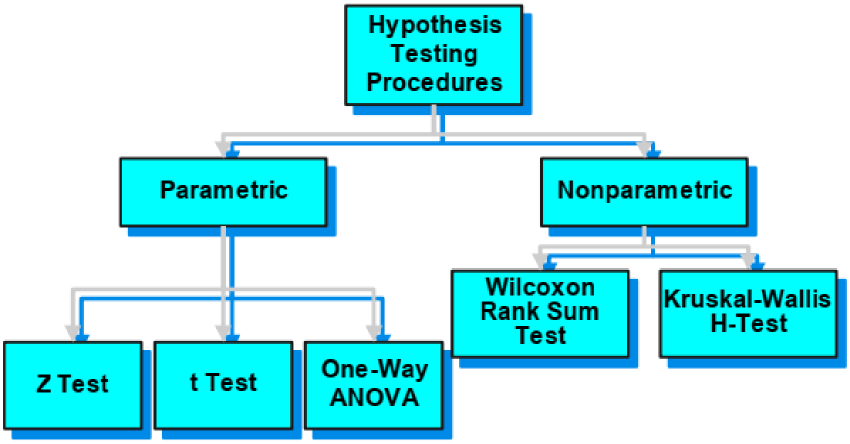
NON PARAMETRIC TEST

1. Hypothesis of Testing Procedure :



Many more parametric/ nonparametric test exists.

2. Nonparametric Test:
   - the term nonparametric was first used by Wolfowitz, 1942)
   - Do Not Involve Population Parameters. Example: Probability Distributions, Independence
   - Data Measured on Any Scale (Ratio or Interval, Ordinal or Nominal) Example: Wilcoxon Rank Sum Test
3. Advantages of Nonparametric Tests
   - Used With All Scales
   - Easier to Compute
   - Make Fewer Assumptions
   - Need Not Involve Population Parameters
   - Results May Be as Exact as Parametric Procedures
4. Disadvantages of Nonparametric Tests
   - May Waste Information
   - Parametric model more efficient if data permit
   - Difficult to Compute by hand for Large Samples
   - Tables Not Widely Available

5. Popular Nonparametric Tests
    - Sign Test
    - Wilcoxon Signed Test
    - Median Test
    - Mann Whitney Test
    - Kruskal wallis H Test

SIGN TEST :

The **Sign test** is a non-parametric test that is used to test whether or not two groups are equally sized. The sign test is used when dependent samples are ordered in pairs, where the bivariate random variables are mutually independent It is based on the direction of the plus and minus sign of the observation, and not on their numerical magnitude. It is also called the binominal sign test, with p =0 .5. The sign test is considered a weaker test, because it tests the pair value below or above the median and it does not measure the pair difference.

- Tests One Population Median, h
- Corresponds to t-Test for 1 Mean
- Assumes Population Is Continuous
- Small Sample Test Statistic: # Sample Values Above (or Below) Median
- Can Use Normal Approximation If $n \geq 10$

Assumptions:

- **Data distribution:** The Sign test is a non–parametric (distribution free) test, so we do not assume that the data is normally distributed.
- **Two samples:** Data should be from two samples. The population may differ for the two samples.
- **Dependent sample:** Dependent samples should be a paired sample or matched. Also known as 'before–after' sample.

Types of sign test:

- **One sample:** We set up the hypothesis so that + and – signs are the values of random variables having equal size.

- **Paired sample:** This test is also called an alternative to the paired sample t-test. This test uses the + and – signs in paired sample tests or in before-after study. In this test, null hypothesis is set up so that the sign of + and – are of equal size, or the population means are equal to the sample mean.

Procedure:

- Calculate the + and – sign for the given distribution. Put a + sign for a value greater than the mean value, and put a – sign for a value less than the mean value. Put 0 as the value is equal to the mean value; pairs with 0 as the mean value are considered ties.
- Denote the total number of signs by 'n' (ignore the zero sign) and the number of less frequent signs by 'S.'
- Obtain the critical value (K) at .05 of the significance level by using the following formula in case of small samples:

$$K = \frac{n-1}{2} - 0.98\sqrt{n}$$

Sign test in case of large sample:

$$Z = \frac{S - np}{\sqrt{np(1-p)}}$$

Binominal distribution formula = $n^{C_x}\, q^{n-x} p^x$ with p =1/2

- Compare the value of 'S' with the critical value (K). If the value of S is greater than the value of K, then the null hypothesis is accepted. If the value of the S is less than the critical value of K, then the null hypothesis is accepted. In the case of large samples, S is compared with the Z value.

Solved problems:

Example 1:

Consider the following data:

24.1 22.9 23 26.1 25 30.8 27.1 23.2 22.8 23.7 24.6 30.3

23.9 21.8 28.1 25.4 31.2 30.9

Test whether 30 can be regarded as population median.

