

Lecture 11: Nodal Analysis – Introduction

Definition of Essential Nodes

The essential nodes of the circuit are labeled “0”, “1”, “2”, “3”, “4”

All points that are connected by a short circuit belong to the same essential node.

All points in the lower part of the circuit are connected by a *short circuit*, they all belong to node “0”

The same applies to nodes “1”, “2”, “3”, “4”

For this circuit \Rightarrow 5 essential nodes

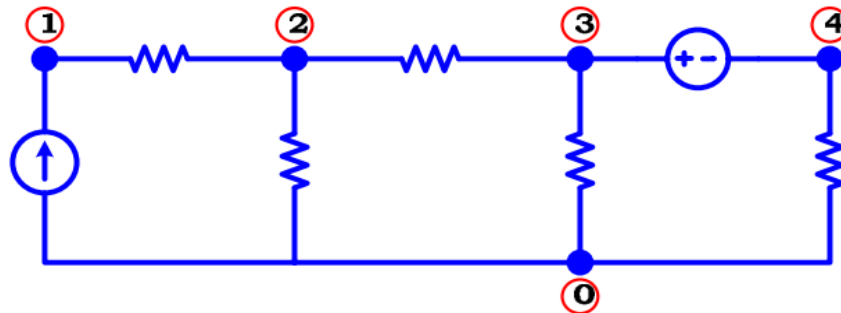


Figure 1

Example 1:

Label the essential nodes starting from “0” onwards.

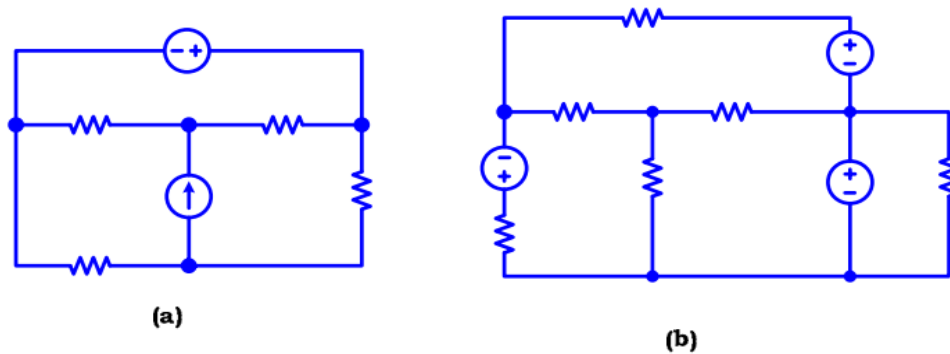


Figure 2

Solution:

- a) 4 essential nodes.
- b) 6 essential nodes.

We can label the essential nodes as we like.

Lecture 11: Nodal Analysis - Introduction

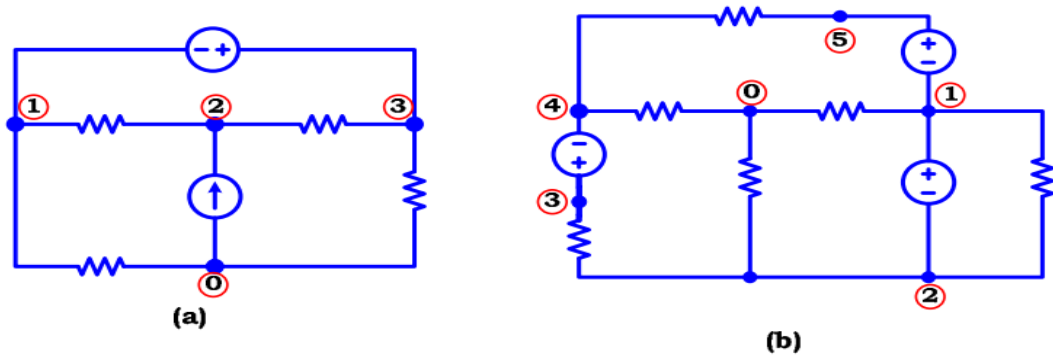


Figure 3

The Reference Node

The node labeled “0” is called a *reference node*.

