

# LIMIT STATE DESIGN OF BEAMS FOR FLEXURE

## REVIEW OF BASIC CONCEPTS

Concrete is a mixture of cement, aggregates, water and admixtures in an adequate proportion. It is a rock like material.

**Properties of Fresh Concrete:** Concrete should be such that it can be transported, placed, compacted and finished without harmful segregation. The mix should maintain its uniformity and not bleed excessively; these two are collectively called as workability .Bleeding is movement and appearance of water at the surface of freshly-placed concrete, due to settlement of heavier particles. Consistency is a measure of its wetness and fluidity .Measured by the slump test and Workability dependent on water content, fineness of cement, and surface area of aggregates.

**Properties of Hardened Concrete:** The important properties of concrete are

- Compressive strength
- Tensile strength
- Shear strength
- Bond strength
- Density
- Impermeability
- Durability
- Ductility

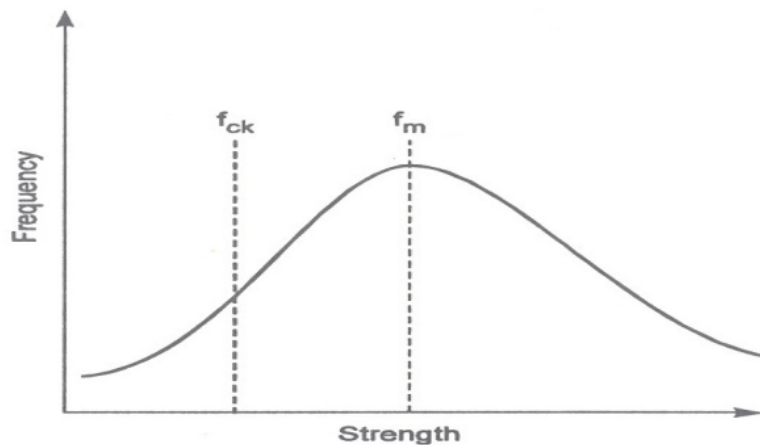
Among these properties, compressive strength of concrete is the most valuable and can be easily tested in laboratory. This is generally measured on concrete cubes or cylinders.

- Many of the properties of concrete can be inferred from compressive strength, using correlation that has been established.
- Quality of concrete depends on the compressive strength.

- Dependent on strength (compressive, tension and flexure), Modulus of elasticity, Durability, Creep and shrinkage
- Concrete is classified under different grades depending upon its characteristic strength.

***“Characteristic strength is defined as the strength of material below which not more than 5 percent of the test results are expected to fall”.***

- Strength of concrete varies for the same concrete mix, which give different compressive strength in laboratory tests.
- Variability in strength evidently depends on degree of quality control.
- Variability in strength is measured in terms of either the “Standard Deviation” or the Coefficient of Variation (COV), which is the ratio of standard deviation to mean strength( $f_{cm}$ ).



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- Due to significant variability in strength, it is necessary to ensure that the designer has a reasonable assurance of a certain minimum strength of concrete.
- Characteristic strength provides minimum guaranteed strength.

### **Different methods of design of RCC**

1. Working Stress Method
2. Limit State Method
3. Ultimate Load Method
4. Probabilistic Method of Design

### **Working Stress Method:**

- The sections of the members of the structure are designed assuming straight line stress-strain relationships ensuring that at service loads the stresses in the steel and concrete do not exceed the allowable working stresses.
- The allowable stresses are taken as fixed proportions of the ultimate or yield strength of the materials.
- The B.Ms and forces that act on statically indeterminate structures are calculated assuming linear – elastic behaviour.
- Reinforced concrete sections behave in elastically at high loads. Hence elastic theory cannot give a reliable prediction of the ultimate strength of the members because inelastic strains are not taken into account.
- For structures designed by the working stress method, the exact load factor is unknown and varies from structures to structure.