Solution

H_O: M=30 H_1: M≠30 (Two Tailed) LOS: $\alpha = 0.05$.

The given data replacing the values exceeding 30 by a+ sign and values less than 30 by a- sign, we get

- - - - - + - - - - - + -

- - - + +

The number of +ve signs=S_+=4, The number of -ve signs=S_-=14.

X=min(S_+, S_-)=4.

P=P($S_+ \leq 4$)=0.0155 [from the Binomial Prob. Table for 18, 4, 0.5]

Since P<0.025 H_O is rejected.

Example 2:

Consider the following data:

5.1625	5.6735	5.2323	5.9192	5.2324	5.1212
5.50000	5.5251	5.4999	5.9998	5.0002	5.5001
5.3434	5.2129	5.1219			

Can 5.5 be regarded as population median?

Solution

H_O: M=5.5 H_1: M≠5.5 (Two Tailed) LOS: $\alpha = 0.05$.

The given data replacing the values exceeding 5.5 by a+ sign and values less than 5.5 by a- sign, we get

- + - + - - + - + - + - - -

The value of 5.5 which is equal to the hypothetical median is discarded.

The number of +ve signs=S_+=5, The number of -ve signs=S_-=9.

X=min(S_+, S_-)=5.

P=P($S_+ \leq 5$)=0.2121 [from the Binomial Prob. Table for 14, 5, 0.5]

Since P>0.025 H_O is accepted. Hence 5.5 can be regarded as population media.

Sign Test for one sided alternative:

The procedure for sign test one sided alternative is same as the procedure for two sided alternative except in "If P< α reject H_O''.

Example 3:

Consider the following data:

7 11 15 12 4 10 19 22 25 15 17 20 23 28.

Test the hypothesis H_O: M=15 against H_1: M>15 at 5% LOS.

Solution

H_O: M=15 H_1: M>15 LOS: α = 0.05.

The given data replacing the values exceeding 15 by a+ sign and values less than 5.5 by a- sign, we get

\- - - - - + + + + + + +

The value of 15 which is equal to the hypothetical median is discarded.

The number of +ve signs=S_+=7, The number of -ve signs=S_-=5.

X=min(S_+, S_-)=5.

P=P($S_- \leq 5$)=0.3871 [from the Binomial Prob. Table for 12, 5, 0.5]

Since P>0.05 H_O is accepted.

WILCOXON SIGNED TEST

The Wilcoxon sign test is the non-parametric alternative of the **dependent samples t-test.**

The Wilcoxon Sign test is a statistical comparison of the average of two dependent samples. The Wilcoxon sign test is a sibling of the t-tests. It is, in fact, a non-paracontinuous-level alternative to the dependent samples t-test. Thus the Wilcoxon signed rank test is used in similar situations as the Mann-Whitney U-test. The main difference is that the Mann-Whitney U-test tests two independent samples, whereas the Wilcox sign test tests two dependent samples.

The Wilcoxon Sign test is a test of dependency. All dependence tests assume that the variables in the analysis can be split into independent and dependent variables. A dependence tests that compares the averages of an independent and a dependent variable assumes that differences in the average of the dependent variable are caused by the independent variable. Sometimes the independent variable is also called factor because the factor splits the sample in two or more groups, also called factor steps.

Dependence tests analyze whether there is a significant difference between the factor levels. The t-test family uses mean scores as the average to compare the differences, the Mann-Whitney U-test uses mean ranks as the average, and the Wilcoxon Sign test uses signed ranks.

Unlike the t-test and F-test the Wilcoxon sign test is a non-paracontinuous-level test. That means that the test does not assume any properties regarding the distribution of the underlying variables in the analysis. This makes the Wilcoxon sign test the analysis to conduct when analyzing variables of ordinal scale or variables that are not multivariate normal.

The Wilcoxon sign test is mathematically similar to conducting a Mann-Whitney U-test (which is sometimes also called Wilcoxon 2-sample t-test). It is also similar to the basic principle of the dependent samples t-test, because just like the dependent samples t-test the Wilcoxon sign test, tests the difference of observations.

However, the Wilcoxon signed rank test pools all differences, ranks them and applies a negative sign to all the ranks where the difference between the two observations is negative. This is called the signed rank. The Wilcoxon signed rank test is a non-paracontinuous-level test, in contrast to the dependent samples t-tests. Whereas the dependent samples t-test tests whether the average difference between two observations is 0, the Wilcoxon test tests whether the difference between two observations has a mean signed rank of 0. Thus it is much more robust against outliers and heavy tail distributions. Because the Wilcoxon sign test is a non-paracontinuous-level test it does not require a special distribution of the dependent variable in the analysis. Therefore it is the best test to compare mean scores when the dependent variable is not normally distributed and at least of ordinal scale.

