FINAL EXAMINATION – ANSWER SHEET

ATTEMPT ALL THE QUESTIONS

SECTION A

QUESTION 1. (20 Marks)

Solve the initial value problem.

$$y^{(4)} = -\sin t + \cos t; \quad y'''(0) = 7, \quad y''(0) = y'(0) = -1,$$
$$y(0) = 0$$

SOLUTION: Since we are working with the fourth derivative, we will have to go through the two steps four times.

STEP 1:

$$y''' = \int(-\sin t + \cos t)\,dt \quad \rightarrow \quad y''' = \cos t + \sin t + c$$

STEP 2: When t = 0, y ''' = 7.

$$7 = \cos 0 + \sin 0 + c \quad \rightarrow \quad 7 = 1 + c \quad \rightarrow \quad c = 6$$
$$y''' = \cos t + \sin t + 6$$

STEP 1:

$$y'' = \int(\cos t + \sin t + 6)\,dt \quad \rightarrow \quad y'' = \sin t - \cos t + 6t + c$$

STEP 2: When t = 0, y '' = -1

$$-1 = \sin 0 - \cos 0 + 6(0) + c \quad \rightarrow \quad -1 = -1 + c \quad \rightarrow \quad c = 0$$
$$y'' = \sin t - \cos t + 6t$$

STEP 1:

$$y' = \int(\sin t - \cos t + 6t)\,dt \quad \rightarrow \quad y' = -\cos t - \sin t + \frac{6t^2}{2} + c$$

STEP 2: When t = 0, y ' = -1.

$$-1 = -\cos 0 - \sin 0 + 3(0)^2 + c \quad \rightarrow -1 = -1 + c \quad \rightarrow \quad c = 0$$

$$y' = -\cos t - \sin t + 3t^2$$

STEP 1:

$$y = \int (-\cos t - \sin t + 3t^2)\, dt \quad \rightarrow \quad y = -\sin t + \cos t + \frac{3t^3}{3} + c$$

STEP 2: When t = 0, y = 0.

$$0 = -\sin 0 + \cos 0 + 0^3 + c \quad \rightarrow \quad 0 = 1 + c \quad \rightarrow \quad c = -1$$

$$\text{SOLUTION}: \quad y = -\sin t + \cos t + t^3 - 1$$

QUESTION 2. (20 Marks)

Given the velocity,

$$v = \frac{ds}{dt} = 32t - 2,$$

and the initial position of the body as s (1/2) = 4. Find the body's position at time t.

SOLUTION:

STEP 1:

$$s = \int (32t - 2)\, dt \quad \rightarrow \quad s = \frac{32t^2}{2} - 2t + c$$

STEP 2: When t = 1/2, s = 4.

$$4 = 16\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + c \quad \rightarrow \quad 4 = 4 - 1 + c \quad \rightarrow \quad c = 1$$

SOLUTION: $s = 16t^2 - 2t + 1$

QUESTION 3. (20 Marks)

Given the acceleration, $a = d^2s/dt^2 = -4\sin 2t$, initial velocity v (0) = 2, and the initial position of the body as s (0) = -3. Find the body's position at time t.

SOLUTION:

STEP 1:

$$v = \int -4\sin 2t\, dt \quad \rightarrow \quad v = \frac{4\cos 2t}{2} + c$$

STEP 2: When t = 0, v = 2.

$$2 = 2\cos 0 + c \quad \rightarrow \quad 2 = 2 + c \quad \rightarrow \quad c = 0$$

$$v = 2\cos 2t$$

STEP 1:

$$s = \int 2\cos 2t\, dt \quad \rightarrow \quad s = \frac{2\sin 2t}{2} + c$$

STEP 2: When t = 0, s = -3.

$$-3 = \sin 0 + c \quad \rightarrow c = -3$$

SOLUTION: $s = \sin 2t - 3$

SECTION B

QUESTION 4. (10 Marks)

Solve the initial value problem.

$$\frac{dy}{dx} = 10 - x, \qquad y(0) = -1$$

SOLUTION:

STEP 1:

$$\frac{dy}{dx} = 10 - x \quad \rightarrow \quad dy = (10 - x)\,dx$$

$$\int dy = \int (10 - x)\,dx \quad \rightarrow \quad y = 10x - \frac{x^2}{2} + c$$

STEP 2: When x = 0, y = -1.

$$-1 = 10(0) - \frac{0}{2} + c \quad \rightarrow \quad c = -1$$

$$\text{SOLUTION}: \quad y = 10x - \frac{x^2}{2} - 1$$

QUESTION 5. (10 Marks)

Solve the initial value problem.

$$\frac{dy}{dx} = 9x^2 - 4x + 5, \qquad y(-1) = 0$$

SOLUTION:

STEP 1:

$$\frac{dy}{dx} = 9x^2 - 4x + 5 \quad \rightarrow \quad dy = (9x^2 - 4x + 5)\,dx$$

$$\int dy = \int (9x^2 - 4x + 5)\,dx \quad \rightarrow \quad y = \frac{9x^3}{3} - \frac{4x^2}{2} + 5x + c$$

STEP 2: When x = -1, y = 0.

$$0 = 3(-1)^3 - 2(-1)^2 + 5(-1) + c \quad \rightarrow \quad 0 = -3 - 2 - 5 + c \quad \rightarrow \quad c = 10$$

$$\text{SOLUTION}: \quad y = 3x^3 - 2x^2 + 5x + 10$$

QUESTION 6. (10 Marks)

Solve the initial value problem.

$$\frac{ds}{dt} = \cos t + \sin t, \qquad s(\pi) = 1$$

SOLUTION:

STEP 1:

$$\frac{ds}{dt} = \cos t + \sin t \quad \rightarrow \quad ds = (\cos t + \sin t)\,dt$$

$$\int ds = \int (\cos t + \sin t)\,dt \quad \rightarrow \quad s = \sin t - \cos t + c$$

STEP 2: When t = π , s = 1.

$$1 = \sin \pi - \cos \pi + c \quad \rightarrow \quad 1 = 0 - (-1) + c \quad \rightarrow \quad c = 0$$

$$\text{SOLUTION}: \quad s = \sin t - \cos t$$

QUESTION 7. (10 Marks)

Solve the initial value problem.

$$\frac{d^2y}{dx^2} = 2 - 6x; \quad y'(0) = 4, \quad y(0) = 1$$

SOLUTION: We will have to do the two steps twice to find the solution to this initial value problem. The first time through will give us y ' and the second time through will give us y.

STEP 1:

$$y' = \int (2 - 6x)\,dx \quad \rightarrow \quad y' = 2x - \frac{6x^2}{2} + c$$

STEP 2: When x = 0, y ' = 4.

$$4 = 2(0) - 3(0)^2 + c \quad \rightarrow \quad c = 4$$

$$y' = 2x - 3x^2 + 4$$

STEP 1:

$$y = \int (2x - 3x^2 + 4)\,dx \quad \rightarrow \quad y = \frac{2x^2}{2} - \frac{3x^3}{3} + 4x + c$$

STEP 2: When x = 0, y = 1.

$$1 = 0^2 - 0^3 + 4(0) + c \quad \rightarrow \quad c = 1$$

$$\text{SOLUTION:} \quad y = x^2 - x^3 + 4x + 1$$