We will see later that the *reference node* always has *zero Nodal Voltage*.

Other possible *labels* for the reference node are shown in the figure



Figure 4

Nodal Voltages

We associate a voltage with *every essential node*. These voltages are called Nodal Voltages.

V_0, V_1, V_2, V_3 \Leftrightarrow Nodal Voltages of essential nodes 0, 1, 2, 3

The nodal voltage V_i is the voltage drop *from* node “ i ” *to* the reference node “0”

\Downarrow

$$V_i \equiv V_{i,0}$$

\Downarrow

$$V_0 \equiv V_{0,0}$$

$$V_1 \equiv V_{1,0}$$

$$V_2 \equiv V_{2,0}$$

$$V_3 \equiv V_{3,0}$$

Lecture 11: Nodal Analysis - Introduction

Reference node $\Rightarrow V_0 \equiv V_{0,0} = 0 \Rightarrow$ Reference node *always* has zero nodal voltage.

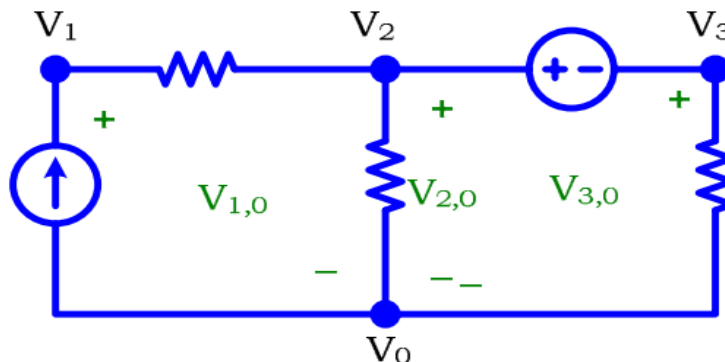


Figure 5

Relationship between Nodal Voltages and Voltages across Elements

Nodal voltages (NV) $\Rightarrow V_1, V_2, V_3$

The reference *nodal voltage* $V_0 = 0$ is not shown (it is always not shown)

Another label is used to mark the reference node in this case.

Voltages across elements (VAE) $\Rightarrow V_a, V_b, V_c, V_d, V_e$

KVL

\Downarrow

$$-V_a + V_{10} = 0 \Rightarrow -V_a + V_1 = 0 \Rightarrow V_a = V_1 \Rightarrow V_a = V_1 - V_0$$

$$-V_1 + V_b + V_2 = 0 \Rightarrow V_b = V_1 - V_2$$

$$V_c = V_2 \Rightarrow V_c = V_2 - V_0$$

$$-V_2 + V_d + V_3 = 0 \Rightarrow V_d = V_2 - V_3$$

$$V_e = V_3 \Rightarrow V_e = V_3 - V_0$$

Lecture 11: Nodal Analysis - Introduction

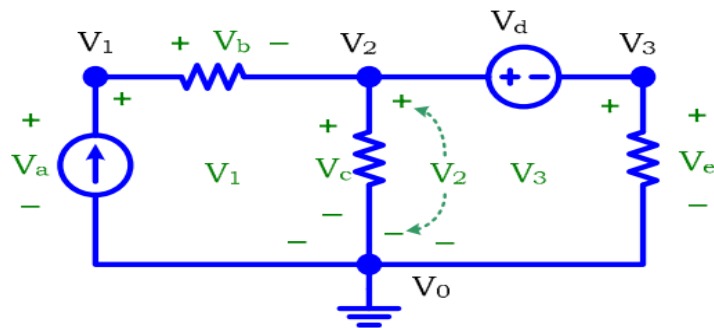


Figure 6

V_x = voltage across element

V_i = Nodal voltage on the (+) side of V_x

V_j = Nodal voltage on the (-) side of V_x

⇓

$$V_x = V_i - V_j$$

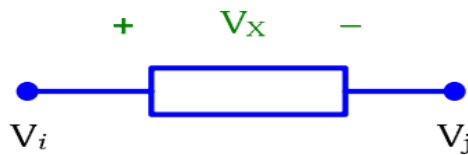


Figure 7

Example 2:

Express the VAE V_a, V_b, V_c, V_d, V_e in terms of the NV V_1, V_2, V_3 .

