INCOMING TRAFFIC AND SERVICE TIME CHARACTERISATION

As we know whenever a subscriber originates a call, he adds one to the number of calls arriving at the network and no way by which he can reduce the number of calls that have already arrived. This process can be treated as a special case of B-D process in which the death rate is equal to zero, i.e. no death occurring in the process. Such a process is known as renewal process.

It is a pure birth process in the sense that it can only add to the population as the time goes by and cannot delete the population by itself. The equation arrived in renewal process by setting $\mu_k = 0$ in B-D process.

i.e. $\frac{d P_k(t)}{dt} = P_{k-1}(t)\lambda_{k-1} - \lambda_k P_k(t)$ for $k \geq 1$ ………….. 3.17

$\frac{dP_0(t)}{dt} = -\lambda_0 P_0(t)$ for k=0 ……………………….3.20

$$Let\ the\ time\ t = 0$$

In equation (3.17) and (3.20), the birth rate is dependent upon the state of the system. Let us assume a constant birth rate λ which is independent of the state of the system and then we get a Poisson process. The governing equations of a Poisson process are —

$$\frac{d P_k(t)}{dt} = \lambda P_{k-1}(t) - \lambda P_k(t) \text{ for } k \geq 1 \quad \text{... (3.21)}$$

$$\frac{d P_o(t)}{dt} = -\lambda P_o(t) \text{ for } k = 0 \quad \text{... (3.22)}$$

Now, we assume certain boundary conditions *i.e.*, at time $t = 0$, the system is in state zero or no births have taken place. So, we have,

$$P_k(0) = \begin{cases} 1 & \text{for} \quad k = 0 \\ 0 & \text{for} \quad k \neq 0 \end{cases}$$

Thus, equation (3.22) can be written as,

$$P_o(t) = e^{-\lambda t} \quad \text{... (3.23)}$$

From equations (3.21) and (3.23) we get, for $k = 1$,

$$\frac{d P_1(t)}{dt} = -\lambda P_1(t) + \lambda e^{-\lambda t}$$

$\Rightarrow$ $P_1(t) = \lambda t e^{-\lambda t}$

for $k = 2$,

$$P_2(t) = \frac{(\lambda t)^2 e^{-\lambda t}}{2!}$$

Thus, the general solution or equation is,

$$\boxed{P_k(t) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}} \quad \text{... (3.24)}$$

The equation represents the probability of k arrivals in the time interval t and equation (3.23) express the probability of zero arrival in a given time interval or probability distribution of inter arrival times *i.e.*, the time that elapses between two arrivals. So, in a Poisson Process, the inter arrival time is exponentially distributed as interstate transition times in case of Markov process.

BLOCKING MODELS AND LOSS ESTIMATES

The behavior of loss system is studied by using blocking models. In loss system the over flow of traffic is rejected i.e. the overflow of traffic experiences a blocking from the network[2] . There are three waysin which overflow traffic may be handled:

1. The traffic rejected by one set of resources may be cleared by another set of resources in the network.
2. The traffic may return to the same resource after some time.
3. The traffic may be held by the resource as if being serviced but actually serviced only after the resources become available.

Corresponding to the above three cases, we consider three models of loss systems:

a. Lost calls cleared (LCC)
b. Lost calls returned (LCR)
c. Lost calls held (LCH)

a. Lost calls cleared (LCC):

a.1. Lost calls cleared system with infinite sources :

Let's assume the LCC system using infinite number of subscribers. This model is well suited for study of the behavior of the trunk transmission systems. Usually there are many trunk groups emanating from a switching office and terminating on adjacent switching offices. Whenever a direct trunk group between two switching offices is busy, it is possible to divert the traffic via other switching offices using different trunk groups. In this way the blocked calls in one trunk group are cleared via other trunk group

The LCC model assumes that the subscriber on hearing the engaged tone, hangs up and waits for some length of time before reattempting. He does not reattempt immediately or within a short time. Such calls are considered to have been cleared from the system and the reattempts are treated as new calls.

