

BIOLOGICAL CONTROL SYSTEMS

TIME RESPONSE ANALYSIS OF CONTROL SYSTEM

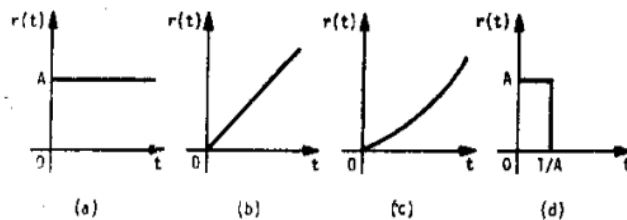
In time-domain analysis the response of a dynamic system to an input is expressed as a function of time. It is possible to compute the time response of a system if the nature of input and the mathematical model of the system are known.

Usually, the input signals to control systems are not known fully ahead of time. In a radar tracking system, the position and the speed of the target to be tracked may vary in a random fashion. It is therefore difficult to express the actual input signals mathematically by simple equations. The characteristics of actual input signals are a sudden shock, a sudden change, a constant velocity, and constant acceleration. The dynamic behavior of a system is therefore judged and compared under application of standard test signals – an impulse, a step, a constant velocity, and constant acceleration. Another standard signal of great importance is a sinusoidal signal.

The time response of any system has two components: transient response and the steady-state response. Transient response is dependent upon the system poles only and not on the type of input. It is therefore sufficient to analyze the transient response using a step input. The steady-state response depends on system dynamics and the input quantity. It is then examined using different test signals by final value theorem.

STANDARD TEST SIGNALS

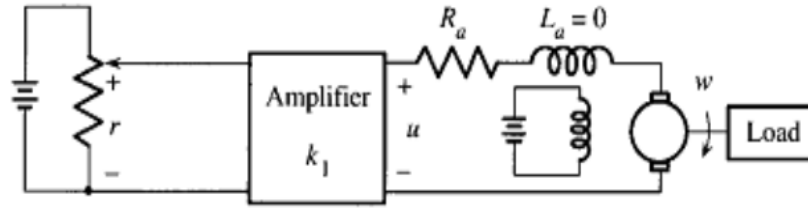
- a) Step signal:
- b) Ramp signal:
- c) Parabolic signal:
- d) Impulse signal:



TIME-RESPONSE OF FIRST-ORDER SYSTEMS

Let us consider the armature-controlled dc motor driving a load, such as a video tape. The objective is to drive the tape at constant speed. Note that it is an open-loop system.

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; If ,

;

is the steady-state final speed. If the desired speed is , choosing the motor will eventually reach the desired speed.

We are interested not only in final speed, but also in the speed of response. Here, is the time constant of motor which is responsible for the speed of response.

The time response is plotted in the Figure in next page. A plot of is shown, from where it is seen that, for the value of is less than 1% of its original value. Therefore, the speed of the motor will reach and stay within 1% of its final speed at 5 time constants.

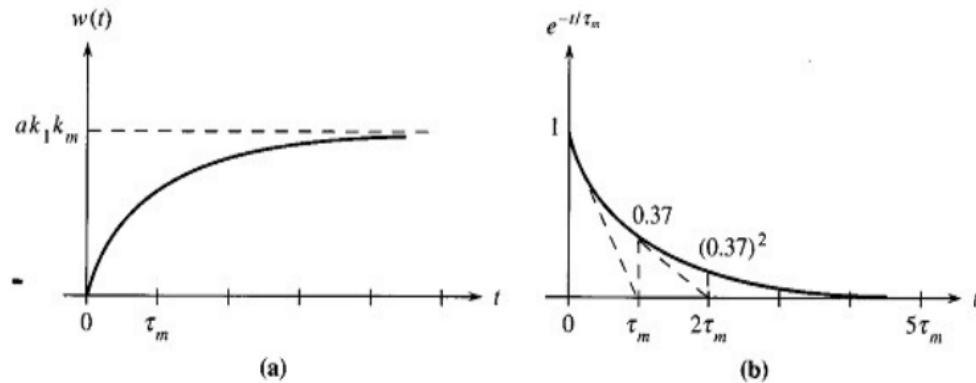
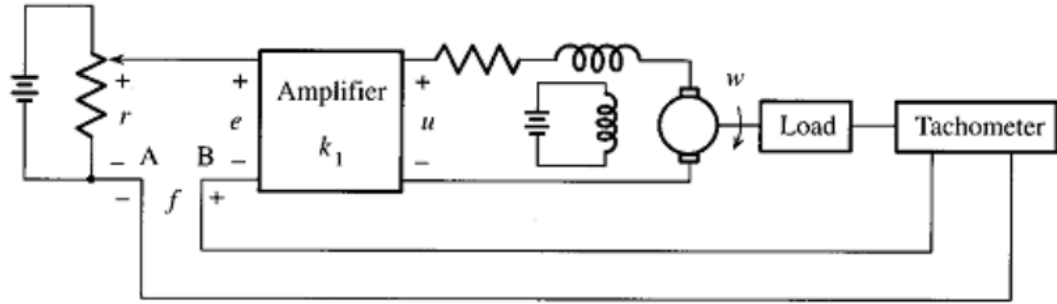


