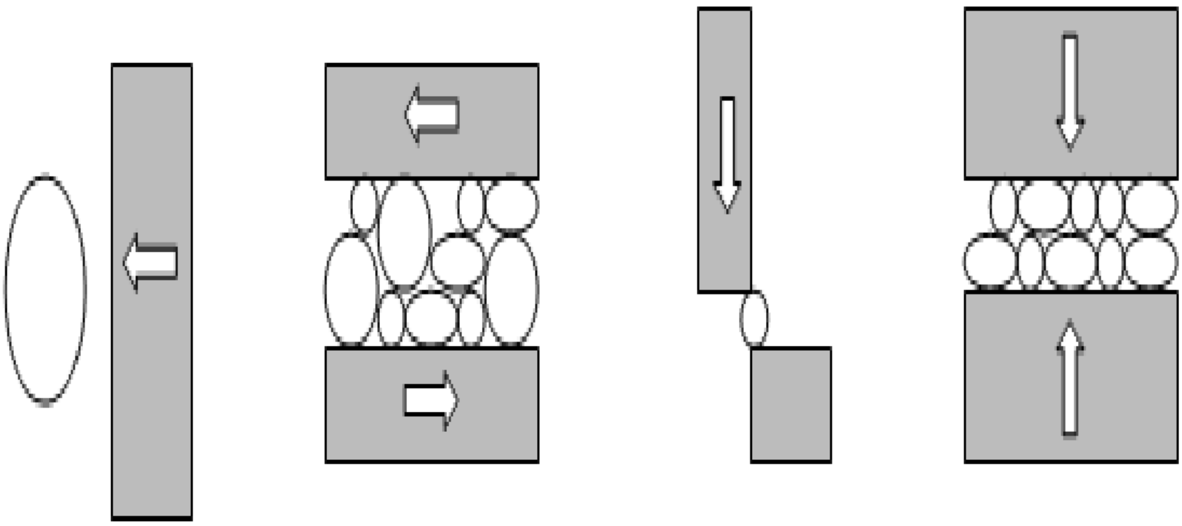
## Differential and Cumulative Analysis Continued

# SIZE REDUCTION OR COMMINUTION

## Introduction

- Solid particles are cut or broken into small pieces throughout the process.
- Sizes of the solids are reduced into different methods for various process.
- It may be broken into 8 or 9 ways.
- Size reduction machines by applying forces like
    - Compression force -> Not crackers (coarse)
    - Impact force -> Pounding with hammer(coarse, intermediate, fine)
    - Attrition or rubbing force -> file (ultra fine)
    - cutting force -> pair of shears
    - Tensile or shear force



(a) (b) (c) (d)

(a) Impact
(b) Attrition
(c) Shear
(d) Compression

## Principle of comminution

- Criteria for comminution.
- Character of comminution.
- Energy and power requirements for comminution.

## Criteria for comminution

Comminution is a generic term for size reduction crusher and grinders are types of comminuting equipments. An ideal crusher or grinder would have

- Large capacity
- Require small power input for unit of product
- Yield a Product of single size

### Characteristics of comminuted products

- Objective of crushing and grinding into produce small particles from larger ones.
- Energy measured by the new surface area created by reduction in size.
- Feed is homogenous -> shape, chemical and physical structure of the product may be quite uniform.

### Energy and power requirements for comminution

- Cost of power is a major expense in crushing and grinding operation. Therefore we must consider the factor which controls the cost.

## Crushing efficiency ($\eta_c$)

It is defines as the ration of product of surface area created to the energy absorbed by the solids

$\eta_c = [e_s(A_{wb} - A_{wa})]/W_n$

$e_s$ - surface energy

$A_{wb}$ - product size

$A_{wa}$ -Feed size

$w_n$- energy absorbed by solids

Mechanical effiency ($\eta_m$)

Energy absorbed by solids ($w_n$) is less than that of feed into the machinery parts of total energy (w) which is used for rotating the machine, bearing and in other cooling parts.

Rest of energy used for material crushing. It is defined as the ratio of surface energy and new surface area created to the energy input

$\eta_m=[e_s(A_{wb}-A_{wa})]/_cW$

$\eta_c W$=energy input

Total energy $W= [e_s(A_{wb}-A_{wa})]/\eta_c\eta_m$

If m is the feedway then the power required by machinery is given by

$P=[me_s(A_{wb}-A_{wa})]/n.n$

We know that

Total surface area $(A_w) = 6m/\phi_s \rho_p D_p$

$P= me_s/\eta_c\eta_m[(6/\phi_s \rho_p D_{sb})-( 6/\phi_s \rho_p D_{sa})]$

$P=6\ me_s/\eta_c\eta_m\phi_s \rho_p[1/D_{sb}- 1/D_{sa}]$

## Law of comminution or size reduction laws

### Rittingers law

The work required for crushing is directly proportional to the new surface area created

we know that,

$P=6\ me_s/\eta_c\eta_m\phi \rho_p[1/\phi_{sb}D_{sb}- 1/D_{sa}\phi_{sa}]$

If the $\phi_{sb}$ and $\phi_{sa}$ are equal, mechanical effiency is also constant for particular machine, then the various constant in the above equation can be combined as a single constant $K_r$ (Rittingers constant).

