistor

VDR $\Rightarrow$ $V_{oc} = \dfrac{2}{2+4}(12) = 4V$

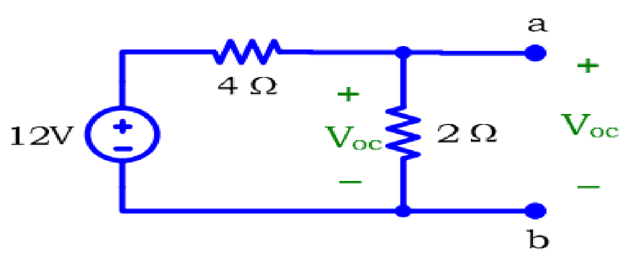

Figure 6

Place a short circuit

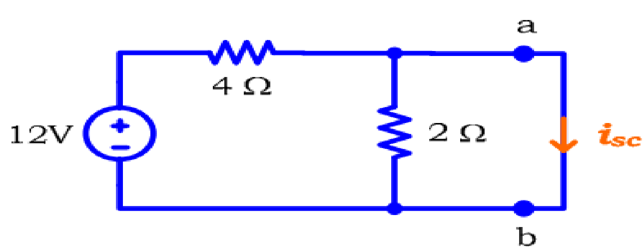

Figure 7

$$2\Omega \| short\ circuit \quad \Rightarrow \quad R_{eq} = \frac{0 \times 2}{0+2} = 0\Omega \ \text{(short circuit)}$$

[A short circuit in parallel with *any resistance* is equivalent to a short circuit]

$$\therefore i_{sc} = \frac{12}{4} = 3A$$

$$\therefore R_{eq} = \frac{V_{oc}}{i_{sc}} = \frac{4}{3} = 1.333\Omega$$

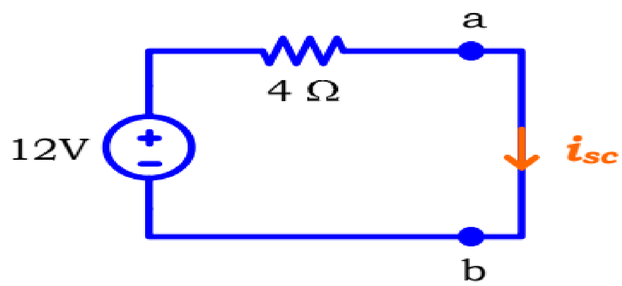

Figure 8

$\therefore$ the TEC seen by the 3Ω resistance is:

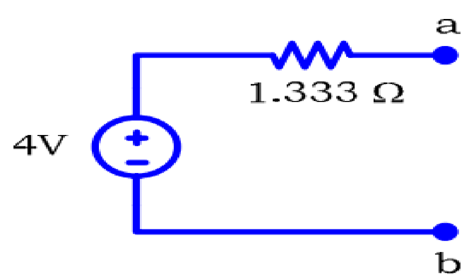

Figure 9

Example 2:

Calculate the TEC to the left of "$a-b$" [load has already been removed].

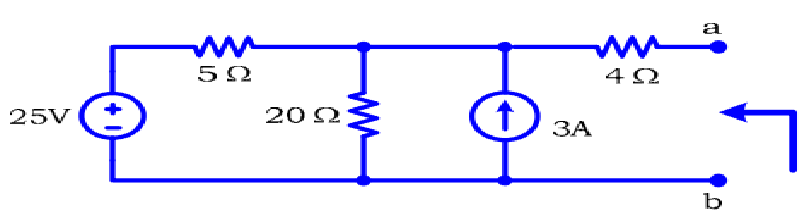

Figure 10

The current through the 4Ω resistor is zero, because of the open circuit.

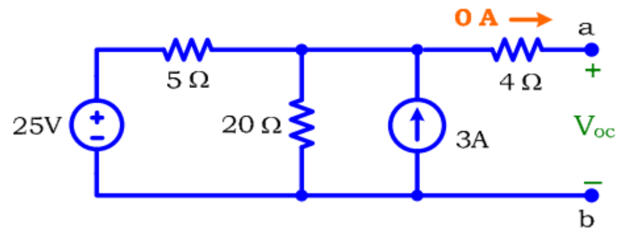

Figure 11

Voltage drop across 4Ω is zero (why?)

