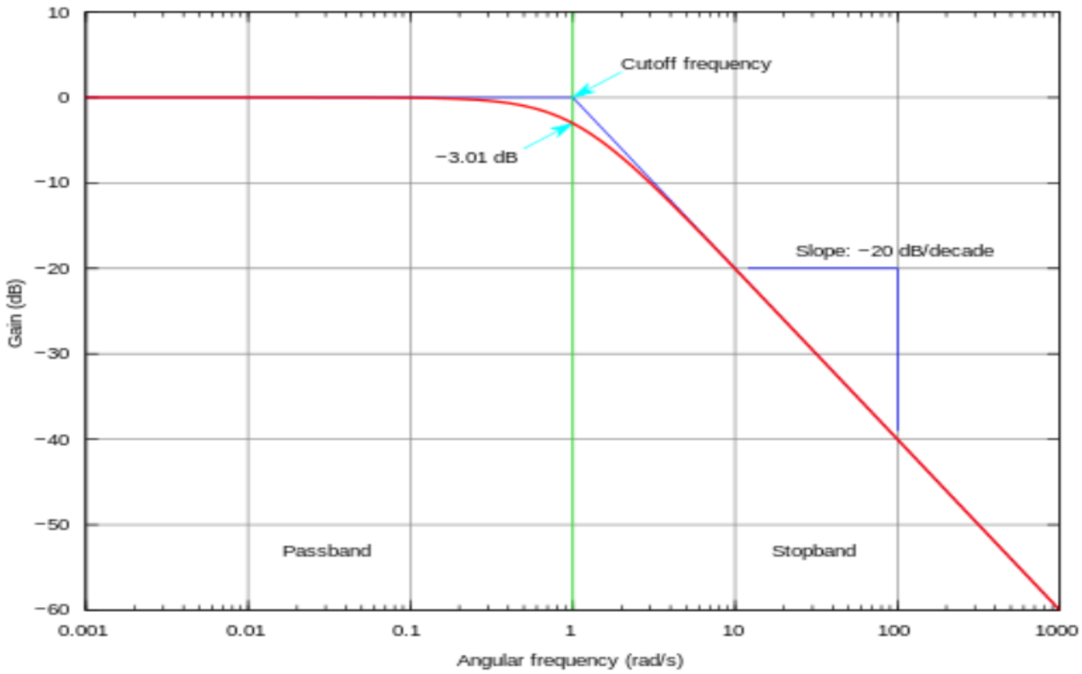
FREQUENCY RESPONSE ANALYSIS

Frequency response is the quantitative measure of the output spectrum of a system or device in response to a stimulus, and is used to characterize the dynamics of the system. It is a measure of magnitude and phase of the output as a function of frequency, in comparison to the input. In simplest terms, if a sine wave is injected into a system at a given frequency, a linear system will respond at that same frequency with a certain magnitude and a certain phase angle relative to the input. Also for a linear system, doubling the amplitude of the input will double the amplitude of the output. In addition, if the system is time-invariant (so LTI), then the frequency response also will not vary with time. Thus for LTI systems, the frequency response can be seen as applying the system's transfer function to a purely imaginary number argument representing the frequency of the sinusoidal excitation.[1]

Two applications of frequency response analysis are related but have different objectives. For an audio system, the objective may be to reproduce the input signal with no distortion. That would require a uniform (flat) magnitude of response up to the bandwidth limitation of the system, with the signal delayed by precisely the same amount of time at all frequencies. That amount of time could be seconds, or weeks or months in the case of recorded media. In contrast, for a feedback apparatus used to control a dynamic system, the objective is to give the closed-loop system improved response as compared to the uncompensated system. The feedback generally needs to respond to system dynamics within a very small number of cycles of oscillation (usually less than one full cycle), and with a definite phase angle relative to the commanded control input. For feedback of sufficient amplification, getting the phase angle wrong can lead to instability for an open-loop stable system, or failure to stabilize a system that is open-loop unstable. Digital filters may be used for both audio systems and feedback control systems, but since the objectives are different, generally the phase characteristics of the filters will be significantly different for the two applications.