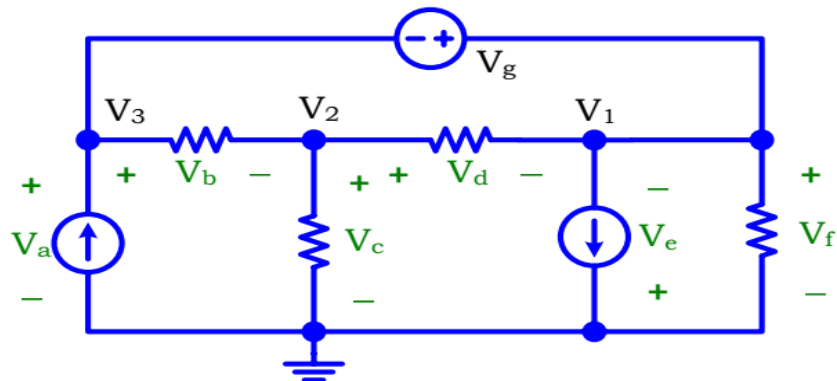


Figure 8

Solution:

Lecture 11: Nodal Analysis - Introduction

$$V_a = V_3 - V_0 = V_3 - 0 = V_3$$

$$V_b = V_3 - V_2$$

$$V_c = V_2 - 0 = V_2$$

$$V_d = V_2 - V_1$$

$$V_e = 0 - V_1 = -V_1 \quad (\text{why?})$$

$$V_f = V_1 - 0 = V_1$$

$$V_g = V_1 - V_3$$

if we know *all* NV \Rightarrow we know *all* VAE

Nodal Analysis Procedure (without voltage sources)

If the circuit contains *no voltage* sources, the nodal analysis is very easy to apply.

The Nodal analysis *procedure* is best illustrated by an example.

Example 3:

Calculate the nodal voltages V_1 & V_2

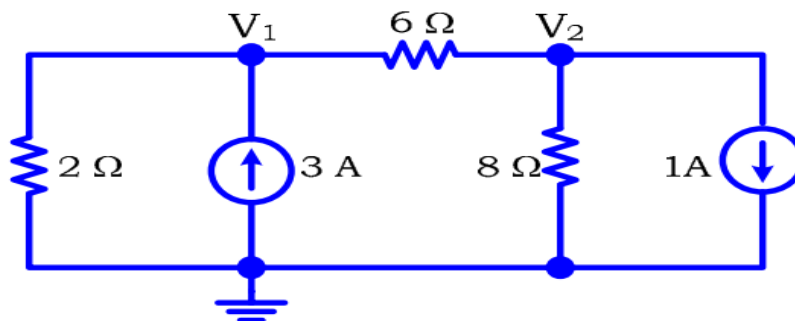


Figure 9

Solution:

The procedure is started with KCL.

Current leaving a node we use (+) & Current entering a node we use (-).

$$\text{Step 1: KCL at node 1} \quad \Rightarrow \quad i_a - 3 + i_b = 0$$

Lecture 11: Nodal Analysis - Introduction

Step 2: Ohm's Law $\Rightarrow \frac{V_a}{2} - 3 + \frac{V_b}{6} = 0$

Step 3: Replace V_{AE} with NV $\Rightarrow \frac{V_1 - 0}{2} - 3 + \frac{V_1 - V_2}{6} = 0$

Step 4: Simplify $\Rightarrow 3V_1 - 18 + V_1 - V_2 = 0 \Rightarrow 4V_1 - V_2 = 18 \quad (1)$

Repeat the same procedure for the remaining nodes:

KCL at node 2 $\Rightarrow -i_b + i_c + 1 = 0$

Ohm's Law $\Rightarrow -\frac{V_b}{6} + \frac{V_c}{8} + 1 = 0$

Replace VAE with NV $\Rightarrow -\frac{V_1 - V_2}{6} + \frac{V_2 - 0}{8} + 1 = 0$

