ASSESSMENT TEST

OBSERVE SILENCE IN THE EXAM ROOM

QUESTION 1.

Solve the initial value problem.

$$y^{(4)} = -\sin t + \cos t; \quad y'''(0) = 7, \quad y''(0) = y'(0) = -1,$$
$$y(0) = 0$$

SOLUTION: Since we are working with the fourth derivative, we will have to go through the two steps four times.

STEP 1:

$$y''' = \int (-\sin t + \cos t)\, dt \quad \rightarrow \quad y''' = \cos t + \sin t + c$$

STEP 2: When t = 0, y ''' = 7.

$$7 = \cos 0 + \sin 0 + c \quad \rightarrow \quad 7 = 1 + c \quad \rightarrow \quad c = 6$$
$$y''' = \cos t + \sin t + 6$$

STEP 1:

$$y'' = \int (\cos t + \sin t + 6)\, dt \quad \rightarrow \quad y'' = \sin t - \cos t + 6t + c$$

STEP 2: When t = 0, y '' = -1

$$-1 = \sin 0 - \cos 0 + 6(0) + c \quad \rightarrow \quad -1 = -1 + c \quad \rightarrow \quad c = 0$$
$$y'' = \sin t - \cos t + 6t$$

STEP 1:

$$y' = \int (\sin t - \cos t + 6t)\, dt \quad \rightarrow \quad y' = -\cos t - \sin t + \frac{6t^2}{2} + c$$

STEP 2: When t = 0, y ' = -1.

$$-1 = -\cos 0 - \sin 0 + 3(0)^2 + c \rightarrow -1 = -1 + c \rightarrow c = 0$$

$$y' = -\cos t - \sin t + 3t^2$$

STEP 1:

$$y = \int (-\cos t - \sin t + 3t^2)\, dt \rightarrow y = -\sin t + \cos t + \frac{3t^3}{3} + c$$

STEP 2: When t = 0, y = 0.

$$0 = -\sin 0 + \cos 0 + 0^3 + c \rightarrow 0 = 1 + c \rightarrow c = -1$$

$$\text{SOLUTION: } y = -\sin t + \cos t + t^3 - 1$$

QUESTION 2.

Solve the initial value problem.

$$\frac{dy}{dx} = 10 - x, \qquad y(0) = -1$$

SOLUTION:

STEP 1:

$$\frac{dy}{dx} = 10 - x \rightarrow dy = (10 - x)\, dx$$

$$\int dy = \int (10 - x)\, dx \rightarrow y = 10x - \frac{x^2}{2} + c$$

STEP 2: When x = 0, y = -1.

$$-1 = 10(0) - \frac{0}{2} + c \rightarrow c = -1$$

$$\text{SOLUTION: } y = 10x - \frac{x^2}{2} - 1$$