KVL $\Rightarrow$ Voltage across the $3A$ & 20Ω is V_{oc}

We can calculate V_{oc} by any method we choose, let us use KVL, KCL & Ohm's law.

[Also, the mesh analysis is *efficient* in this case, because we have only *one actual* unknown. Why?].

The $3A$ current completely goes to the left (why?)

Assume current i through 5Ω

KCL $\Rightarrow$ current through 20Ω is $(i+3)$

KVL $\Rightarrow$ $-25+5i+20(i+3)=0 \Rightarrow i=-\frac{7}{5}A$

Ohm's law $\Rightarrow$ $V_{oc}=20(i+3)=20(-\frac{7}{5}+3)=32V$

$\therefore V_{th}=V_{oc}=32V$

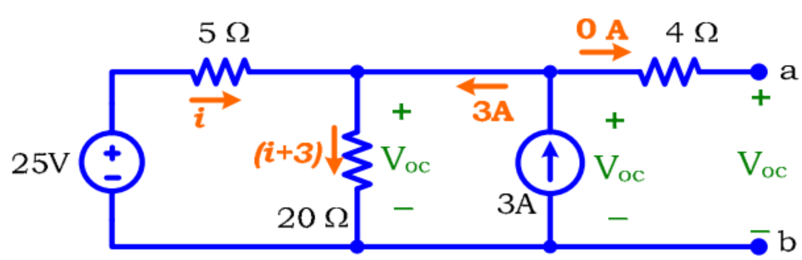

Figure 12

Place a short circuit across "$a-b$"

We can calculate i_{sc} by any method we choose

Let us use the NA

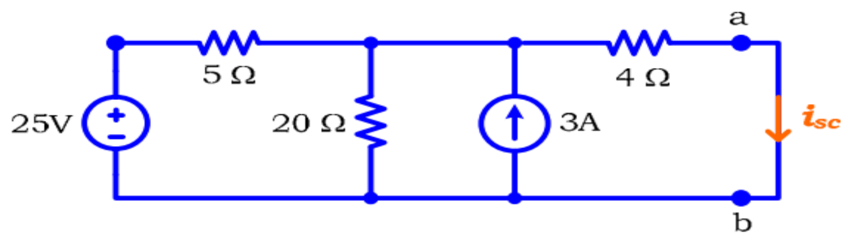

Figure 13

Using the reference node as shown $\Rightarrow$ only V_2 is unknown.

KCL at node 2 $\Rightarrow$ $\frac{V_2 - 25}{5} + \frac{V_2}{20} - 3 + \frac{V_2 - 0}{4} = 0$ $\Rightarrow$ $V_2 = 16V$

$\therefore i_{sc} = \frac{V_2 - 0}{4} = \frac{16}{4} = 4A$

$\therefore R_{eq} = \frac{V_{oc}}{i_{sc}} = \frac{32}{4} = 8\Omega$

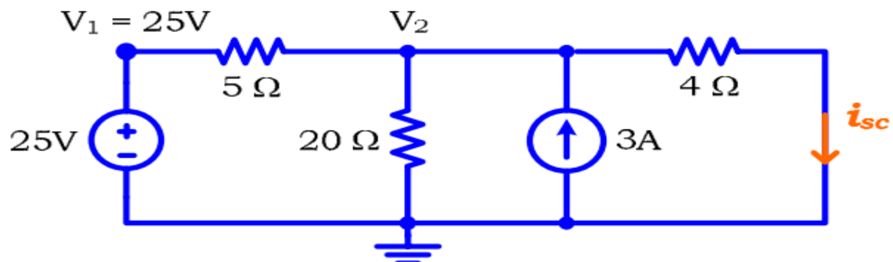

Figure 14

The resulting TEC is:

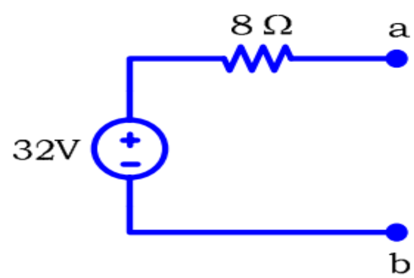

Figure 15

Finding the TEC (Method 2):

We can also use ST to find the TEC. This is the second method.

