

## Concepts of Stress and Strain

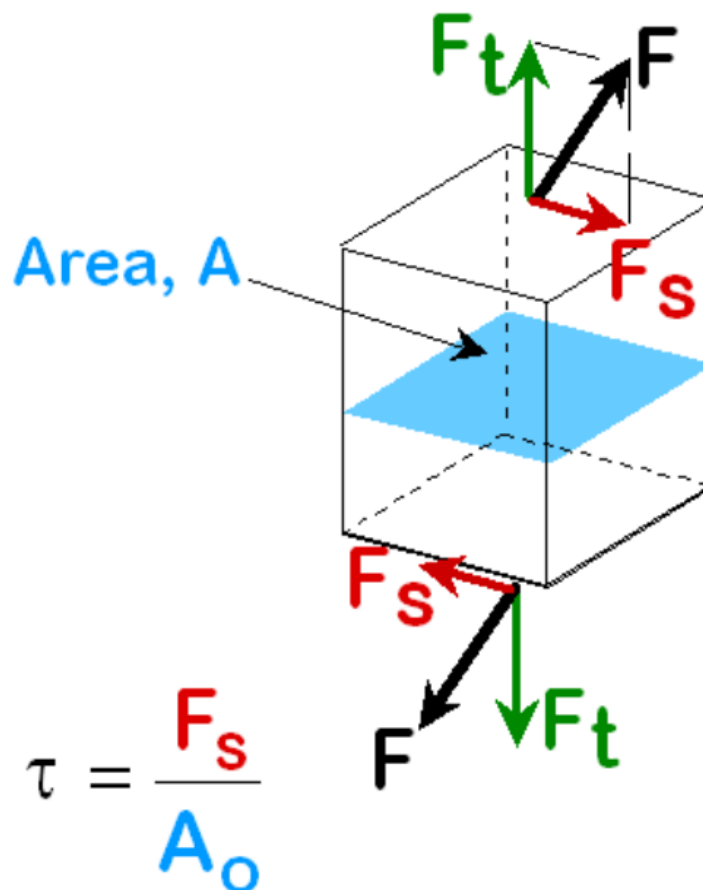
To compare specimens of different sizes, the load is calculated per unit area, also called normalization to the area. Force divided by area is called stress. In tension and compression tests, the relevant area is that perpendicular to the force. In shear or torsion tests, the area is perpendicular to the axis of rotation.

$$= F/A_0 \text{ tensile or compressive stress}$$

$$= F/A_0 \text{ shear stress}$$

Shear stress – shear – applied load is in plane with the object or parallel to the object.

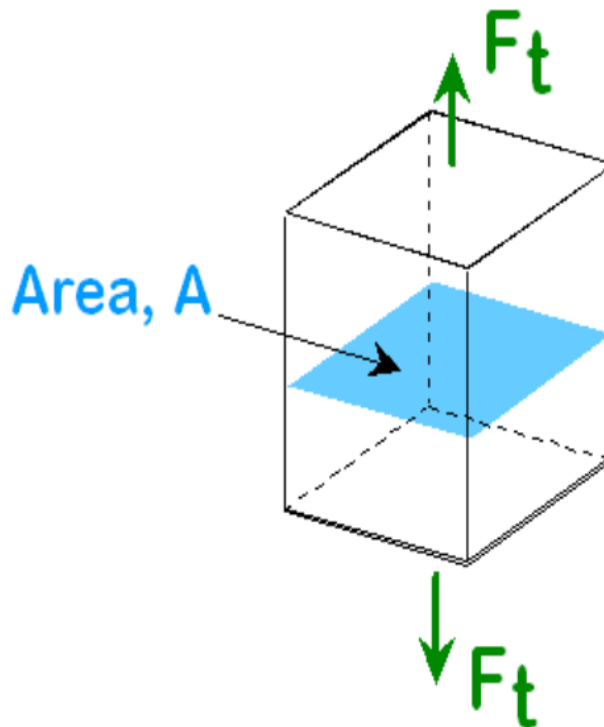
Applied load is parallel to area supporting it and its dimension change is perpendicular to reference dimension is shear strain.