The LCC model was first studied by A. K. Erlang in 1917.The main purpose of the analysis is to estimate the blocking probability and the grade of service.Consider the Erlang loss system with N fully accessible lines and exponential holding times. The Erlang loss system can be modeled by birth and death process with birth and death rate as follows.

$$\lambda_k = \begin{cases} \lambda, k = 0, 1, \ldots\ldots N-1 \\ 0 \quad k \geq N \end{cases} \qquad \text{equation 1}$$

$$\mu_k = \begin{cases} k\mu, k = 0, 1, \ldots\ldots N \\ 0, \quad k > N \end{cases} \qquad \text{equation 2}$$

From $$\mathrm{P}(k) = \frac{\lambda_0 \lambda_1 \ldots\ldots \lambda_{k-1}}{\mu_1 \mu_2 \ldots\ldots \mu_k} \mathrm{P}(0), k = 1, 2, 3$$

Substituting equation 1 and 2 in the above equation, we get

$$P(k) = \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k P(0),\ k = 1, 2, 3, \ldots\ldots N$$

From equation (8.4), the offered traffic is

$$A = \frac{\lambda}{\mu}$$

$$P(k) = \frac{1}{k!}(A)^k P(0),\ k = 1, 2, 3, \ldots\ldots N$$

The probability P(0) is determined by the normalization condition

$$\sum_{k=0}^{N} P(k) = P(0) \sum_{k=0}^{N} \frac{A^k}{k!} = 1 \quad \text{equation 3}$$

$$P(0) = \frac{1}{\sum_{k=0}^{N} \frac{A^k}{k!}} \quad \text{equation 4}$$

Substituting equation 3 in 4 we get

$$P(k) = \frac{A^k / k!}{\sum_{k=0}^{N} \frac{A^k}{k!}}$$

The probability distribution is called the truncated **Poisson distribution** or **Erlang's loss distribution.** In particular when k = N, the probability of loss is given by

$$P(N) = B(N, A) = \frac{A^N}{N! \sum_{k=0}^{N} \left(\frac{A^k}{k!}\right)} \quad \text{where } A = \lambda/\mu .$$

This result is variously referred to as **Erlang's formula of the first kind, the Erlangs-B formula or Erlangs loss formula.**

A.2 Lost Calls Cleared system with finite subscribers:

In case of LCC model with finite subscribers, the call arrival rate is dependent on the number of subscribers, who are not occupied as the busy subscribers do not generate new calls. The traffic in this case is known as Engest traffic or pure chance traffic type 2.

Let, λ_R = arrival rate per subscriber

K = number of busy subscribers.

S = number of server

N = total number of subscriber.

The offered traffic (arrival rate) when the system is in state k is given by,

$$R_k = (N - K)\lambda_R \qquad \text{for} \quad k \leq 0 \leq S$$

The mean offered traffic rate is given by,

$$R = \sum_{k=0}^{S} (N-k)\lambda_R P_k = N\lambda r \sum_{k=0}^{S} P_k - \lambda_R \sum_{k=0}^{S} kP_k$$

$$= \lambda_R \left(N - \sum_{K=0}^{S} kP_k \right)$$

$$\left[\sum kP_k \text{ represents the average number of busy servers} \right]$$

$$= \lambda_R (N - A_o)$$

The offered traffic is,

$$A = Rt_n = \lambda_R t_h (N - A_o)$$

when the system is in state S, the offered traffic rate is $(N - S)\lambda_R$, but all the arrivals are rejected. so, the lost traffic.

$$A - A_o = (N - S)\lambda_R P_R t_h$$

Therefore, the grade of service,

$$GOS = \frac{N - S}{N - A_o} P_R$$

So, we see that Engest traffic, the blocking probability and the GOS are not same or we can say that the time congestion and the call congestion values are different.

By analyzing the steady state characteristics of B-D process, we can calculate the blocking probability and grade of service. The expressions are as follows—

$$P_B = \frac{P^R \binom{N}{S}}{\sum_{k=0}^{R} P^k \binom{N}{K}}$$

and

$$GOS = \frac{P^R \binom{N-1}{S}}{\sum_{k=0}^{R} P^k \binom{N-1}{K}}$$

Lost Calls Returned System:

LCR model based on the assumption that the blocked calls return to the system as a form of retries.So that the offered traffic comprisestwo components:

Offered traffic = new traffic + retry traffic

The following assumptions are made to analyse the LCR model with regard to the nature of returning calls [1] .

I. Retries calls get the service even if multiple retries are required.

II. No new call is generated when a blocked call is being retried.
III. Time between call blocking and regeneration is random stastistically independent of each other.
IV. Typical working time before a retry is longer than the average holding time .
V. If retries are immidiate ,congestion may occur or the network operation becomes delay system.

Suppose a system with first attempt call arrival ratio of λ=100 (say).If a percentage B of the call is blocked,B times λ retries.