### **Ultimate Load Method**

- Sections of members of the structures are designed taking inelastic strains into account to reach ultimate (maximum) strength when an ultimate load, equal to the sum of each service load multiplied by its respective load factor, is applied to the structure.
- The beginning moments and forces that act as statically indeterminate structures at the ultimate load are calculated assuming non linear elastic behaviour of the structure up to the ultimate load. i.e., redistribution of same actions are taking place due to nonlinear relationship between actions and deformations.
- Ultimate strength design makes more efficient use of high strength reinforcement and smaller beam depths can be used without compression steel.
- Ultimate strength design allows the designer to assess the ductility of the structure in the post-elastic range.
- If the sections are designed based on ultimate strength design, there is a danger that although the load factor is adequate. The cracking and the deflections at the service loads may be excessive.
- Cracking may be excessive if the steel stresses are high or if the bars are badly distributed.
- Deflections may be critical if the shallow sections, which are possible in USD, are used and the stresses are high.
- To ensure a satisfactory design, the crack widths and deflections at service loads must be checked to make sure that this lies within reasonable limiting values, as per functional requirements of the structure. This is done by use of elastic theory.

### Limit state method of design

The object of the design based on the limit state concept is to achieve an acceptable probability, that a structure will not become unsuitable in its lifetime for the use for which it is intended, i.e. It will not reach a limit state

- A structure with appropriate degree of reliability should be able to withstand safely.
- All loads, that are reliable to act on it throughout its life and it should also satisfy the sustainability requirements, such as limitations on deflection and cracking.
- It should also be able to maintain the required structural integrity, during and after accident, such as fires, explosion & local failure. i.e. limit state must be considered in design to ensure an adequate degree of safety and serviceability
- The most important of these limit states, which must be examined in design are as follows
  - Limit state of collapse - Flexure, Compression, Shear and Torsion
  - Limit state of serviceability
- This state corresponds to the maximum load carrying capacity.

### Deflection Criteria

Deflection of structure or part thereof shall not adversely affect the appearance or efficiency of structure or finishes or partitions.

Deflection shall generally be limited to the following:

- i. Final deflection due to all loads including the effects of temperature, creep and shrinkage and measured from as-cast level of supports of floors, roofs and all other horizontal members should not normally exceed  $\text{span}/250$ .
- ii. Deflection including effects of temperature, creep and shrinkage occurring after erection of partitions and application of finishes should not normally exceed  $(\text{span}/350)$  or 20mm whichever is less.

Factors influencing limits on deflection in flexural members:

- Aesthetic/psychological discomfort
- Crack width limitation
- Effect on attached structural and non structural elements
- Ponding in (roof) slabs

### Basic Span-to-depth Ratios as per IS 456 – 2000

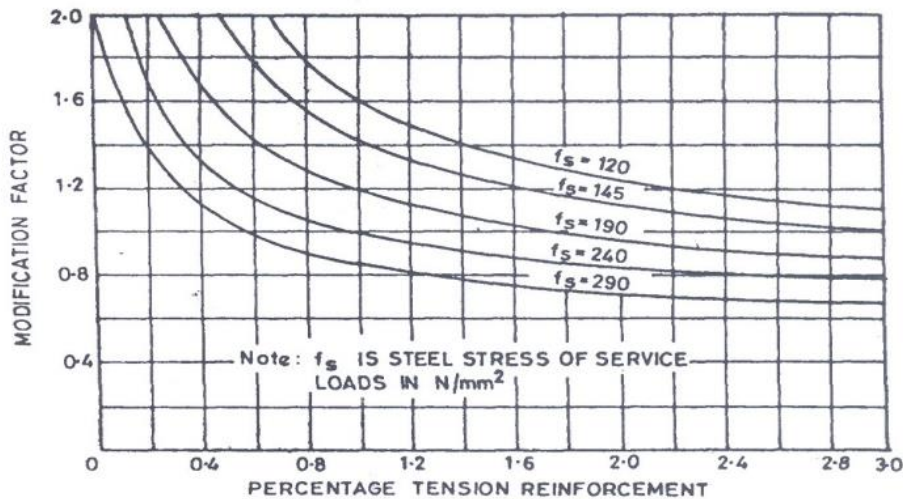
- a. Basic values of span-to-effective depth ratios for spans up to or less than 10m are
  1. Cantilever 7
  2. Simply supported 20

3. Fixed or Continuous

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- b. For spans above 10m, values in (a) may be multiplied by (10/span in meters), except for cantilever. For cantilever beams, the actual deflections should be calculated and the requirement for limit state of deflection be checked.