Strain – the amount of deformation that takes place in a stressed body eg. Tensile test

The extension for a given load varies with the geometry of the specimen and its composition. It is difficult to compare stiffness, load carrying capacity. To resolve the load and deformation is normalized. There is a change in dimensions, or deformation elongation,  $L$  as a result of a tensile or compressive stress. To enable comparison with specimens of different length, the elongation is also normalized, this time to the length  $L$ . This is called strain,  $\epsilon$ .

Tensile stress – the load applied is divided by the cross sectional area available to support the load. The extension is divided by original length of the specimen (Normalized load)  $\sigma = F / A$

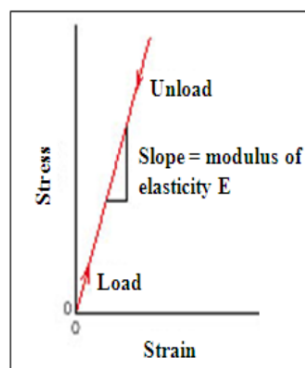


Strain – The deformation is reported as elongation per unit of the original length over which the elongation occurred

Elastic behavior – The basic experiment to determine the mechanical property is tensile test. In 1678, Robert Hooke – showed when a material is subjected to tensile (distraction) force would extend in the direction of traction by an amount that was proportional to the load. This is Hooke’s law. Most solids behave in an elastic manner (like a spring) if the loads are not too great.

### 1. Stress—Strain Behavior

**Elastic deformation.** When the stress is removed, the material returns to the dimension it had before the load was applied. Valid for small strains (except the case of rubbers).



Higher E → higher “stiffness”

Deformation is *reversible, non permanent*

**Plastic deformation.** When the stress is removed, the material does not return to its previous dimension but there is a *permanent, irreversible* deformation.

Tension and compression – if weights are used to provide the applied load, stress is calculated by adding the total number of pounds used and dividing the perpendicular cross sectional area.

A micrometer is used to determine the dimension unit is  $N/m^2 = F / A$

Strain – It is measured by applying reference marks to the specimen and measuring the distance between with calipers. If original length is  $l$ , a load is applied and the distance between the marks is measured again to find the final length  $l_0$ . Strain

$$e = \frac{\Delta L}{L} = \frac{l - L}{L}$$

Eg – rubber, polymer, soft tissues

E & G are linked to spring constants.

E – Tensile / young's modulus, G – shear modulus

Elastic strain – achieved by actually increasing the interatomic distances in the crystal (stretching the bond). For diamond  $Al_2O_3$ , tungsten – moduli are increased for material with strong bond and given stress produce only a small strain. For material with weaker bonds (polymer and gold) the moduli are lower.

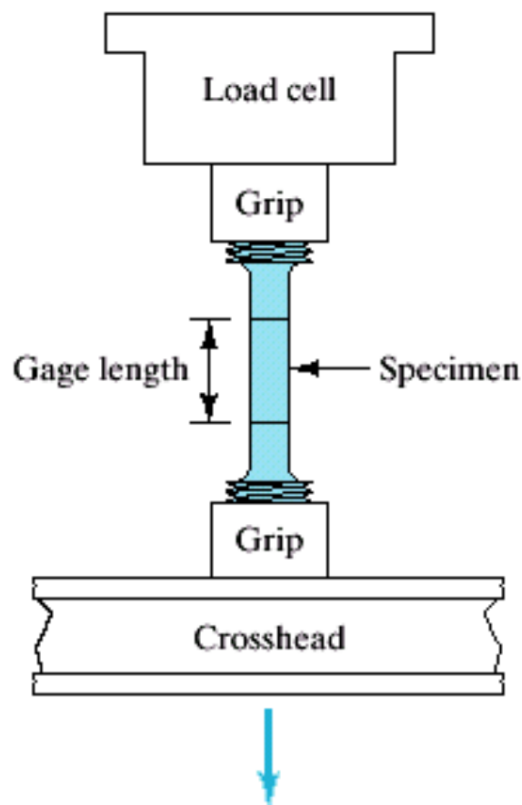
Elastic deformation – it regain its original position the load is removed. Eg. Rubber band, any synthetic material.