Example 3:

Repeat the previous example using ST to find the TEC.

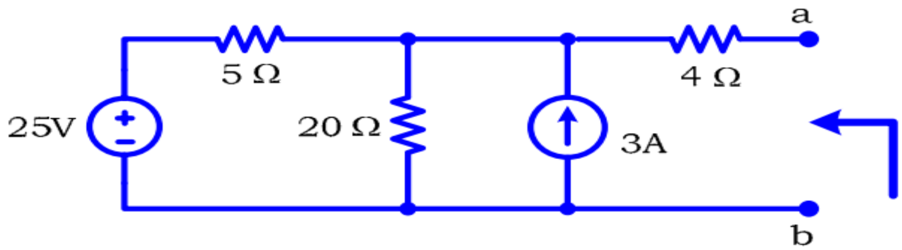

Figure 10

Solution:

ST $\Rightarrow$ $I = \frac{25}{5} = 5A$

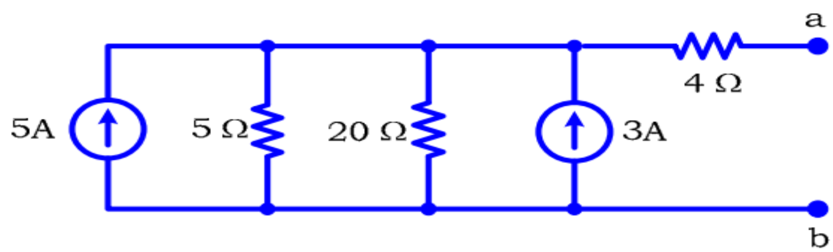

Figure 16

$3A \| 5A$ $\Rightarrow$ $8A$

$5\Omega \| 25\Omega$ $\Rightarrow$ $\frac{5 \times 20}{5+20} = \frac{100}{25} = 4\Omega$

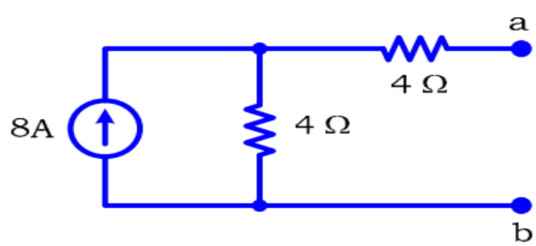

Figure 17

ST $\Rightarrow$ $V = 8 \times 4 = 32V$

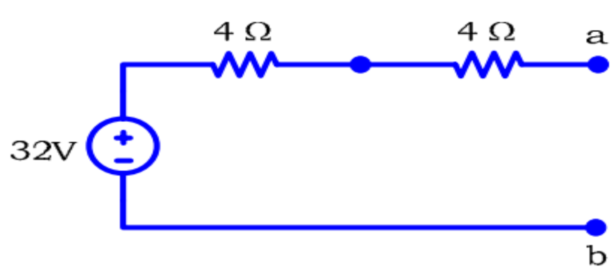

Figure 18

$4+4=8\Omega$

The circuit is reduced to a voltage source in series with a resistor.

$\therefore V_{th}=32V$ & $R_{th}=8\Omega$

Same answer as before.

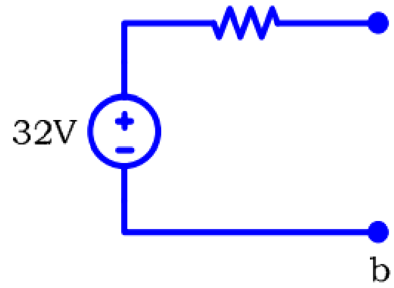

Figure 19

Using ST, we are able to find V_{th} & R_{th} *simultaneously*.

Methods 1 & 2 are *not always* applicable. They have certain limitations.

In the coming classes, we will present the *other two* methods for finding the TEC and also discuss the *limitations* and *advantages* of the *four* methods.