Depending on the area and the stress in steel reinforcement in tension, the values in (a) or (b) shall be modified by multiplying with the modification factor obtained as per Fig. 4.



$$f_s = 0.58 f_y \frac{\text{Area of cross-section of steel required}}{\text{Area of cross-section of steel provided}}$$

FIG. 4 MODIFICATION FACTOR FOR TENSION REINFORCEMENT

Depending on the area of compression reinforcement, the value of span-to-depth ratio is further modified by multiplying with the modification factor obtained as per Fig. 5.

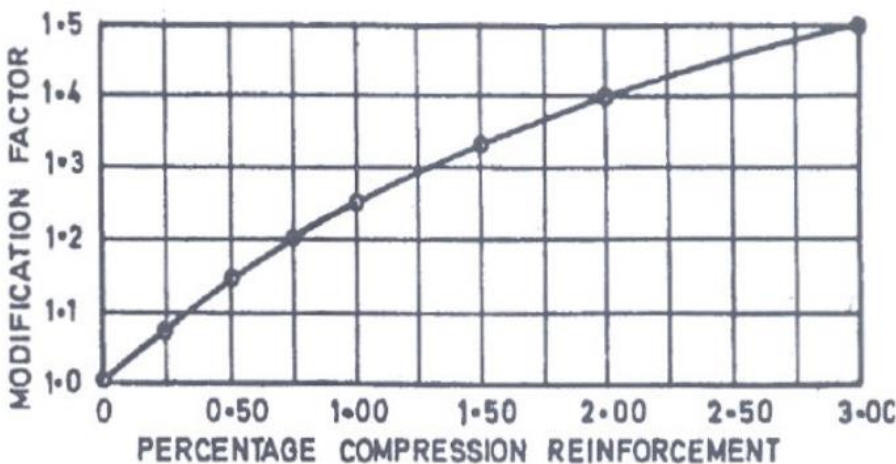


FIG. 5 MODIFICATION FACTOR FOR COMPRESSION REINFORCEMENT

For flanged beam, values of (a) or (b) be modified as per Fig. 6 and the reinforcement percentage for use in Fig.4 and 5 should be based on area of section equal to  $b_f d$ .

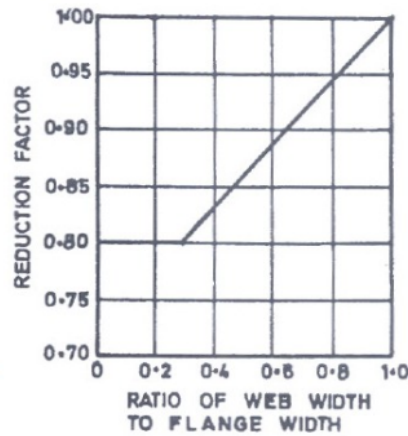


FIG. 6 REDUCTION FACTORS FOR RATIOS OF SPAN TO EFFECTIVE DEPTH FOR FLANGED BEAMS

Final Span-to-Effective Depth Ratio

- The final allowable span-to-effective depth ratio
  
- The ratio ( $L/d$ ) obtained should be as follows

### DESIGN FOR BOND IN RC

- Bond stress: stress developed between surface of steel reinforcement and surrounding concrete by which slip occurs when concrete and steel act together in structural member by transferring stress between each other.
- Slip: relative displacement between reinforcement and surrounding concrete.
- Bond stresses are, in effect, longitudinal shearing stresses developed on surface between steel and surrounding concrete wherever stress in a bar changes.
- Importance of determining bond stress in tensile reinforcing bars has increased due to the following reasons
  - more frequent use of high grade steel with larger bar diameter and
  - Adopting refined ultimate strength design procedures

### Distribution of Bond Stress

- Bond stress in a beam or wall is neither uniform nor gradually varying from point to point

- Very large bond stresses develop adjacent to tension crack
- Essentially ultimate bond stresses exist close by on the same bar, even reversed in direction in many cases.
- Thus there is a practical problem as to how to describe or measure or evaluate such a fluctuating stress condition.

