THEORY OF SAMPLING AND TEST OF HYPOTHESIS

Sampling theory is a study of relationships existing between a population and samples drawn from the population. Sampling theory is applicable only to random samples. For this purpose the population or a universe may be defined as an aggregate of items possessing a common trait or traits. In other words, a universe is the complete group of items about which knowledge is sought. The universe may be finite or infinite. Finite universe is one which has a definite and certain number of items, but when the number of items is uncertain and infinite, the universe is said to be an infinite universe. Similarly, the universe may be hypothetical or existent. In the former case the universe in fact does not exist and we can only imagine the items constituting it. Tossing of a coin or throwing a dice are examples of hypothetical universe. Existent universe is a universe of concrete objects i.e., the universe where the items constituting it really exist. On the other hand, the term sample refers to that part of the universe which is selected for the purpose of investigation. The theory of sampling studies the relationships that exist between the universe and the sample or samples drawn from it. The main problem of sampling theory is the problem of relationship between a parameter and a statistic. The theory of sampling is concerned with estimating the properties of the population from those of the sample and also with gauging the precision of the estimate. This sort of movement from particular (sample) towards general (universe) is what is known as statistical induction or statistical inference. In more clear terms "from the sample we attempt to draw inference concerning the universe. In order to be able to follow this inductive method, we first follow a deductive argument which is that we imagine a population or universe (finite or infinite) and investigate the behavior of the samples drawn from this universe applying the laws of probability.The methodology dealing with all this is known as sampling theory.

(*i*) *Statistical estimation:* Sampling theory helps in estimating unknown population parameters from a knowledge of statistical measures based on sample studies. In other words, to obtain an estimate of parameter from statistic is the main objective of the sampling theory. The estimate can either be a point estimate or it may be an interval estimate. Point estimate is a single estimate expressed in the form of a single figure, but interval estimate has two limits viz., the upper limit and the lower limit within which the parameter value may lie. Interval estimates are often used in statistical induction.

(*ii*) *Testing of hypotheses:* The second objective of sampling theory is to enable us to decide whether to accept or reject hypothesis; the sampling theory helps in determining whether observed differences are actually due to chance or whether they are really significant.

(*iii*) *Statistical inference:* Sampling theory helps in making generalisation about the population/ universe from the studies based on samples drawn from it. It also helps in determining the accuracy of such generalisations.

WHAT IS A HYPOTHESIS?

Ordinarily, when one talks about hypothesis, one simply means a mere assumption or some supposition to be proved or disproved. But for a researcher hypothesis is a formal question that he intends to resolve. Thus a hypothesis may be defined as a proposition or a set of proposition set forth as an explanation for the occurrence of some specified group of phenomena either asserted merely as a provisional njecture to guide some investigation or accepted as highly probable in the light of established facts. Quite often a research hypothesis is a predictive statement, capable of being tested by scientific methods, that relates an independent variable to some dependent variable. For example, consider statements like the following ones: "Students who receive counselling will show a greater increase in creativity than students not receiving counselling" Or "the automobile A is performing as well as automobile B." These are hypotheses capable of being objectively verified and tested. Thus, we may conclude that a hypothesis states what we are looking for and it is a proposition which can be put to a test to determine its validity.

Characteristics of hypothesis: Hypothesis must possess the following characteristics:
(i) Hypothesis should be clear and precise. If the hypothesis is not clear and precise, the inferences drawn on its basis cannot be taken as reliable.
(ii) Hypothesis should be capable of being tested. In a swamp of untestable hypotheses, many a time the research programmes have bogged down. Some prior study may be done by researcher in order to make hypothesis a testable one. A hypothesis "is testable if other deductions can be made from it which, in turn, can be confirmed or disproved by observation."
(iii) Hypothesis should state relationship between variables, if it happens to be a relational hypothesis.
(iv) Hypothesis should be limited in scope and must be specific. A researcher must remember that narrower hypotheses are generally more testable and he should develop such hypotheses.
(v) Hypothesis should be stated as far as possible in most simple terms so that the same is easily understandable by all concerned. But one must remember that simplicity of hypothesis has nothing to do with its significance.
(vi) Hypothesis should be consistent with most known facts i.e., it must be consistent with a substantial body of established facts. In other words, it should be one which judges accept as being the most likely.
(vii) Hypothesis should be amenable to testing within a reasonable time. One should not use even an excellent hypothesis, if the same cannot be tested in reasonable time for one cannot spend a life-time collecting data to test it.
(viii) Hypothesis must explain the facts that gave rise to the need for explanation. This means that by using the hypothesis plus other known and accepted generalizations, one should be able to deduce the original problem condition. Thus hypothesis must actually explain what it claims to explain; it should have empirical reference.