Thus, the infinite series, total arrival rate λ′ is given as,

$$\lambda' = \lambda + B\lambda + B^2\lambda + B^3\lambda + \ldots\ldots$$

$$\boxed{\lambda' = \frac{\lambda}{1-B}}$$

where B is the blocking probability.

Similarly we can say

$$\boxed{\lambda' = \frac{\lambda}{1-GOS}}$$

The effect of returning traffic is insignificant when operating at low blocking probabilities or grade of service.

C. Lost Calls Held System:

I. In this case, the blocked calls are held in a queue and serviced when necessary facilities become available.
II. In this system, the total time is not dependent on the waiting time. The total time is determined by the average service time required.
III. The sources need service continuously for a period of time whether or not the services are available. As the service becomes free from other calls it handles the queued calls.
IV. If a number of calls blocked, a portion of it is lost until a server become free to service a call as shown in figure

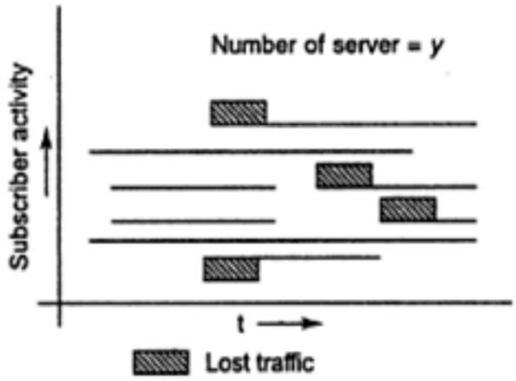

Fig22: Lost traffic in LCH Model. [1]

LCH model is used to find the probability of the total number of calls in the system at a particular time. The number of active sources is identical to the number of call arrivals in that particular time.
i.e. given as Poisson distribution equation

$$P(x) = \frac{\mu^x}{x!} e^{-\mu}$$

DELAY SYSTEMS:

1. In telecommunication network, such as data network places the call or messages in a queue in the absence of resources and services them when the resources become available. Such systems are known as delay systems, which are also called lost call delayed systems [1]. For example:
   - Message switching
   - Packet switching
   - Digital receiver access
   - Automatic call distribution
   - Call processing

2. Delay systems are analyzed using queuing theory or waiting line theory. The elements of a queuing system are shown in figure

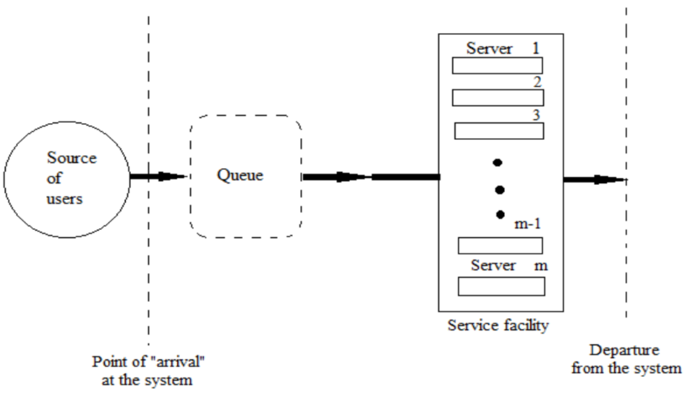

Fig23: Queuing diagram of users waiting to be served. [1]

There is a large number of populations of sources that generates traffic or service requests to the network. There is a service facility that contains a number of identical servers, each of which is capable of providing the desired service to a request. When all the servers are busy, a request arriving at the network is placed in a queue until a server becomes available. Hence
$K = k_q + R$, i.e. the mean time a call or a request spends in the system is the sum of the mean wait time t_q and mean service or holding time t_h.

3. If a delay system has infinite queue capacity during operation then a necessary condition for the system is stable is:
$\frac{mean\ arrival\ rate}{mean\ service\ rate} < 1$ or $\frac{offered\ traffic}{number\ of services} < 1$

4. A queuing system is characterized by a set of six parameters such as A/B/c/K/m/Z and the parameter specifications are
A= Arrival process specification (values are GI, G, ER, M, D, Hk)
B= Service time distribution (values are GI, G, E_R, M, D, H_k)
c = number of servers (nonzero positive finite number)
K= queue capacity (may or be finite an infinite number)
m = number of sources (input population) (may or be finite an infinite number)
Z=service discipline (first-come-first-served (FCFS))

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