Simplify $-4V_1 + 4V_2 + 3V_2 + 24 = 0 \Rightarrow -4V_1 + 7V_2 = -24 \quad (2)$

Solve the resulting equations simultaneously:

$(1) + (2) \Rightarrow 6V_2 = -6 \Rightarrow V_2 = -1V$

Substitute in (1) $\Rightarrow 4V_1 - (-1) = 18 \Rightarrow V_1 = \frac{17}{4}V$

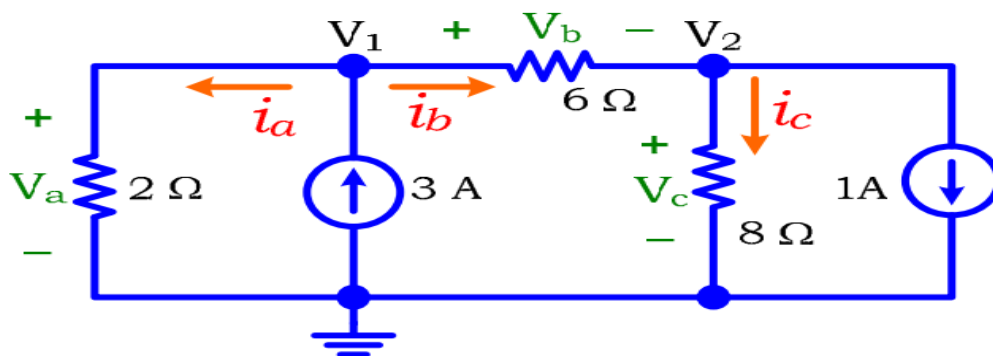


Figure 10

Nodal analysis \Rightarrow step 1) KCL \Rightarrow step 2) Ohm's Law \Rightarrow step 3) KVL

Nodal analysis \Rightarrow COV

Example 3:

Lecture 11: Nodal Analysis - Introduction

Derive the nodal equations (do not simplify and do not solve)

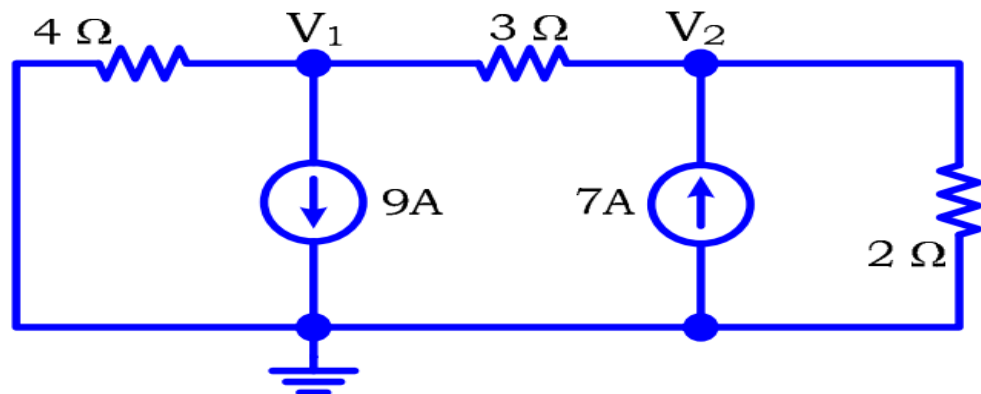


Figure 11

Solution:

This time, we will combine steps 2 & 3 into *a single step*.

The voltages across resistances will not be shown explicitly.

