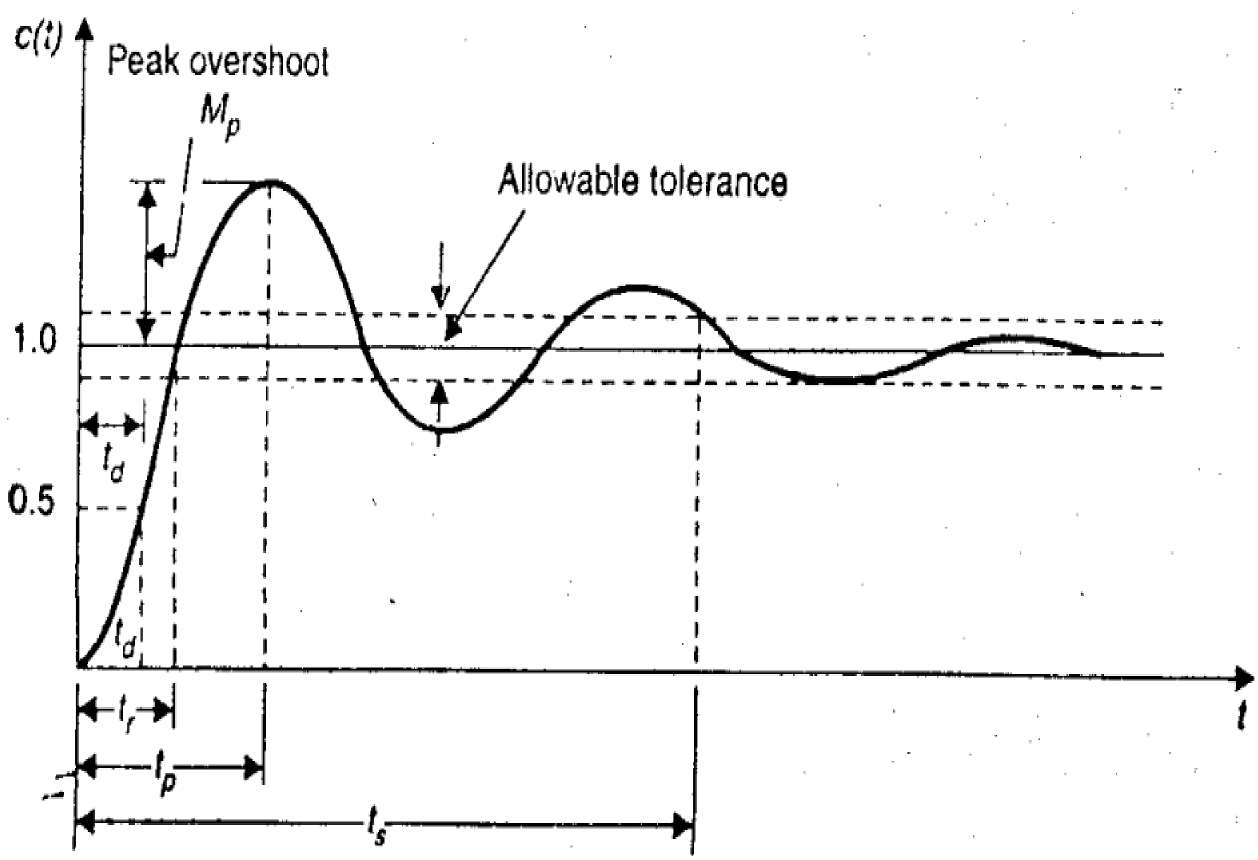
TIME DOMAIN SPECIALIZATION

Control systems are generally designed with damping less than one, i.e., oscillatory step response. Higher order control systems usually have a pair of complex conjugate poles with damping less than unity that dominate over the other poles. Therefore the time response of second- and higher-order control systems to a step input is generally of damped oscillatory nature as shown in Figure next (next page).

In specifying the transient-response characteristics of a control system to a unit step input, we usually specify the following:

1. Delay time,
2. Rise time,
3. Peak time,
4. Peak overshoot,
5. Settling time,
6. Steady-state error,



1. ***Delay time, :*** It is the time required for the response to reach 50% of the final value in first attempt.
2. ***Rise time, :*** It is the time required for the response to rise from 0 to 100% of the final value for the underdamped system.
3. ***Peak time, :*** It is the time required for the response to reach the peak of time response or the peak overshoot.
4. ***Settling time, :*** It is the time required for the response to reach and stay within a specified tolerance band (2% or 5%) of its final value.
5. ***Peak overshoot, :*** It is the normalized difference between the time response peak and the steady output and is defined as,
6. ***Steady-state error, :*** It indicates the error between the actual output and desired output as ‘t’ tends to infinity.

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Let us now obtain the expressions for the rise time, peak time, peak overshoot, and settling time for the second order system.

1. ***Rise time, :*** Put at ,,;.
2. ***Peak time, :*** Put and solve for ; .

Peak overshoot occurs at *k = 1*. .

3. ***Settling time, :*** For 2% tolerance band, , .
4. ***Steady-state error, :*** It is found previously that steady-state error for step input is zero.

Let us now consider ramp input, .

Then,

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Therefore, the steady-state error due to ramp input is.

