

**DESIGN OF A SECTION**

**Design of rectangular beam to resist a bending moment equal to 45 kNm using (i) M15 mix and mild steel.**

The beam will be designed so that under the applied moment both materials reach their maximum stresses.

**Assume ratio of overall depth to breadth of the beam equal to 2.**

Breadth of the beam =  $b$

Overall depth of beam =  $D$

therefore,  $D/b = 2$

**For a balanced design,**

Factored BM = moment of resistance with respect to concrete

= moment of resistance with respect to steel

= load factor X B.M

=  $1.5 \times 45$

= 67.5 kNm

**For balanced section,**

Moment of resistance  $M_u = 0.36 f_{ck} b x_m (d - 0.42 x_m)$

$f_y$	$x_m$
250	0.53d
415	0.48d

Grade for mild steel is Fe250

**For Fe250 steel,**

$$x_m = 0.53d ; \quad \mathbf{Mu} = 0.36 f_{ck} b (0.53 d) (1 - 0.42 \times 0.53) d$$

$$= 2.22bd$$

Since  $D/b = 2$  or,  $d/b = 2$  or,  $b = d/2$

$$\mathbf{Mu} = 1.11 d$$

$$\mathbf{Mu} = 67.5 \times 10 \text{ Nmm}$$

$d = 394 \text{ mm}$  and  $b = 200 \text{ mm}$

Adopt  $D = 450 \text{ mm}$ ,  $b = 250 \text{ mm}$ ,  $d = 415 \text{ mm}$

$$\begin{aligned} \text{Area of tensile steel } A_t &= \frac{\text{Factored BM}}{0.87 f_y (d - 0.42 x_m)} \\ &= \frac{67.5 \times 10^6}{0.87 \times 250 (1 - 0.42 \times 0.53) 415} \\ &= 962 \text{ mm}^2 \\ &= 9.62 \text{ cm}^2 \\ \text{Minimum area of steel } A_{o\ominus} &= 0.85 \frac{bd}{f_y} \end{aligned}$$

$$= (0.85 \times 250 \times 415) / 250$$

$$= 353 \text{ mm}$$

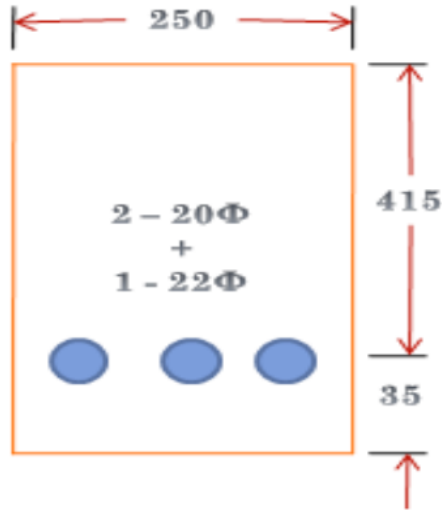
$$353 \text{ mm} < 962 \text{ mm}$$

∴ beams the diameter of main reinforced bars is usually selected between 12 mm and 25 mm.

∴ provide 2-20mm and 1-22mm bars giving total area

$$= 6.28 + 3.80$$

$$= 10.08 \text{ cm} > 9.62 \text{ cm}$$

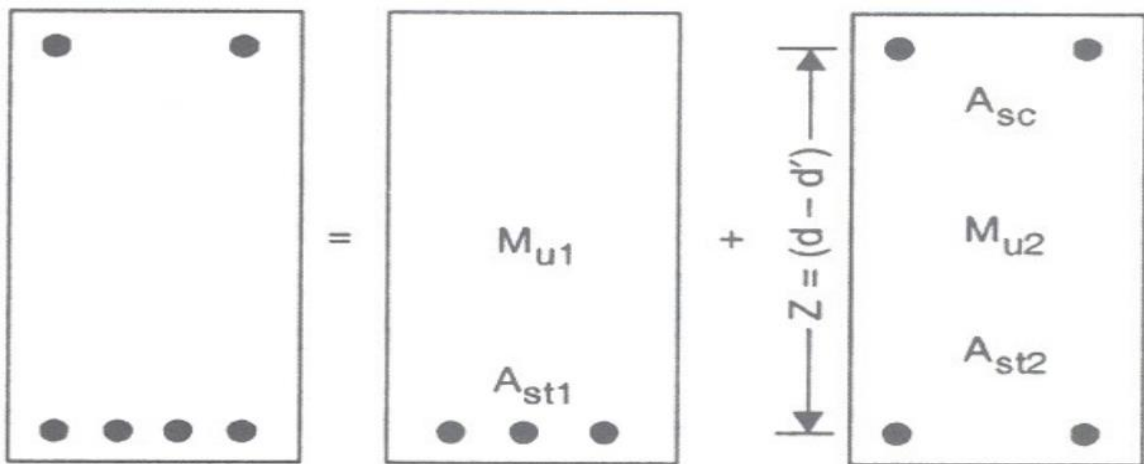
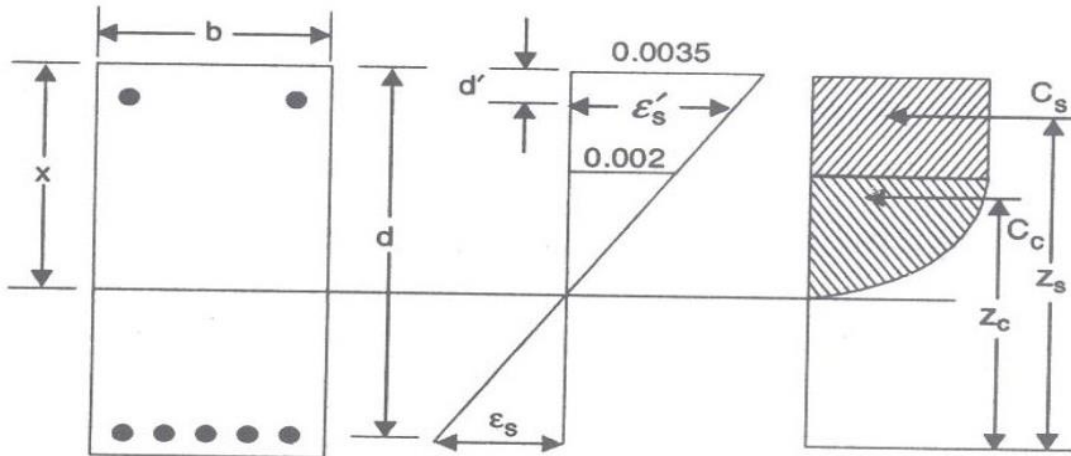