Nonlinear frequency response

If the system under investigation is nonlinear then applying purely linear frequency domain analysis will not reveal all the nonlinear characteristics. To overcome these limitations, generalized frequency response functions and nonlinear output frequency response functions have been defined that allow the user to analyze complex nonlinear dynamic effects.[2] The nonlinear frequency response methods reveal complex resonance, inter modulation, and energy transfer effects that cannot be seen using a purely linear analysis and are becoming increasingly important in a nonlinear world.

TIME DOMAIN AND FREQUENCY DOMAIN:

The frequency domainrefers to the analysis of mathematical functions or signals with respect to frequency, rather than time.[1] Put simply, a time-domain graph shows how a signal changes over time, whereas a frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies. A frequency-domain representation can also include information on the phase shift that must be applied to each sinusoid in order to be able to recombine the frequency components to recover the original time signal. the frequency domainrefers to the analysis of mathematical functions or signals with respect to frequency, rather than time.[1] Put simply, a time-domain graph shows how a signal changes over time, whereas a frequency-domain graph shows how much of the signal lies within each given frequency band over a range of frequencies. A frequency-domain representation can also include information on the phase shift that must be applied to each sinusoid in order to be able to recombine the frequency components to recover the original time signal. Time domain and frequency domain are two modes used to analyze data. Both time domain analysis and frequency domain analysis are widely used in fields such as electronics, acoustics, telecommunications, and many other fields.

- Frequency domain analysis is used in conditions where processes such as filtering, amplifying and mixing are required.
- Time domain analysis gives the behavior of the signal over time. This allows predictions and regression models for the signal.
- Frequency domain analysis is very useful in creating desired wave patterns such as binary bit patterns of a computer.
- Time domain analysis is used to understand data sent in such bit patterns over time.

Time Domain:

Time domain analysis is analyzing the data over a time period. Functions such as electronic signals, market behaviors, and biological systems are some of the functions that are analyzed using time domain analysis. For an electronic signal, the time domain analysis is mainly based on the voltage – time plot or the current – time plot. In a time domain analysis, the variable is always measured against time. There are several devices used to analyze data on a time domain basis. The cathode ray oscilloscope (CRO) is the most common device when analyzing electrical signals on a time domain.

Frequency Domain:

Frequency domain is a method used to analyze data. This refers to analyzing a mathematical function or a signal with respect to the frequency. Frequency domain analysis is widely used in fields such as control systems engineering, electronics and statistics. Frequency domain analysis is mostly used to signals or functions that are periodic over time. This does not mean that frequency domain analysis cannot be used in signals that are not periodic.

The most important concept in the frequency domain analysis is the transformation. Transformation is used to convert a time domain function to a frequency domain function and vice versa. The most common transformation used in the frequency domain is the Fourier transformations. Fourier transformation is used to convert a signal of any shape into a sum of infinite number of sinusoidal waves. Since analyzing sinusoidal functions is easier than analyzing general shaped functions, this method is very useful and widely used.

All signals have a frequency domain representation and Fourier detailed the theory that any real world waveform can be generated by the addition of sinusoidal waves. The following diagram shows an example of this process:

There are a number of different mathematical transforms which are used to analyze time functions and are referred to as frequency domain methods. The following are some most common transforms, and the fields in which they are used:

- Fourier series – repetitive signals, oscillating systems
- Fourier transform – nonrepetitive signals, transients
- Laplace transform – electronic circuits and control systems
- Z transform – discrete signals, digital signal processing .

GAIN AND PHASE MARGINS:

The gain margin is the amount of gain increase or decrease required to make the loop gain unity at the frequency Wgm where the phase angle is –**180°** (modulo 360°). In other words, the gain margin is 1/g if g is the gain at the –**180°** phase frequency. n electronic amplifiers, the **phase margin** (PM) is the difference between the phase and 180°, for an amplifier's output signal (relative to its input), at a certain frequency.