Isotropy – Two constants E & G – needed to fully characterize the stiffness of an isotropic material (i.e) a material whose properties are same in all directions. Single crystals are anisotropic – the stiffness varies as the orientation of applied force changes relative to the interatomic bond directions in crystal.

Polycrystalline material – eg. Most metallic & ceramic specimen) a great multitude of grains (crystallites) are aggregated with multiple distributed orientations. These aggregates exhibit isotropic behavior at macroscopic level and values of E & G are highly reproducible for metal, alloy or ceramic.

Polymeric material - tissue samples are anisotropic (not same in all directions) even at macroscopic level. Bone, ligament, sutures are stronger and stiffer in fiber (longitudinal) direction than in transverse direction. For such materials more than 2 elastic constants are required to relate stress and strain properties.

Mechanical testing – to conduct controlled load – deflection (stress –strain) tests a load frame is used that is much stiffer and stronger than the specimen to be tested. One cross bar / cross head is moved up & down by a screw or a hydraulic piston. Jaws that provide attachment to the specimen are connected to the frame and to the movable cross head. Load cell is placed in series with the specimen to monitor the force being applied. It functions like a stiff spring scale to measure the applied loads. For compression testing, the direction of cross head movement is reversed and cylindrical or prismatic specimens are simply squeezed between flat anvils. Standardized specimens should be used for all mechanical testing to ensure reproducibility of results.



Bend test – Outside of the bowed specimen is in tension and inside in compression. The outer fiber stresses can be calculated from the load and specimen geometry. Strain gauge is used for outer fiber strain. Some mechanisms apply torsion (rotational) loads, where torque versus angular deflection is determined and used to calculate torsion properties of materials.

Elasticity – tensile elastic modulus  $E$ , is determined by strain gauge, load cell

$$E = \sigma / \epsilon$$

Brittle fracture – In real materials, elastic behavior does not persist indefinitely. The microscopic defects will grow rapidly under the influence of applied tensile or shear stress and the specimen will fail suddenly by brittle fracture.

Stress at which failure occurs is called fracture stress. Eg. Glass, ceramic, graphite, bone cement, PVC. The number and size of defects particularly pores, affects the strength of brittle materials.

Plastic deformation – large scale displacement of atoms without complete rupture of a material is possible in the presence of metallic bond so only metals and alloys exhibit true plastic deformation.

For metals, alloys plastic deformation occurs before fracture occurs. Once plastic deformation starts, strain produced is very greater than those during elastic deformation and they are not recovered when stress is removed.

This happens because whole array of atoms under the influence of an applied stress are forced to move to new locations in crystal.

Since long distance re-arrangement of atoms under the influence of an applied stress cannot occur in ionic / convolutedly bonded materials, ceramic and many polymers exhibit only brittle behavior.

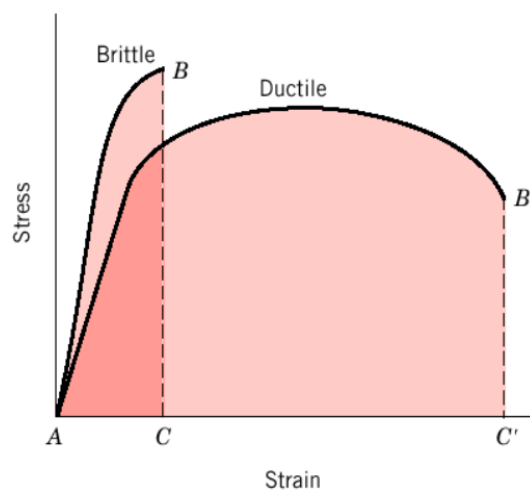
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Plastic deformation is useful in shaping metals and alloys and is called ductility or malleability. The permanent strain (plastic) exhibited upto fracture by a material is a quantitative measure of its ductility.