#### Bond Mechanism:

Bond can be thought of as shearing stress or force between a bar and surrounding concrete

Bond resistance is offered by

- Chemical Adhesion – gel like hydration products
- Friction–surface roughness and concrete shrinkage
- Mechanical Interaction - ribs in deformed bars

Importance of Bond

- Ultimate Limit State
  - Anchorage of Reinforcement
  - Control of Hinge rotations in flexural members
  - Maintenance of Composite action
- Serviceability Limit State
  - Control of Flexural Cracking
  - Control of Member Deformations

#### Factors affecting bond strength:

- 1.1. Type of Reinforcement – Plain or Deformed
- 1.2. Diameter of bars
- 1.3. Grade of Concrete
- 1.4. Cover to reinforcement
- 1.5. Confinement of Concrete
- 1.6. Direction of Casting
- 1.7. Top bar effect
- 1.8. Spacing of bars
- 1.9. Relative rib area of bars
- 1.10. Quality of reinforcement
- 1.11. Coating to reinforcement

## Modes of Bond failure

### **Types of Bond**

Flexural bond : arises in flexural members on account of shear or variation in bending moment-critical at points where shear is significant.

Anchorage bond : arises over a length of anchorage provided for a bar or near the end of a reinforcing bar and resists pulling out of bar in tension.

### **Types of reinforced concrete beams**

- a) Singly reinforced beam
- b) Doubly reinforced beam
- c) Singly or Doubly reinforced flanged beams

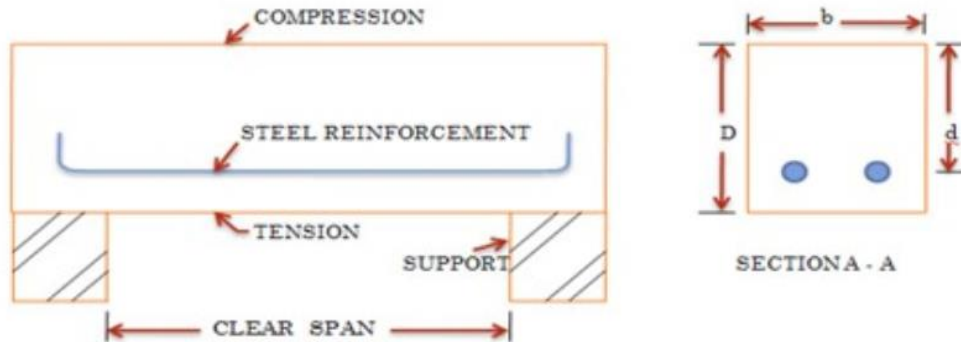
### **Singly reinforced beam**

In singly reinforced simply supported beams or slabs reinforcing steel bars are placed near the bottom of the beam or slabs where they are most effective in resisting the tensile stresses.

### **REINFORCEMENT DETAILING**

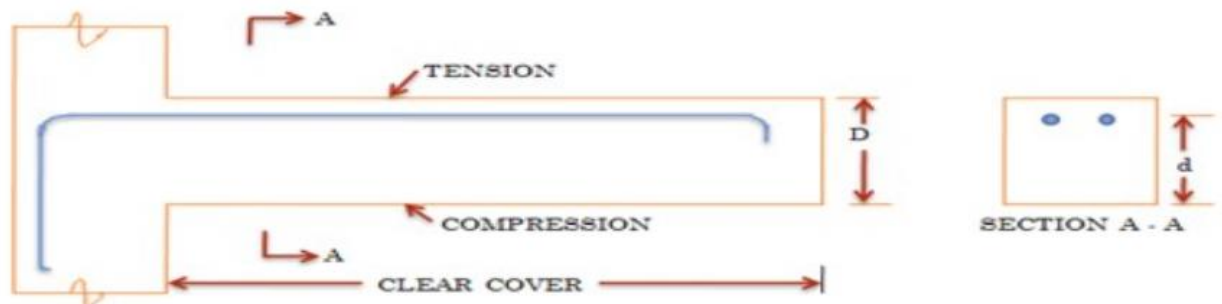
Reinforcement is provided at bottom in case of simply supported beam since bottom fibers are subjected to tension and top fibers are subjected to compression.