STEADY STATE ERRORS

The steady-state performance of a stable control system is generally judged by its steady-state error to step, ramp and parabolic inputs. For a unity feedback system,

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It is seen that steady-state error depends upon the input and the forward transfer function. The steady-state errors for different inputs are derived as follows:

ALGEBRIC CRITERIA

1. For unit-step input:

; is called position error constant.

2. For unit-ramp input:

; is called velocity error constant.

3. For unit-parabolic input:

; is called acceleration error const.

Types of Feedback Control System

The open-loop transfer function of a system can be written as,

If n = 0, the system is called type-0 system, if n = 1, the system is called type-1 system, if n = 2, the system is called type-2 system, etc. Steady-state errors for various inputs and system types are tabulated below.

Type of input	Steady-state error		
	Type-0 system	Type-1 system	Type-2 system
Unit-step	$1/(1+K_p)$	0	0
Unit-ramp	∞	$1/K_v$	0
Unit-parabolic	∞	∞	$1/K_a$
	$K_p = \lim_{s\to 0} G(s)$	$K_v = \lim_{s\to 0} sG(s)$	$K_a = \lim_{s\to 0} s^2G(s)$

ERROR CONSTANTS

The error constants for non-unity feedback systems may be obtained by replacing G(s) by G(s)H(s). Systems of type higher than 2 are not employed due to two reasons:

1. The system is difficult to stabilize.
2. The dynamic errors for such systems tend to be larger than those types-0, -1 and -2.

Effect of Adding a Zero to a System

Let a zero at s = -z be added to a second order system. Then we have,

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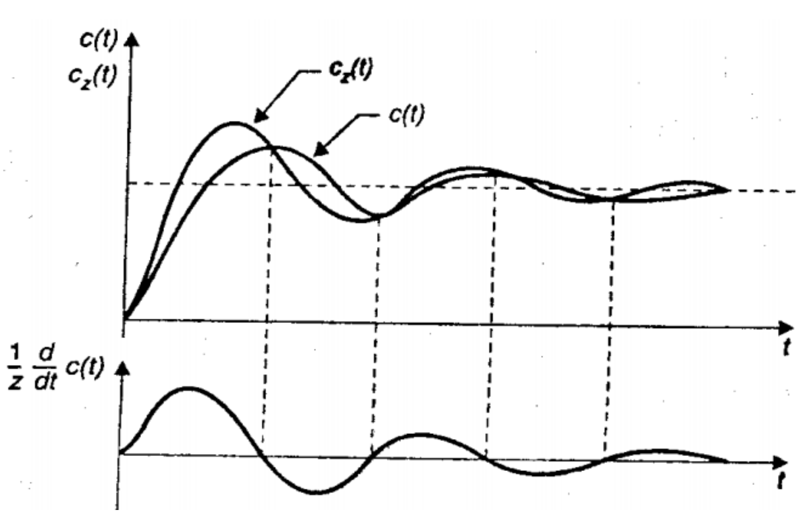
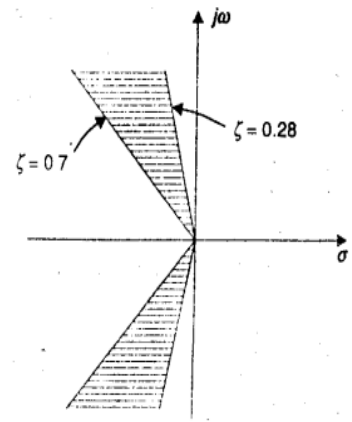
The multiplication term is adjusted to make the steady-state gain of the system unity. This gives $c_{ss} = 1$ when the input is unit step. Let $c_z(t)$ be the response of the system given by the above equation and *c(t)* is the response without adding the pole. Manipulation of the above equation gives,

The effect of added derivative term is to produce a pronounced early peak to the system response which will be clear from the figure in the next page. Closer the zero to origin, the more pronounce the peaking phenomenon. Due to this fact, *the zeros on the real axis near the origin are generally avoided in design*. However, in a sluggish system the artful introduction of a zero at the proper position can improve the transient response. We can see from equation (03) that as z increases, i.e., the zero moves further into the left half of the s-plane, its effect becomes less pronounced.



Design Specifications of Second-order Systems

A control system is generally required to meet three time response specifications: steady-state accuracy, damping factor ζ (or peak overshoot, M_p) and settling time t_s. Steady-state accuracy requirement is met by suitable choice of *Kp, Kv, or Ka* depending on the type of the system. For most control systems ζ in the range of 0.7 – 0.28 (or peak overshoot of 5 – 40%) is considered acceptable. For this range of ζ, the closed-loop pole locations are restricted to the shaded region of the s-plane as shown in Figure.



For the antenna position control system,;;;. Here, is only the adjustable parameter. If we increase, will increase and thus settling time will decrease. At the same time, will decrease, this indicates the increase in peak overshoot. Thus by merely increasing gain, we cannot improve both transient and steady-state error specifications. We need to add additional components to the system. These are called compensators. It will allow improvement of both transient and steady-state specifications.

CONCEPTS OF STABILITY

BIBO stability: A system is said to be BIBO stable if for any bounded input, its output is also bounded. • Absolute stability: Stable /Unstable • Relative stability: Degree of stability (i.e. how far from instability) • A stable linear system described by a T.F. is such that all its poles have negative real parts