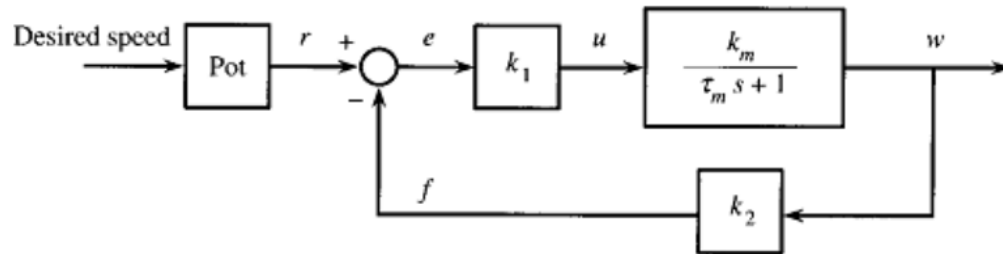
Figure: Time responses

Let us now consider the closed-loop system shown below.

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(a)



(b)

Here,

where, and .

If , the response would be, .

If a is properly chosen, the tape can reach a desired speed. It will reach the desired speed in 5seconds. Here, . Thus, we can control the speed of response in feedback system.

Although the time-constant is reduced by a factor , in the feedback system, the motor gain constant is also reduced by the same factor. In order to compensate for this loss of gain, the applied reference voltage must be increased by the same factor.

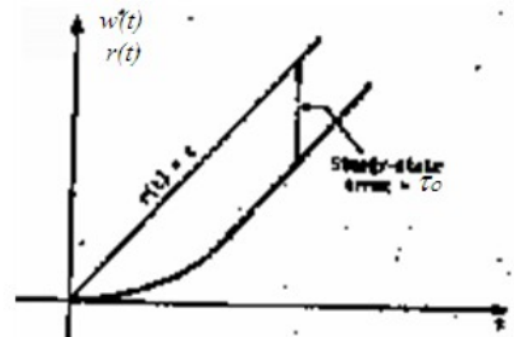
Ramp response of first-order system

Let, for simplicity. Then, . Also, let, .

Then, ;

The error signal is,

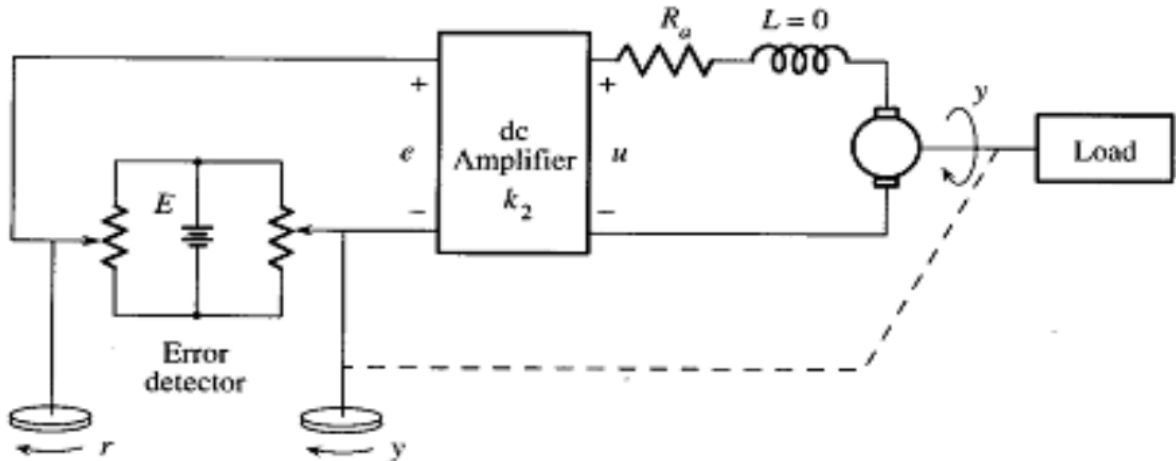
Or,



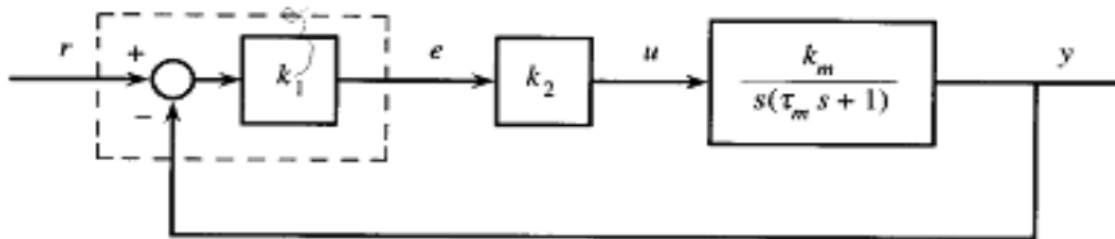
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Thus, the first-order system will track the unit ramp input with a steady-state error, which is equal to the time-constant of the system.

TIME-RESPONSE OF SECOND-ORDER SYSTEMS



(a)



(b)

Consider the antenna position control system. Its transfer function from r to y is,

where, we define, and . The constant is called the *damping ratio* and is called the *natural frequency*. The system above is in fact a standard second order system.

The transfer function has two poles and no zero. Its poles are,

Here, is called the *damping factor*, is called *damped or actual frequency*.

The location of poles for different are plotted in Figure below. For, the two poles are purely imaginary. If, the two poles are complex conjugate. All possible cases are described in a table shown below.

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