○ Reinforcement in simply supported beam



Reinforcement is provided at top in case of cantilever beam since bottom fibers are subjected to compression and top fibers are subjected to tension.

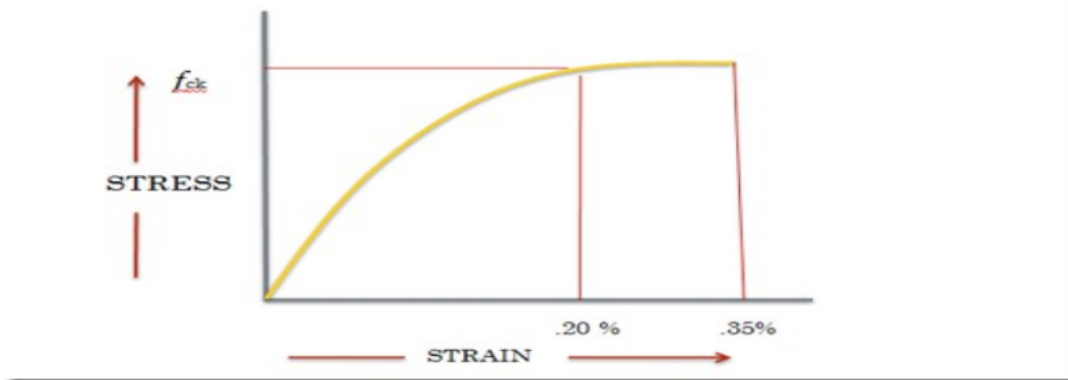
○ Reinforcement in a cantilever beam



## STRESS STRAIN CURVE

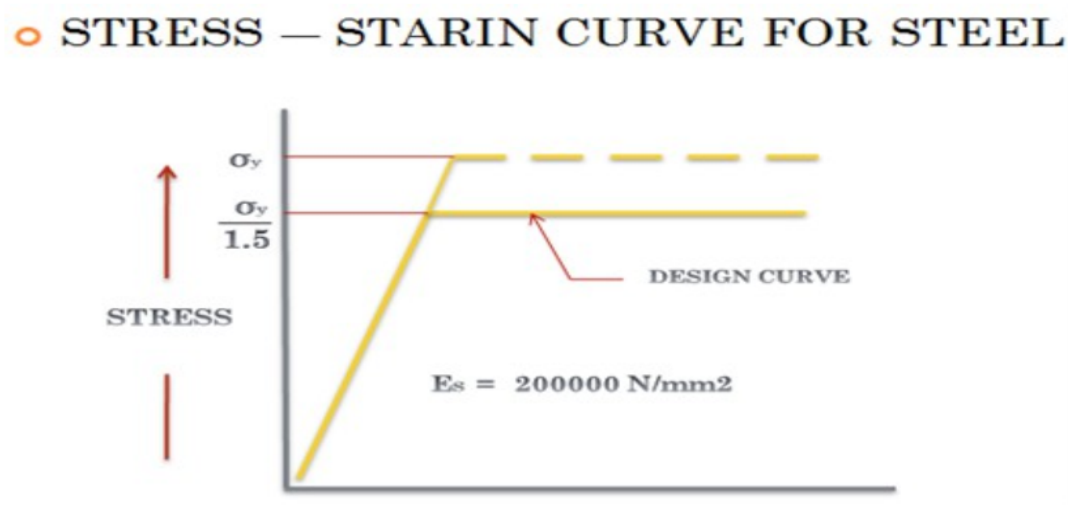
Stress strain curve for concrete has been shown in figure. Since it is a brittle material it reaches the breaking point when the ultimate stress attained.