Basic concepts in the context of testing of hypotheses need to be explained.

(a) *Null hypothesis and alternative hypothesis:* In the context of statistical analysis, we often talk about null hypothesis and alternative hypothesis. If we are to compare method *A* with method *B* about its superiority and if we proceed on the assumption that both methods are equally good, then this assumption is termed as the null hypothesis. As against this, we may think that the method *A* is superior or the method *B* is inferior, we are then stating what is termed as alternative hypothesis. The null hypothesis is generally symbolized as H_0 and the alternative hypothesis as H_a. Suppose we want to test the hypothesis that the population mean (μ) is equal to the hypothesised mean $\left(\mu_{H_0}\right) = 100$.

Then we would say that the null hypothesis is that the population mean is equal to the hypothesised mean 100 and symbolically we can express as:

$$H_0 : \mu = \mu_{H_0} = 100$$

If our sample results do not support this null hypothesis, we should conclude that something else is true. What we conclude rejecting the null hypothesis is known as alternative hypothesis. In other words, the set of alternatives to the null hypothesis is referred to as the alternative hypothesis. If we accept H_0, then we are rejecting H_a and if we reject H_0, then we are accepting H_a. For $H_0 : \mu = \mu_{H_0} = 100$, we may consider three possible alternative hypotheses as follows*:

Alternative hypothesis	*To be read as follows*
H_a : H_0	(The alternative hypothesis is that the population mean is not equal to 100 i.e., it may be more or less than 100)
H_a : H_0	(The alternative hypothesis is that the population mean is greater than 100)
H_a : H_0	(The alternative hypothesis is that the population mean is less than 100)

The null hypothesis and the alternative hypothesis are chosen before the sample is drawn (the researcher must avoid the error of deriving hypotheses from the data that he collects and then testing the hypotheses from the same data). In the choice of null hypothesis, the following considerations are usually kept in view:

(a) Alternative hypothesis is usually the one which one wishes to prove and the null hypothesis is the one which one wishes to disprove. Thus, a null hypothesis represents the hypothesis we are trying to reject, and alternative hypothesis represents all other possibilities.

(b) If the rejection of a certain hypothesis when it is actually true involves great risk, it is taken as null hypothesis because then the probability of rejecting it when it is true is α (the level of significance) which is chosen very small.

(c) Null hypothesis should always be specific hypothesis i.e., it should not state about or approximately a certain value.

Generally, in hypothesis testing we proceed on the basis of null hypothesis, keeping the alternative hypothesis in view. Why so? The answer is that on the assumption that null hypothesis is true, one can assign the probabilities to different possible sample results, but this cannot be done if we proceed with the alternative hypothesis. Hence the use of null hypothesis (at times also known as statistical hypothesis) is quite frequent.

(b) *The level of significance:* This is a very important concept in the context of hypothesis testing. It is always some percentage (usually 5%) which should be chosen with great care, thought and reason. In case we take the significance level at 5 per cent, then this implies that H_0 will be rejected

*If a hypothesis is of the type $\mu = \mu_{H_0}$, then we call such a hypothesis as simple (or specific) hypothesis but if it is of the type $\mu \neq \mu_{H_0}$ or $\mu > \mu_{H_0}$ or $\mu < \mu_{H_0}$, then we call it a composite (or nonspecific) hypothesis.

when the sampling result (i.e., observed evidence) has a less than 0.05 probability of occurring if H_0 is true. In other words, the 5 per cent level of significance means that researcher is willing to take as much as a 5 per cent risk of rejecting the null hypothesis when it (H_0) happens to be true. Thus the significance level is the maximum value of the probability of rejecting H_0 when it is true and is usually determined in advance before testing the hypothesis.