For the test of significance of Wilcoxon signed rank test it is assumed that with at least ten paired observations the distribution of the W-value approximates a normal distribution. Thus we can normalize the empirical W-statistics and compare this to the tabulated z-ratio of the normal distribution to calculate the confidence level.

Assumptions

- Data comes from two matched, or dependent, populations.
- The data is continuous.

  Because it is a non-parametric test it does not require a special distribution of the dependent variable in the analysis.
- The Wilcoxon Sign test is a repeated measures test of dependency. This test is mathematically similar to conducting a Mann-Whitney U-test (which is sometimes also called Wilcoxon 2-sample t-test). It is also similar to the basic principle of the dependent samples t-test, because just like the

dependent samples t-test the Wilcoxon sign test, tests the difference of observations when the observations are matched.

- However this test pools all differences, ranks them and applies a negative sign to all the ranks where the difference between the two observations is negative. This is called the signed rank. Whereas the dependent samples t-test tests whether the average difference between two observations is 0 the Wilcoxon test tests whether the difference between two observations has a mean signed rank of 0. Thus it is much more robust against outliers and heavy tail distributions. Thus it is the best test to compare mean scores when the dependent variable is not normally distributed.

Procedure:

1. The first step of the Wilcoxon sign test is to calculate the differences of the repeated measurements and to calculate the absolute differences.
2. The next step of the Wilcoxon sign test is to order the cases by increasing absolute differences. For the Wilcoxon signed rank test we can ignore cases where the difference is zero. For all other cases we assign their relative rank. In case of tied ranks the average rank is calculated. That is if rank 10 and 11 have the same observed differences both are assigned rank 10.5.
3. The next step of the Wilcoxon sign test is to sign each rank. If the original difference < 0 then the rank is multiplied by -1; if the difference is positive the rank stays positive.
4. The W-statistic is simply the sum of the signed ranks. For large samples with n>10 the W-statistics approximates normal distribution, with Since the Wilcoxon signed rank test has the null hypothesis that there is on average no difference between the two measurements, it is assumed that $m_w=0$ with $\sigma_W = \sqrt{\frac{n(n+1)(2n+1)}{6}}$
5. If the difference W- m_w is negative the continuity correction is +0.5 (this is the case when W < m_w). If the difference W- m_w is positive the correction is -0.5 (this is the case when W > m_w).
6. The shortcut to the hypothesis testing of the Wilcoxon signed rank-test is knowing the critical z-value for a 95% confidence interval (or a 5% level of significance) $z = \frac{W - m_W \pm 0.5}{\sigma_W}$ which is z = 1.96 for a two-tailed test and directionality.

Whenever a test is based the normal distribution the sample z value needs to be 1.96 or higher to reject the null hypothesis.

Solved problems:

Example 4:

The following data represents the number of hours that a rechargeable instrument operates before recharge is required.

1.5 2.2 0.9 1.3 2.0 1.6 1.8 1.5 2.0 1.2 1.7

Test at 5% level that this particular instrument operates with a mean of 1.8 hours before requiring a recharge.

Solution

H_O: μ=1.8 H_1: μ≠1.8 LOS: $\alpha = 0.05$.

Subtracting 1.8 from each observation and ranking them without regard to sign:

Difference (x_i – 1.8)	-0.3	0.4	-0.9	-0.5	0.2	-0.2	-0.3	0.2	-0.6	-0.1
Rank	5.5	7	10	8	3	3	5.5	3	9	1

W_+=7+3+3=13, W_-=5.5+10+8+3+5.5+9+1=42

X=min(W_+, W_-)=13

Table value for n=10 and $\alpha = 0.05$ is 8.

Since X > Table value

H_O is accepted.

Example 5:

The height of 10 persons is given below:

171 175 177 178 180 182 190 192 195 202

Check whether the data supports the claim that the mean height is 178 cms. Use Wilcoxon's signed rank test.

Solution

H_O: μ=178 H_1: μ≠178 LOS: $\alpha = 0.05$.

Subtracting 178 from each observation and ranking them without regard to sign:

Difference (x_i – 178)	-7	-3	-1	2	4	12	14	17	24
Rank	5	3	1	2	4	6	7	8	9

W_+=2+4+6+7+8+9=36, W_-=5+3+1=9

$X = \min(W_+, W_-) = 9$

Table value for n=9 and $\alpha = 0.05$ is 6.

Since 9>table value, H_O is accepted.