### ○ STRESS – STRAIN CURVE FOR CONCRETE

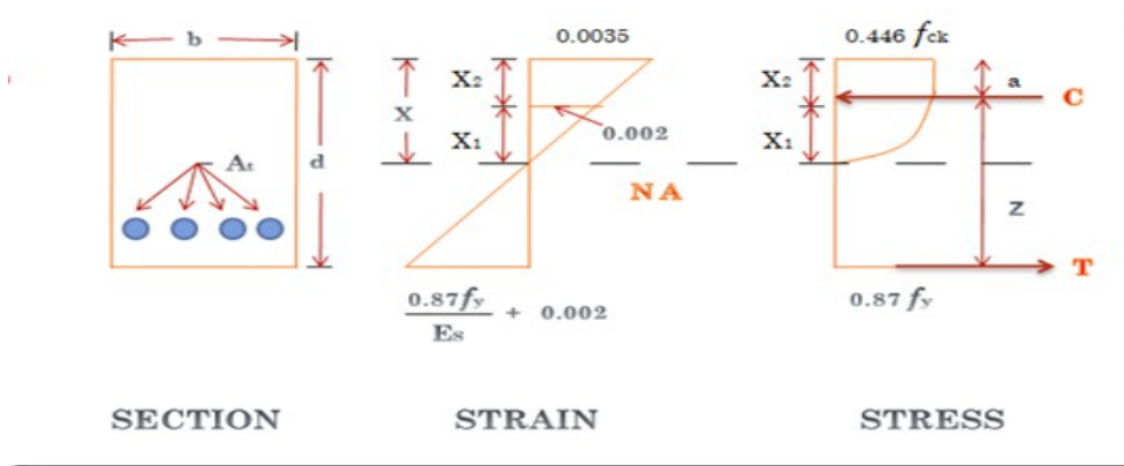


Stress strain curve for steel has been shown in figure. Since it is a ductile material it reaches the yielding point before the failure.

### ○ STRESS – STRAIN CURVE FOR STEEL



Singly reinforced rectangular strain diagram and stress block has shown in figure below.



Where as

$x$  = Depth of Neutral axis

$b$  = breadth of section

$d$  = effective depth of section

The depth of neutral axis can be obtained by considering the equilibrium of the normal forces ,

that is, Resultant force of compression = average stress X area =  $0.36 f_{ck} bx$

Resultant force of tension = average stress X area =  $0.87 f_y A_t$

**Force of compression should be equal to force of tension,**

$$0.36 f_{ck} bx = 0.87 f_y A_t$$

The distance between the lines of action of two forces C & T is called the lever arm and is denoted by  $z$ .

$$\text{Lever arm } z = d - 0.42 x$$

$$z = d - (f_y A_t / f_{ck} b)$$

Moment of resistance with respect to concrete = compressive force x lever arm

$$= 0.36 f_{ck} b x z$$

Moment of resistance with respect to steel = tensile force x lever arm

$$= 0.87 f_y A_t z$$

### Maximum depth of neutral axis

A compression failure is brittle failure. The maximum depth of neutral axis is limited to ensure that tensile steel will reach its yield stress before concrete fails in compression, thus a brittle failure is avoided. The limiting values of the depth of neutral axis  $x_m$  for different grades of steel from strain diagram.

$$\frac{x_m}{d - x_m} = \frac{0.0035}{(0.87 f_y / E_s) + 0.002}$$

$$\frac{x_m}{d} = \frac{0.0035}{(0.87 f_y / E_s) + 0.0055}$$

$f_y$ , N/mm <sup>2</sup>	$x_m$
250	0.53 d
415	0.48 d
500	0.46 d

### Limiting value of tension steel and moment of resistance

Since the maximum depth of neutral axis is limited, the maximum value of moment of resistance is also limited.  $M_{lim}$  with respect to concrete

$$= 0.36 f_{ck} b x z$$

$$= 0.36 f_{ck} b x_m (d - 0.42 x_m)$$

$$M_{lim} \text{ with respect to steel} = 0.87 f_{ck} A_t (d - 0.42 x_m)$$

Limiting moment of resistance values, N mm:

Grade of concrete	Grade of steel		
	Fe 250 steel	Fe 450 steel	Fe 500 steel
General	$0.148 f_{ck} bd^2$	$0.138 f_{ck} bd^2$	$0.133 f_{ck} bd^2$
M20	$2.96 bd^2$	$2.76 bd^2$	$2.66 bd^2$
M25	$3.70 bd^2$	$3.45 bd^2$	$3.33 bd^2$
M30	$4.44 bd^2$	$4.14 bd^2$	$3.99 bd^2$