Area under tensile curve is proportional to work required to deform a specimen unit it fails. Area under entire curve  $\propto$  stress x strain

The work to fracture is a measure of toughness and reflects a material resistance to crack propagation. **Toughness: ability to absorb energy up to fracture (Area under the strain-stress curve up to fracture) Units: the energy per unit volume, e.g.  $J/m^3$**



Creep – When a weight is suspended from a specimen ligament, the ligament continues to elongate for a considerable time even though the load is constant. This continuous, time dependent extension under load is creep.

If the ligament is extended in a tensile mechanism to a fixed elongation and load is monitored, the load drops continuously with time. The continuous drop in load at constant extension is stress relaxation. Creep and stress relaxation are result of viscos flow in the material. The mechanical analog of viscous flow is a dash pot or cylinder and piston.

Fatigue – Progressive and localized structural damage that occurs when a material is subjected to cyclic damage.

Microstructure - structure in solids occurs in a hierarchy of sizes. Internal or electronic structure of atoms occurs at finest scale  $< 10^{-4}$   $\mu\text{m}$  responsible for interatomic bonds. Next higher size level around  $10^{-4}$   $\mu\text{m}$  – 3 dimensional arrangement of atoms in crystal, glass are observed.

Large size  $10^{-3}$  to  $10^{-2}$   $\mu\text{m}$  – important type of structural organization exists. When atoms of molten sample are incorporated into a crystal during freezing, many small crystals are formed, and then grow until they impinge on each other and all liquids are used up. The sample becomes complete solids, thus most crystalline solids have small crystals called grains that are tightly packed called microstructures.

In general grains are too small to be viewed by light microscope. A metallographic or ceramographic reflecting microscope is used. Incident light is reflected from polished metal or ceramic surface.

Grain structure is revealed by itching the surface with a mildly corrosive medium that attacks grain boundaries and grains size and shape is viewed through reflecting microscope.

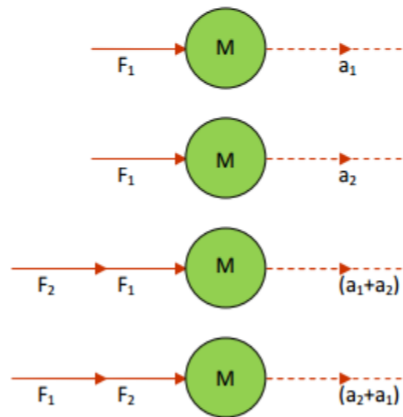
Revealing the microstructure by microscope evaluates 2 features

1. Grain size – fine grained samples stronger than coarse grained specimen of a given material.
2. Co-existence of two or more phases in some solid materials. The grain of a given phase has some chemical composition and crystal structure but the grains in second phase will be different.

Eg. Alloys (first phase have same chemical composition and crystal structure while other phase will differ) stainless steel (Fe+Cr), amalgam (Hg based), implant alloys (Ti & cobalt)

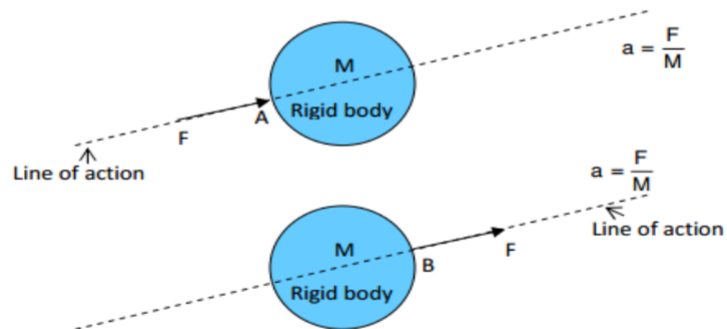
Force – External agency, which overcomes or tends to overcome the inertia of a body. It is a vector quantity. Element of Force – Magnitude, direction, line of action, Point of action / application

Principle of superposition of forces



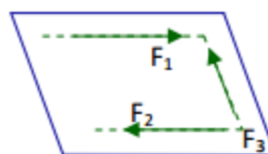
Net effect of forces applied in any sequence on a body is given by algebraic sum of effect of individual forces on the body.