Note:

To test H_O	H_1	Compute
$\mu = \mu_0$	$\mu \neq \mu_0$(Two Tailed)	$X = \min(W_+, W_-)$
	$\mu < \mu_0$(Left Tailed)	W_+
	$\mu > \mu_0$(Right Tailed)	W_-

KRUSKAL WALLIS H TEST

The Kruskal-Wallis Test was developed by Kruskal and Wallis (1952) jointly and is named after them. The Kruskal-Wallis test is a nonparametric (distribution free) test, and is used when the assumptions of ANOVA are not met. They both assess for significant differences on a continuous dependent variable by a grouping independent variable (with three or more groups). In the ANOVA, we assume that distribution of each group is normally distributed and there is approximately equal variance on the scores for each group. However, in the Kruskal-Wallis Test, we do not have any of these assumptions. Like all non-parametric tests, the Kruskal-Wallis Test is not as powerful as the ANOVA.

Assumptions:

- We assume that the samples drawn from the population are random.
- We also assume that the cases of each group are independent.
- The measurement scale for should be at least ordinal.

Hypothesis:

Null hypothesis: Null hypothesis assumes that the samples are from identical populations.

Alternative hypothesis: Alternative hypothesis assumes that the samples come from different populations.

Procedure:

1. Arrange the data of both samples in a single series in ascending order.

2. Assign rank to them in ascending order. In the case of a repeated value, or a tie, assign ranks to them by averaging their rank position.
3. Then sum up the different ranks, e.g. R_1 R_2 $R_{3....}$, for each of the different groups.
4. To calculate the value, apply the following formula:

$$H = \frac{12}{n(n+1)}\left[\frac{R_1^2}{n_1}+\frac{R_2^2}{n_2}+\frac{R_3^2}{n_3}...+\frac{R_k^2}{n_k}\right]-[3(n+1)]$$

Where

H=Kruskal-Wallis Test statistic

N = total number of observations in all samples

R_i = Sum of the ranks assigned

5. The Kruskal-Wallis test statistic is approximately a chi-square distribution, with k-1 degrees of freedom where n_i should be greater than 5. If the calculated value of the Kruskal-Wallis test is less than the critical chi-square value, then the null hypothesis cannot be reject.
6. If the calculated value of Kruskal-Wallis test is greater than the critical chi-square value, then we can reject the null hypothesis and say that the sample comes from a different population.

Solved problems:

Example 6:

The following data represents the final examination grades of samples from 3 groups of students who were taught statistics by 3 different methods.

Method I	94	88	91	74	87	97	
Method II	85	82	79	84	61	72	80
Method III	89	67	72	76	69		

Use the H test at 5% level of test that the 3 methods are equally effective.

Solution

H_O: $\mu_1 = \mu_2 = \mu_3$

H_1: $\mu_1 \neq \mu_2 \neq \mu_3$ LOS: $\alpha = 0.05$ $n_1=6$, $n_2=7$, $n_3=5$ $n=6+7+5=18$

The ranking procedure is given in the following table:

Method I	Rank I	Method II	Rank II	Method III	Rank III
94	17	85	12	89	15
88	14	82	10	67	2
91	16	79	8	72	4.5
74	6	84	11	76	7
87	13	61	1	76	7
97	18	72	4.5	69	3
		80	9		
	R_1=84		**R_2=55.5**		**R_3=31.5**

$$H = \frac{12}{18(19)}\left[\frac{84^2}{6} + \frac{55.5^2}{7} + \frac{31.5^2}{5}\right] - [3(19)] = 6.67$$

Table value χ^2 AT 5% level for 2 d.f. is 5.991.

Since $H > \chi^2_{tab}$ table value. H_O is rejected.

Example 7:

To compare 4 cricket balls, a professional bowler bowled 5 games with each ball and gets the following data:

Ball A	208	220	247	192	229
Ball B	216	196	189	205	210
Ball C	226	218	252	225	202
Ball D	212	198	207	232	221

Use the Kruskal-Wallis H test at 5% level to test whether or not a bowler can expect to score equally with the 4 cricket balls.

Solution

H_O: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

H_1: $\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$

LOS: α = 0.05 $n_1 = n_2 = n_3 = n4 = 5$ n=5+5+5+5=20

The ranking procedure is given in the following table:

Ball A	Rank A	Ball B	Rank B	Ball C	Rank C	Ball D	Rank D
208	8	216	11	226	16	212	10
220	13	196	3	218	12	198	4
247	19	189	1	252	20	207	7
192	2	205	6	225	15	232	18
229	17	210	9	202	2	221	14
	R_1=59		**R_2=30**		**R_3=68**		**R_4=53**

$$H = \frac{12}{20(21)}\left[\frac{59^2}{5} + \frac{30^2}{5} + \frac{68^2}{5} + \frac{53^2}{5}\right] - [3(21)] = 4.51$$

Table value χ^2 AT 5% level for 3 d.f. is 7.815.

Since $H < \chi^2_{tab}$ table value, H_O is accepted.

REFERENCES

1. Conover WJ. Practical Nonparametric Statistics, 2nd edition, New York: John Wiley and Sons.
2. Wikipedia - https://en.wikipedia.org/wiki/Sign_test