$p/m= K_r[1/D_{sb}- 1/D_{sa}]$

## Kick's law

The energy required for crushing a given mass of material is directly proportional to the logarthamic reduction ratio.

### Reduction ratio

A ratio of initial particle size to final particle size, also this law states energy required the same when the reduction ratio remains the same.

$p/m= K_b \ln [D_{sa} / D_{sb}]$

## Bond's law

The work required to form particle of size $D_p$ from a very large feed is proportional to the square root of surface to volume ration of the particle in product($S_p/V_p$).

$p/m \, \alpha \sqrt{sp / vp}$

$S_p=\pi D_p^2 \quad V_p=D_p^3/6$

$S_p/V_p=\pi D_p^3/6$

$p/m \, \alpha \sqrt{6 / Dp}$

$p/m=K_b[1/\sqrt{Dsb} -1/\sqrt{Dsa}]$__________ 1

$K_b$=Bonds constant depends on type of machine

## Work Index ($W_i$)

It is defined as gross energy in Kw-hr / tonne for a particle of very large feed is reduced to such a size that 80% of product pass through 100micron screen

$p/m=K_b[1/\sqrt{Dp}]$

$W_i= K_b[1/0.3162]$

$K_b=0.3162W_i$_________ 2

Substitute 2 in 1

$p/m=0.3162W_i[1/\sqrt{Dsb} -1/\sqrt{Dsa}]$

$W_i$ includes the friction in crusher and power given by the above equation is gross power.

**List of Crushing equipments**

| Coarse Crushers | Intermediate crushers | Fine grinders |
|---|---|---|
| Stage or Blake | Crushing rolls | Colloidal mill |
| Dodge jaw crusher | Disk crusher | Ball mill |
| Gyratory crusher | Edge runner | Rod mill |
| | Mill, Hammer mill | Tube mill |
| | Pin mill | Hardinge mill, fluid energy mill |

| Type | Feed size | Product size |
|---|---|---|
| Coarse crushers | 150-5 cm | 5cm-0.5cm |
| Intermediate crushers | 5-0.5cm | 0.5cm-200mesh |
| Fine grinders | 0.5cm-0.2cm | 200 mesh |
| Ultra fine grinding | 80 mesh | $0.01\times10^{-6}$ |
| cutters | Defined ore Irregular shape | Defined ore Regular shape |

## Alternative derivation in crushing laws

A number of empirical law have been put forward to estimate the amount of energy required for size reduction,

Rittinger's law, Bond's law, Kick's law

Basic equation for deriving laws is

$dE/dL = -CL^{p}$_______ 1

Above equation states that energy dE requires to effect a small change dL in size for a unit mass of material is a simple power function of size.

Put P=-2 in equation 1

$dE/dL = -CL^{-2}$

$dE/dL = -C/L^{2}$

$$\int_{0}^{E} dE = \int_{L1}^{L2} -C/L^{2}dL$$

$$= -C[(1/L_2)-(1/L_1)]$$

Putting $C= k_r f_c[(1/L_2)-(1/L_1)]$________ 2

**Hence we obtained Rittingers's law**

put P=-1 in 1

$dE/dL= -CL^{-1}$

$dE=-CL^{-1}dL$

$E=C \ln[L]_{L1}^{L2}$

$E=C \ln(L_1/L_2)$

put $C=k_k f_c$

$E=k_k f_c \ln(L_1/L_2)$________ 3

**Hence we obtained Kick's law**

Put P=-3/2

$dE/dL=-CL^{-3/2}$

$dE=-CL^{-3/2}dL$

$$E=C \int_{L1}^{L2} L^{-3/2}dL$$

$=2C[(1/\sqrt{L2}-1/\sqrt{L1})]$

Put $2C = k_b f_c$

$E=k_b f_c[(1/\sqrt{L2}-1/\sqrt{L1})]$________ 4

**Hence we obtained Bond's law**

$dE/dD_p=aD_p^{-b}$(b=2, b=1, b=3/2)

REFERENCES:


- WHO Regional Office for Europe "Health Aspects of Air Pollution" (2003), Particulate matter (PM), Section 5.1 Introduction

- Mechanical Eng. Msc Suphi Yavuz, Başak Yavuz Makine, Technical Notes

- Mining Eng. Msc Necati Yıldız, Ore Dressing and Enrichment Book

- R. Hogg, H. Cho, in Encyclopedia of Materials: Science and Technology, 2001

- Mohamed Rahaman; Mohamed N. Rahaman (7 August 2006). Ceramic Processing. CRC Press. pp. 41–. ISBN 978-0-8493-7285-8.

- John B. Wachtman (28 September 2009). Materials and Equipment - Whitewares Manufacturing: Ceramic Engineering and Science Proceedings, Volume 14. John Wiley & Sons. pp. 264–. ISBN 978-0-470-31618-4.

- Hammer Milling and Jet Milling fundamentals, Gary Liu, CEP, 2017

- Air jet Milling, Chamayou, Elsevier, 2007