Principle of transmissibility of forces.

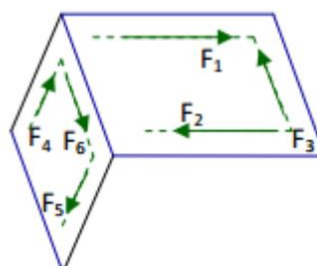


The point of application of a force on a rigid body can be changed along the same line of action maintaining the same magnitude and direction without affecting the effect of force on the body.

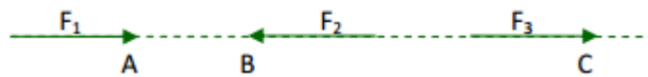
Types – 1. Coplanar force system - if lines of action of forces forming the system lie in same plane.



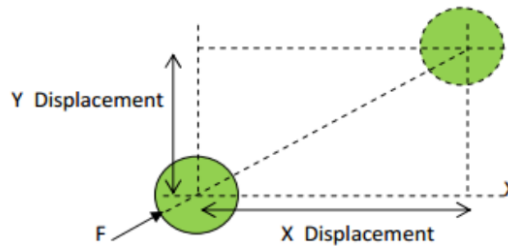
2. Non planar forces – if lines of action of forces forming the system do not lie in same plane.



3. Collinear system – if forces forming the system have a common line of action

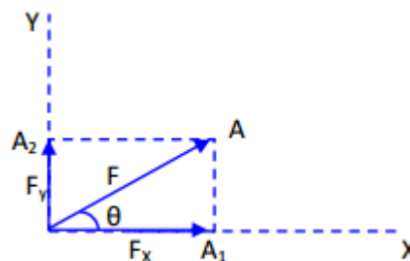


Resolution of a force – the technique of finding a component of a force along any direction is resolution of force. The component of a force along any direction is called resolved component.



The force F is producing, simultaneously X displacement, Y displacement. The part of force F which is producing X displacement is called X component or horizontal component of force ( $F_x$ ). The part of Force F which produces Y displacement is called Y component of force or vertical component of force F ( $F_y$ ).

The components of force determined along two mutually perpendicular direction is called rectangular components. To resolve a force along any direction



$\theta$  is the acute angle made by force with reference to X direction

$$\cos \theta = \frac{OA_1}{OA}$$

$$\cos \theta = \frac{F_x}{F}$$

$$F_x = F \cos \theta \quad (\rightarrow)$$

$$\sin \theta = \frac{AA_1}{OA} \quad \sin \theta = \frac{AA_2}{OA}$$

$$\sin \theta = \frac{F_y}{F} \quad F_y = F \sin \theta \quad (\rightarrow)$$

X component of a force is given by product of magnitude of force and cosine of acute angle made by force with reference to X direction.

Y component of force is given by product of magnitude of force and sine of acute angle made by force with reference to X direction

$$\begin{aligned}\theta &= 0 \\ F_x &= F \cos 0 & F_y &= F \sin 0 \\ &= F & &= 0\end{aligned}$$

The horizontal component or X component of a force acting along X direction is the force itself, whereas its vertical component is '0'.

$$\begin{aligned}F_x &= F \cos 90 & F_y &= F \sin 90 \\ &= 0 & &= F\end{aligned}$$

X component of force acting along Y direction is zero whereas Y component is equal to itself (F). If a force is inclined at 45° with reference to XY axis then its component is equal to Y component.

$$F_x = F_y$$

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