$$\mathrm{PM} = 180^\circ - |\Delta\phi|.$$

Typically the open-loop phase lag (relative to input) varies with frequency, progressively increasing to exceed 180°, at which frequency the output signal becomes inverted, or antiphase in relation to the input. The PM will be positive but decreasing at frequencies less than the frequency at which inversion sets in (at which PM = 0), and PM is negative (PM < 0) at higher frequencies. In the presence of negative feedback, a zero or negative PM at a frequency where the loop gain exceeds unity (1) guarantees instability. Thus positive PM is a "safety margin" that ensures proper (non-oscillatory) operation of the circuit. This applies to amplifier circuits as well as more generally, to active filters, under various load conditions (e.g. reactive loads). In its simplest form, involving ideal negative feedback *voltage* amplifiers with non-reactive feedback, the phase margin is measured at the frequency where the open-loop voltage gain of the amplifier equals the desired closed-loop DC voltage gain.[1]

More generally, PM is defined as that of the amplifier and its feedback network combined (the "loop", normally opened at the amplifier input), measured at a frequency where the loop gain is unity, and prior to the closing of the loop, through tying the output of the open loop to the input source, in such a way as to subtract from it.

In the above loop-gain definition, it is assumed that the amplifier input presents zero load. To make this work for non-zero-load input, the output of the feedback network needs to be loaded with an equivalent load for the purpose of determining the frequency response of the loop gain.

It is also assumed that the graph of gain vs. frequency crosses unity gain with a negative slope and does so only once. This consideration matters only with reactive and active feedback networks, as may be the case with active filters.

Phase margin and its important companion concept, gain margin, are measures of stability in closed-loop, dynamic-control systems. Phase margin indicates relative stability, the tendency to oscillate during its damped response to an input change such as a step function. Gain margin indicates absolute stability and the degree to which the system will oscillate, without limit, given any disturbance.

The output signals of all amplifiers exhibit a time delay when compared to their input signals. This delay causes a phase difference between the amplifier's input and output signals. If there are enough stages in the amplifier, at some frequency, the output signal will lag behind the input signal by one cycle period at that frequency. In this situation, the amplifier's output signal will be in phase with its input signal though lagging behind it by 360°, i.e., the output will have a phase angle of −360°. This lag is of great consequence in amplifiers that use feedback. The reason: the amplifier will oscillate if the fed-back output signal is in phase with the input signal at the frequency at which its open-loop voltage gain equals its closed-loop voltage gain and the open-loop voltage gain is one or greater. The oscillation will occur because the fed-back output signal will then reinforce the input signal at that frequency.[2] In conventional operational amplifiers, the critical output phase angle is −180° because the output is fed back to the input through an inverting input which adds an additional −180°.

In practice, feedback amplifiers must be designed with phase margins substantially in excess of 0°, even though amplifiers with phase margins of, say, 1° are theoretically stable. The reason is that many practical factors can reduce the phase margin below the theoretical minimum. A prime example is when the amplifier's output is connected to a capacitive load. Therefore, operational amplifiers are usuallycompensated to achieve a minimum phase margin of 45° or so. This means that at the frequency at which the open and closed loop gains meet, the phase angle is −135°. The calculation is: -135° - (-180°) = 45°. See Warwick[3] or Stout[4] for a detailed analysis of the techniques and results of compensation to insure adequate phase margins. See also the article "Pole splitting". Often amplifiers are designed to achieve a typical phase margin of 60 degrees. If the typical phase margin is around 60 degrees then the minimum phase margin will typically be greater than 45 degrees. A phase margin of 60 degrees is also a magic number because it allows for the fastest settling time when attempting to follow a voltage step input (a Butterworth design). An amplifier with lower phase margin will ring[nb 1] for longer and an amplifier with more phase margin will take a longer time to rise to the voltage step's final level.

A related measure is gain margin. While phase margin comes from the phase where the loop gain equals one, the gain margin is based upon the gain where the phase equals -180 degrees.

BODE PLOTS:

Bode plot is a graph of the frequency response of a system. It is usually a combination of a Bode magnitude plot, expressing the magnitude (usually in decibels) of the frequency response, and a Bode phase plot, expressing the phase shift. Both quantities are plotted against a horizontal axis proportional to thelogarithm of frequency. Given that the decibel is itself a logarithmic scale, the Bode amplitude plot is log–log plot, whereas the Bode phase plot is a lin-logplot.[1]

As originally conceived by Bode in the 1930s, the plot is only an asymptoticapproximation of the frequency response, using straight line segments.[2]However, with the advent of low cost computing, it is often taken nowadays to mean the precise plot of the actual frequency response.