(c) *Decision rule or test of hypothesis:* Given a hypothesis H_0 and an alternative hypothesis H_a, we make a rule which is known as decision rule according to which we accept H_0 (i.e., reject H_a) or reject H_0 (i.e., accept H_a). For instance, if (H_0 is that a certain lot is good (there are very few defective items in it) against H_a) that the lot is not good (there are too many defective items in it), then we must decide the number of items to be tested and the criterion for accepting or rejecting the hypothesis. We might test 10 items in the lot and plan our decision saying that if there are none or only 1 defective item among the 10, we will accept H_0 otherwise we will reject H_0 (or accept H_a). This sort of basis is known as decision rule.

(d) *Type I and Type II errors:* In the context of testing of hypotheses, there are basically two types of errors we can make. We may reject H_0 when H_0 is true and we may accept H_0 when in fact H_0 is not true. The former is known as Type I error and the latter as Type II error. In other words, Type I error means rejection of hypothesis which should have been accepted and Type II error means accepting the hypothesis which should have been rejected. Type I error is denoted by α (alpha) known as α error, also called the level of significance of test; and Type II error is denoted by β (beta) known as β error. In a tabular form the said two errors can be presented as follows:

	Decision	
	Accept H	Reject H
H_0 (true)	Correct decision	Type I error (error)
H_0 (false)	Type II error (error)	Correct decision

The probability of Type I error is usually determined in advance and is understood as the level of significance of testing the hypothesis. If type I error is fixed at 5 per cent, it means that there are about 5 chances in 100 that we will reject H_0 when H_0 is true. We can control Type I error just by fixing it at a lower level. For instance, if we fix it at 1 per cent, we will say that the maximum probability of committing Type I error would only be 0.01.

But with a fixed sample size, n, when we try to reduce Type I error, the probability of committing Type II error increases. Both types of errors cannot be reduced simultaneously. There is a trade-off between two types of errors which means that the probability of making one type of error can only be reduced if we are willing to increase the probability of making the other type of error. To deal with this trade-off in business situations, decision-makers decide the appropriate level of Type I error by examining the costs or penalties attached to both types of errors. If Type I error involves the time and trouble of reworking a batch of chemicals that should have been accepted, whereas Type II error means taking a chance that an entire group of users of this chemical compound will be poisoned, then in such a situation one should prefer a Type I error to a Type II error. As a result one must set very high level for Type I error in one's testing technique of a given hypothesis.[2] Hence, in the testing of hypothesis, one must make all possible effort to strike an adequate balance between Type I and Type II errors.

(e) *Two-tailed and One-tailed tests:* In the context of hypothesis testing, these two terms are quite important and must be clearly understood. A two-tailed test rejects the null hypothesis if, say, the sample mean is significantly higher or lower than the hypothesised value of the mean of the population. Such a test is appropriate when the null hypothesis is some specified value and the alternative hypothesis is a value not equal to the specified value of the null hypothesis.

TESTS OF HYPOTHESES

As has been stated above that hypothesis testing determines the validity of the assumption (technically described as null hypothesis) with a view to choose between two conflicting hypotheses about the value of a population parameter. Hypothesis testing helps to decide on the basis of a sample data, whether a hypothesis about the population is likely to be true or false. Statisticians have developed several tests of hypotheses (also known as the tests of significance) for the purpose of testing of hypotheses which can be classified as: (a) Parametric tests or standard tests of hypotheses; and

(b) Non-parametric tests or distribution-free test of hypotheses.

Parametric tests usually assume certain properties of the parent population from which we draw samples. Assumptions like observations come from a normal population, sample size is large, assumptions about the population parameters like mean, variance, etc., must hold good before parametric tests can be used. But there are situations when the researcher cannot or does not want to make such assumptions. In such situations we use statistical methods for testing hypotheses which are called non-parametric tests because such tests do not depend on any assumption about the parameters of the parent population. Besides, most non-parametric tests assume only nominal or ordinal data, whereas parametric tests require measurement equivalent to at least an interval scale. As a result, non-parametric tests need more observations than parametric tests to achieve the same size of Type I and Type II errors.[4] We take up in the present chapter some of the important parametric tests, whereas non-parametric tests will be dealt with in a separate chapter later in the book.

IMPORTANT PARAMETRIC TESTS

The important parametric tests are: (1) z-test; (2) t-test; (*3) χ^2-test, and (4) F-test. All these tests are based on the assumption of normality i.e., the source of data is considered to be normally distributed.