Node 1:

$$\text{KCL} \quad \Rightarrow \quad i_a + 9 + i_b = 0$$

$$\text{Ohm's Law then KVL} \quad \Rightarrow \quad \frac{V_1 - 0}{4} + 9 + \frac{V_1 - V_2}{3} = 0 \quad \Rightarrow \quad \text{eqn. (1)}$$

Node 2:

$$\text{KCL} \quad \Rightarrow \quad -i_b - 7 + i_c = 0$$

$$\text{Ohm's Law then KVL} \quad \Rightarrow \quad -\frac{V_1 - V_2}{3} - 7 + \frac{V_2 - 0}{2} = 0 \quad \Rightarrow \quad \text{eqn. (2)}$$

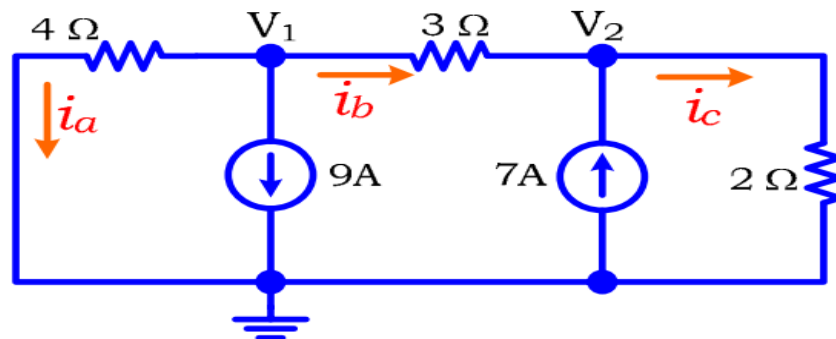


Figure 12

Lecture 11: Nodal Analysis – Introduction

Example 4:

Repeat the previous example by combining steps 1, 2, 3 into a single step.

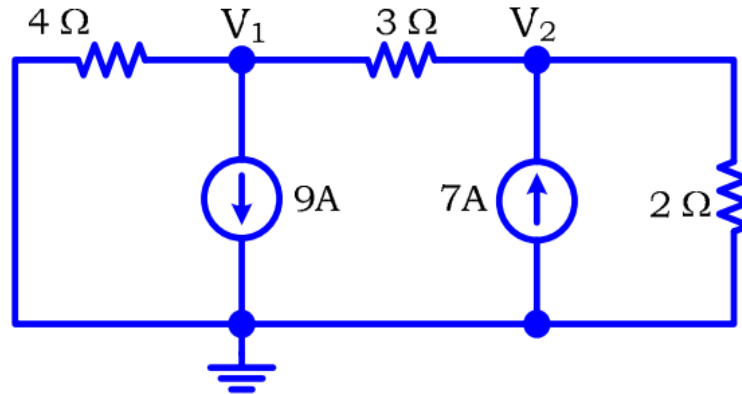


Figure 13

Solution:

This time we will not show currents through resistances or voltages across resistances.

Important: we will *imagine* currents through *resistors* to be **leaving** the node under consideration.

Node 1:

$$\frac{V_1 - 0}{4} + 9 + \frac{V_1 - V_2}{3} = 0 \quad \Rightarrow \quad \text{eqn. (1)}$$

Node 2:

$$\frac{V_2 - V_1}{3} - 7 + \frac{V_2 - 0}{2} = 0 \quad \Rightarrow \quad \text{eqn. (2)}$$

We obtained the same equations directly.

Example 5:

Write the nodal equations directly (do not simplify or solve)

Lecture 11: Nodal Analysis - Introduction

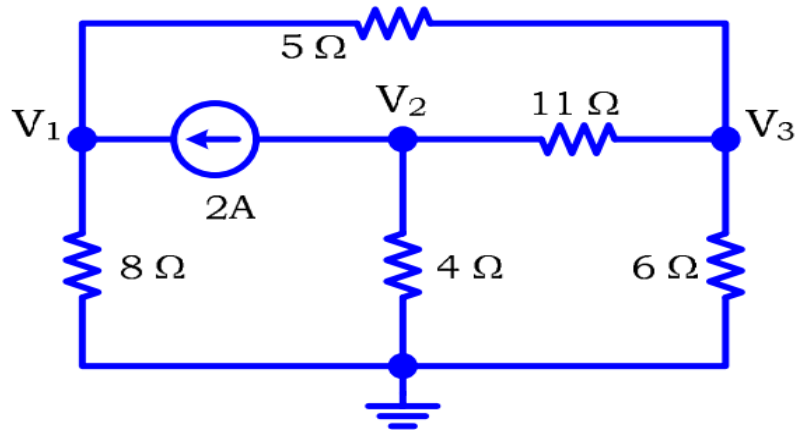


Figure 14

Solution:

Again imagine currents through resistors to be *leaving* the node under consideration.

$$\text{Node 1} \Rightarrow \frac{V_1}{8} - 2 + \frac{V_1 - V_3}{5} = 0$$

$$\text{Node 2} \Rightarrow 2 + \frac{V_2}{4} + \frac{V_2 - V_3}{11} = 0$$

$$\text{Node 3} \Rightarrow \frac{V_3 - V_2}{11} + \frac{V_3 - V_1}{5} + \frac{V_3}{6} = 0$$

The last equation has *no constant* term, because *no current sources* are connected to node 3.

Example 6:

a) Calculate the nodal voltages V_1 , V_2 and V_3

b) Use the above results to calculate i_1 & i_2

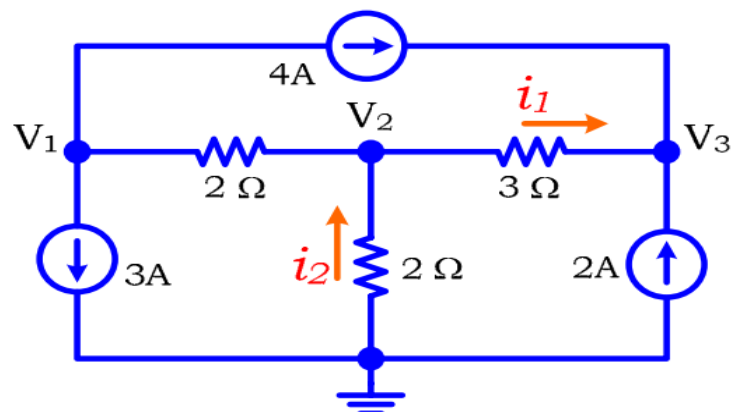


Figure 15

Lecture 11: Nodal Analysis - Introduction

Solution:

a)

We will *ignore* the currents i_2 & i_3 and again *imagine* currents through resistors to be *leaving* the node *under consideration*.

After all if i_1 and i_2 were *not shown* in the circuit, we should still obtain the *same* nodal voltages.

$$\text{Node 1} \Rightarrow 3 + 4 + \frac{V_1 - V_2}{2} = 0 \quad \Rightarrow \quad V_1 - V_2 = -14 \quad (1)$$

$$\text{Node 2} \Rightarrow \frac{V_2 - V_1}{1} + \frac{V_2 - 0}{2} + \frac{V_2 - V_3}{3} = 0 \quad \Rightarrow \quad -6V_1 + 11V_2 - 2V_3 = 0 \quad (2)$$

$$\text{Node 3} \Rightarrow \frac{V_3 - V_2}{3} - 4 - 2 = 0 \quad \Rightarrow \quad -V_2 + V_3 = 18 \quad (3)$$

Solving (1), (2), (3) simultaneously:

$$V_1 = -30V \quad \& \quad V_2 = -16V \quad \& \quad V_3 = 2V$$

[After you solve the equations, you *should always* check that the results satisfy those equations]

b)

$$i_1 = \frac{V_2 - V_3}{3} = \frac{-16 - 2}{3} = -6A$$

$$i_2 = \frac{0 - V_2}{2} = \frac{0 - (-16)}{2} = 8A$$

From the examples done in this class it easy to conclude the following:

$$1) N_u = N_{ess} - 1$$

N_u = Number of unknown nodal voltages

N_{ess} = Number of essential nodes

$$2) N_u \leq N_{ele}$$

N_{ele} = Number of unknown voltages across elements

Thus the nodal analysis is efficient because the number of unknown voltages is *reduced*.

3) The procedure remains the *same* if no voltage sources are present in the circuit. We will deal with this case in the next class.