Mean of the population can be tested presuming different situations such as the population may be normal or other than normal, it may be finite or infinite, sample size may be large or small, variance of the population may be known or unknown and the alternative hypothesis may be two-sided or one-sided. Our testing technique will differ in different situations. We may consider some of the important situations.

1. *Population normal, population infinite, sample size may be large or small but variance of the population is known, H_a may be one-sided or two-sided:*

   In such a situation z-test is used for testing hypothesis of mean and the test statistic z is worked our as under:

$$z = \frac{\overline{X} - \mu_{H_0}}{\sigma_p / \sqrt{n}}$$

2. *Population normal, population finite, sample size may be large or small but variance of the population is known, H_a may be one-sided or two-sided:*

   In such a situation z-test is used and the test statistic z is worked out as under (using finite population multiplier):

$$z = \frac{\overline{X} - \mu_{H_0}}{\left(\sigma_p / \sqrt{n}\right) \times \left[\sqrt{(N-n)/(N-1)}\right]}$$

and $$\sigma_s = \sqrt{\frac{\Sigma\left(X_i - \bar{X}\right)^2}{(n-1)}}$$

5. *Population may not be normal but sample size is large, variance of the population may be known or unknown, and H_a may be one-sided or two-sided:*

   In such a situation we use *z*-test and work out the test statistic *z* as under:

$$z = \frac{\bar{X} - \mu_{H_0}}{\sigma_p/\sqrt{n}}$$

(This applies in case of infinite population when variance of the population is known but when variance is not known, we use σ_s in place of σ_p in this formula.)

OR

$$z = \frac{\bar{X} - \mu_{H_0}}{\left(\sigma_p/\sqrt{n}\right) \times \sqrt{(N-n)/(N-1)}}$$

(This applies in case of finite population when variance of the population is known but when variance is not known, we use σ_s in place of σ_p in this formula.)

3. *Population normal, population infinite, sample size small and variance of the population unknown, H_a may be one-sided or two-sided:*

   In such a situation *t*-test is used and the test statistic *t* is worked out as under:

$$t = \frac{\bar{X} - \mu_{H_0}}{\sigma_s/\sqrt{n}} \text{ with d.f.} = (n-1)$$

and $$\sigma_s = \sqrt{\frac{\Sigma\left(X_i - \bar{X}\right)^2}{(n-1)}}$$

4. *Population normal, population finite, sample size small and variance of the population unknown, and H_a may be one-sided or two-sided:*

   In such a situation *t*-test is used and the test statistic '*t*' is worked out as under (using finite population multiplier):

$$t = \frac{\bar{X} - \mu_{H_0}}{\left(\sigma_s/\sqrt{n}\right) \times \sqrt{(N-n)/(N-1)}} \text{ with d.f.} = (n-1)$$

and
$$\sigma_s = \sqrt{\frac{\Sigma\left(X_i - \overline{X}\right)^2}{(n-1)}}$$

5. *Population may not be normal but sample size is large, variance of the population may be known or unknown, and H_a may be one-sided or two-sided:*

In such a situation we use z-test and work out the test statistic z as under:

$$z = \frac{\overline{X} - \mu_{H_0}}{\sigma_p / \sqrt{n}}$$

(This applies in case of infinite population when variance of the population is known but when variance is not known, we use σ_s in place of σ_p in this formula.)

OR

$$z = \frac{\overline{X} - \mu_{H_0}}{\left(\sigma_p/\sqrt{n}\right) \times \sqrt{(N-n)/(N-1)}}$$

(This applies in case of finite population when variance of the population is known but when variance is not known, we use σ_s in place of σ_p in this formula.)

Example 1:

A sample of 400 male students is found to have a mean height 67.47 inches. Can it be reasonably regarded as a sample from a large population with mean height 67.39 inches and standard deviation 1.30 inches? Test at 5% level of significance.

Taking the null hypothesis that the mean height of the population is equal to 67.39 inches, we can write:

$$H_0 : \mu_{H_0} = 67.39''$$
$$H_a : \mu_{H_0} \neq 67.39''$$

and the given information as $\overline{X} = 67.47''$, $\sigma_p = 1.30''$, $n = 400$. Assuming the population to be normal, we can work out the test statistic z as under:

$$z = \frac{\overline{X} - \mu_{H_0}}{\sigma_p/\sqrt{n}} = \frac{67.47 - 67.39}{1.30/\sqrt{400}} = \frac{0.08}{0.065} = 1.231$$

As H_a is two-sided in the given question, we shall be applying a two-tailed test for determining the rejection regions at 5% level of significance which comes to as under, using normal curve area table:

$$R : |z| > 1.96$$

The observed value of z is 1.231 which is in the acceptance region since $R : |z| > 1.96$ and thus H_0 is accepted. We may conclude that the given sample (with mean height = 67.47″) can be regarded

to have been taken from a population with mean height 67.39″ and standard deviation 1.30″ at 5% level of significance.

Example 2:

Suppose we are interested in a population of 20 industrial units of the same size, all of which are experiencing excessive labour turnover problems. The past records show that the mean of the distribution of annual turnover is 320 employees, with a standard deviation of 75 employees. A sample of 5 of these industrial units is taken at random which gives a mean of annual turnover as 300 employees. Is the sample mean consistent with the population mean? Test at 5% level.

$$H_0: \mu_{H_0} = 320 \text{ employees}$$
$$H_a: \mu_{H_0} \neq 320 \text{ employees}$$

and the given information as under:

$$\bar{X} = 300 \text{ employees}, \ \sigma_p = 75 \text{ employees}$$
$$n = 5;\ N = 20$$

Assuming the population to be normal, we can work out the test statistic z as under:

$$z^* = \frac{\bar{X} - \mu_{H_0}}{\sigma_p/\sqrt{n} \times \sqrt{(N-n)/(N-1)}}$$

$$= \frac{300 - 320}{75/\sqrt{5} \times \sqrt{(20-5)/(20-1)}} = -\frac{20}{(33.54)\,(.888)}$$

$$= -0.67$$

As H_a is two-sided in the given question, we shall apply a two-tailed test for determining the rejection regions at 5% level of significance which comes to as under, using normal curve area table:

$$R: |z| > 1.96$$

The observed value of z is –0.67 which is in the acceptance region since $R: |z| > 1.96$ and thus, H_0 is accepted and we may conclude that the sample mean is consistent with population mean i.e., the population mean 320 is supported by sample results.

Example 3:

The mean of a certain production process is known to be 50 with a standard deviation of 2.5. The production manager may welcome any change is mean value towards higher side but would like to safeguard against decreasing values of mean. He takes a sample of 12 items that gives a mean value of 48.5. What inference should the manager take for the production process on the basis of sample results? Use 5 per cent level of significance for the purpose.

$$H_0: \mu_{H_0} = 50$$

$H_a: \mu_{H_0} < 50$ (Since the manager wants to safeguard against decreasing values of mean.)

and the given information as $\bar{X} = 48.5$, $\sigma_p = 2.5$ and $n = 12$. Assuming the population to be normal, we can work out the test statistic z as under:

$$z = \frac{\bar{X} - \mu_{H_0}}{\sigma_p/\sqrt{n}} = \frac{48.5 - 50}{2.5/\sqrt{12}} = -\frac{1.5}{(2.5)/(3.464)} = -2.0784$$

As H_a is one-sided in the given question, we shall determine the rejection region applying one-tailed test (in the left tail because H_a is of less than type) at 5 per cent level of significance and it comes to as under, using normal curve area table:

$$R: z < -1.645$$

The observed value of z is – 2.0784 which is in the rejection region and thus, H_0 is rejected at 5 per cent level of significance. We can conclude that the production process is showing mean which is significantly less than the population mean and this calls for some corrective action concerning the said process.

Example 4:

Raju Restaurant near the railway station at Falna has been having average sales of 500 tea cups per day. Because of the development of bus stand nearby, it expects to increase its sales. During the first 12 days after the start of the bus stand, the daily sales were as under:

550, 570, 490, 615, 505, 580, 570, 460, 600, 580, 530, 526

On the basis of this sample information, can one conclude that Raju Restaurant's sales have increased?

Use 5 per cent level of significance.

$H_0 : \mu = 500$ cups per day

$H_a : \mu > 500$ (as we want to conclude that sales have increased).

As the sample size is small and the population standard deviation is not known, we shall use *t*-test assuming normal population and shall work out the test statistic *t* as:

$$t = \frac{\bar{X} - \mu}{\sigma_s/\sqrt{n}}$$

(To find $\bar{X}$ and σ_s we make the following computations:)

$$\therefore \quad \bar{X} = \frac{\Sigma X_i}{n} = \frac{6576}{12} = 548$$

and
$$\sigma_s = \sqrt{\frac{\Sigma\left(X_i - \bar{X}\right)^2}{n-1}} = \sqrt{\frac{23978}{12-1}} = 46.68$$

Hence,
$$t = \frac{548 - 500}{46.68/\sqrt{12}} = \frac{48}{13.49} = 3.558$$

Degree of freedom $= n - 1 = 12 - 1 = 11$

As H_a is one-sided, we shall determine the rejection region applying one-tailed test (in the right tail because H_a is of more than type) at 5 per cent level of significance and it comes to as under, using table of *t*-distribution for 11 degrees of freedom:

$$R : t > 1.796$$

The observed value of *t* is 3.558 which is in the rejection region and thus H_0 is rejected at 5 per cent level of significance and we can conclude that the sample data indicate that Raju restaurant's sales have increased.

HYPOTHESIS TESTING FOR DIFFERENCES BETWEEN MEANS

In many decision-situations, we may be interested in knowing whether the parameters of two populations are alike or different. For instance, we may be interested in testing whether female workers earn less than male workers for the same job. We shall explain now the technique of hypothesis testing for differences between means. The null hypothesis for testing of difference between means is generally stated as $H_0 : \mu_1 = \mu_2$, where μ_1 is population mean of one population and μ_2 is population mean of the second population, assuming both the populations to be normal populations. Alternative hypothesis may be of not equal to or less than or greater than type as stated earlier and accordingly we shall determine the acceptance or rejection regions for testing the hypotheses. There may be different situations when we are examining the significance of difference between two means, but the following may be taken as the usual situations:

1. *Population variances are known or the samples happen to be large samples:*
In this situation we use z-test for difference in means and work out the test statistic z as under:

$$z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\sigma_{p1}^2}{n_1} + \frac{\sigma_{p2}^2}{n_2}}}$$

In case σ_{p_1} and σ_{p_2} are not known, we use σ_{s_1} and σ_{s_2} respectively in their places calculating

$$\sigma_{s_1} = \sqrt{\frac{\Sigma\left(X_{1i} - \bar{X}_1\right)^2}{n_1 - 1}} \text{ and } \sigma_{s_2} = \sqrt{\frac{\Sigma\left(X_{2i} - \bar{X}_2\right)^2}{n_2 - 1}}$$

2. *Samples happen to be large but presumed to have been drawn from the same population whose variance is known:*
In this situation we use z test for difference in means and work out the test statistic z as under:

$$z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\sigma_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

In case σ_p is not known, we use $\sigma_{s_{1.2}}$ (combined standard deviation of the two samples) in its place calculating

$$\sigma_{s_{1.2}} = \sqrt{\frac{n_1\left(\sigma_{s_1}^2 + D_1^2\right) + n_2\left(\sigma_{s_2}^2 + D_2^2\right)}{n_1 + n_2}}$$

where $D_1 = \left(\bar{X}_1 - \bar{X}_{1.2}\right)$

$D_2 = \left(\bar{X}_2 - \bar{X}_{1.2}\right)$

$$\bar{X}_{1.2} = \frac{n_1\bar{X}_1 + n_2\bar{X}_2}{n_1 + n_2}$$

3. *Samples happen to be small samples and population variances not known but assumed to be equal:*

In this situation we use t-test for difference in means and work out the test statistic t as under:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{\Sigma\left(X_{1i} - \bar{X}_1\right)^2 + \Sigma\left(X_{2i} - \bar{X}_2\right)^2}{n_1 + n_2 - 2}} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

with d.f. = $(n_1 + n_2 - 2)$

Alternatively, we can also state

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1 - 1)\sigma_{s_1}^2 + (n_2 - 1)\sigma_{s_2}^2}{n_1 + n_2 - 2}} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

with d.f. = $(n_1 + n_2 - 2)$

Example 1:

The mean produce of wheat of a sample of 100 fields in 200 lbs. per acre with a standard deviation of 10 lbs. Another samples of 150 fields gives the mean of 220 lbs. with a standard deviation of 12 lbs. Can the two samples be considered to have been taken from the same population whose standard deviation is 11 lbs? Use 5 per cent level of significance.

$$H_0 : \mu = \mu_2$$

$$H_a : \mu_1 \neq \mu_2$$

and the given information as $n_1 = 100$; $n_2 = 150$;

$$\bar{X}_1 = 200 \text{ lbs.}; \qquad \bar{X}_2 = 220 \text{ lbs.};$$

$$\sigma_{s_1} = 10 \text{ lbs.}; \qquad \sigma_{s_2} = 12 \text{ lbs.};$$

and $\sigma_p = 11$ lbs.

Assuming the population to be normal, we can work out the test statistic z as under:

$$z = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\sigma_p^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{200 - 220}{\sqrt{(11)^2\left(\frac{1}{100} + \frac{1}{150}\right)}}$$

$$= -\frac{20}{1.42} = -14.08$$

As H_a is two-sided, we shall apply a two-tailed test for determining the rejection regions at 5 per cent level of significance which come to as under, using normal curve area table:

$$R : |z| > 1.96$$

The observed value of z is – 14.08 which falls in the rejection region and thus we reject H_0 and conclude that the two samples cannot be considered to have been taken at 5 per cent level of significance from the same population whose standard deviation is 11 lbs. This means that the difference between means of two samples is statistically significant and not due to sampling fluctuations.

Example 2:

A group of seven-week old chickens reared on a high protein diet weigh 12, 15, 11, 16, 14, 14, and 1 ounces; a second group of five chickens, similarly treated except that they receive a low protein die weigh 8, 10, 14, 10 and 13 ounces. Test at 5 per cent level whether there is significant evidence tha additional protein has increased the weight of the chickens. Use assumed mean (or A_1) = 10 for th sample of 7 and assumed mean (or A_2) = 8 for the sample of 5 chickens in your calculations.

$H_0 : \mu_1 = \mu_2$

$H_a : \mu_1 > \mu_2$ (as we want to conclude that additional protein has increased the weight of chickens)

Since in the given question variances of the populations are not known and the size of samples is small, we shall use t-test for difference in means, assuming the populations to be normal and thus work out the test statistic t as under:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{(n_1 - 1)\sigma_{s_1}^2 + (n_2 - 1)\sigma_{s_2}^2}{n_1 + n_2 - 2}} \times \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

with d.f. = $(n_1 + n_2 - 2)$

From the sample data we work out $\bar{X}_1, \bar{X}_2, \sigma_{s_1}^2$ and $\sigma_{s_2}^2$ (taking high protein diet sample as sample one and low protein diet sample as sample two) as shown below:

∴

$$\bar{X}_1 = A_1 + \frac{\Sigma(X_{1i} - A_1)}{n_1} = 10 + \frac{28}{7} = 14 \text{ ounces}$$

$$\bar{X}_2 = A_2 + \frac{\Sigma(X_{2i} - A_2)}{n_2} = 8 + \frac{15}{5} = 11 \text{ ounces}$$

$$\sigma_{s_1}^2 = \frac{\Sigma(X_{1i} - A_1)^2 - [\Sigma(X_{1i} - A_1)]^2/n_1}{(n_1 - 1)}$$

$$= \frac{134 - (28)^2/7}{7 - 1} = 3.667 \text{ ounces}$$

$$\sigma_{s_2}^2 = \frac{\Sigma(X_{2i} - A_2)^2 - [\Sigma(X_{2i} - A_2)]^2/n_2}{(n_2 - 1)}$$

$$= \frac{69 - (15)^2/5}{5 - 1} = 6 \text{ ounces}$$

Hence,

$$t = \frac{14 - 11}{\sqrt{\frac{(7 - 1)(3.667) + (5 - 1)(6)}{7 + 5 - 2}} \times \sqrt{\frac{1}{7} + \frac{1}{5}}}$$

$$= \frac{3}{\sqrt{4.6} \times \sqrt{.345}} = \frac{3}{1.26} = 2.381$$

Degrees of freedom = $(n_1 + n_2 - 2) = 10$

As H_a is one-sided, we shall apply a one-tailed test (in the right tail because H_a is of more than type) for determining the rejection region at 5 per cent level which comes to as under, using table of t-distribution for 10 degrees of freedom:

$$R : t > 1.812$$

The observed value of t is 2.381 which falls in the rejection region and thus, we reject H_0 and conclude that additional protein has increased the weight of chickens, at 5 per cent level of significance.

REFERENCES

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