### DOUBLY REINFORCED BEAMS:

- When beam depth is restricted and the moment the beam has to carry is greater than the moment capacity of the beam in concrete failure.
- When B.M at the section can change sign.
- When compression steel can substantially improve the ductility of beams and its use is therefore advisable in members when larger amount of tension steel becomes necessary for its strength.
- Compression steel is always used in structures in earthquake regions to increase their ductility.
- Compression reinforcement will also aid significantly in reducing the long-term deflections of beams.
- A doubly reinforced concrete beam is reinforced in both compression and tension faces.
- When depth of beam is restricted, strength available from a singly reinforced beam is inadequate.
- At a support of a continuous beam, the bending moment changes sign, such a situation may also arise in design of a ring beam.
- Analysis of a doubly reinforced section involves determination of moment of resistance with given beam width, depth, area of tension and compression steels and their covers.
- In doubly reinforced concrete beams the compressive force consists of two parts; both in concrete and steel in compression.

- Stress in steel at the limit state of collapse may be equal to yield stress or less depending on position of the neutral axis.

Doubly reinforced rectangular beam strain diagram and stress block has shown in the figure below.



Design Procedure:

1. Determine the limiting moment of resistance  $M_{um}$  for the given cross-section using the equation for a singly reinforced beam

$$M_{u,lim} = 0.87f_y \cdot A_{st,1} [d - 0.42x_{u,m}] = 0.36 f_{ck} \cdot b \cdot x_{u,m} [d - 0.42x_{u,m}]$$

2. If the factored moment  $M_u$  exceeds  $M_{lim}$ , a doubly reinforced section is required

$$(M_u - M_{lim}) = M_{u2}$$

Additional area of tension steel  $A_{st2}$  is obtained by considering the equilibrium of force of compression in comp. steel and force of tension  $T_2$  in the additional tension steel

$$\sigma_{sc} A_{sc} - \sigma_{cc} A_{sc} = 0.87 f_y A_{st2}$$

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$$A_{sc} = \text{compression steel.}$$

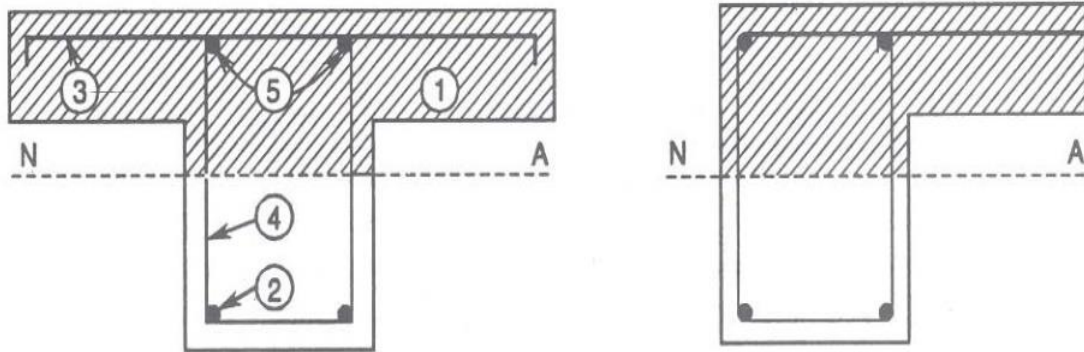
$$\sigma_{cc} = \text{Comp. stress in conc at the level of comp. steel} = 0.446 f_{ck}.$$

### Reasons

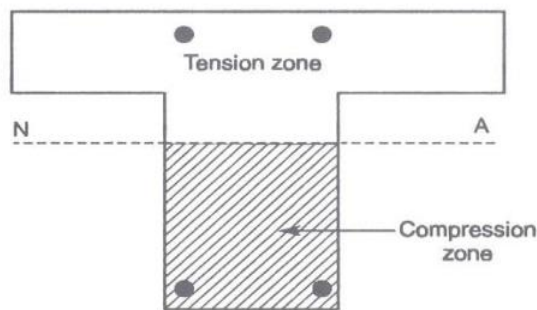
1. When beam section is shallow in depth, and the flexural strength obtained using balanced steel is insufficient i.e. the factored moment is more than the limiting ultimate moment of resistance of the beam section. Additional steel enhances the moment capacity.
2. Steel bars in compression enhance ductility of beam at ultimate strength.
3. Compression steel reinforcement reduces deflection as moment of inertia of the beam section also increases.
4. Long-term deflections of beam are reduced by compression steel.
5. Curvature due to shrinkage of concrete are also reduced.
6. Doubly reinforced beams are also used in reversal of external loading.

### DESIGN OF FLANGED BEAMS

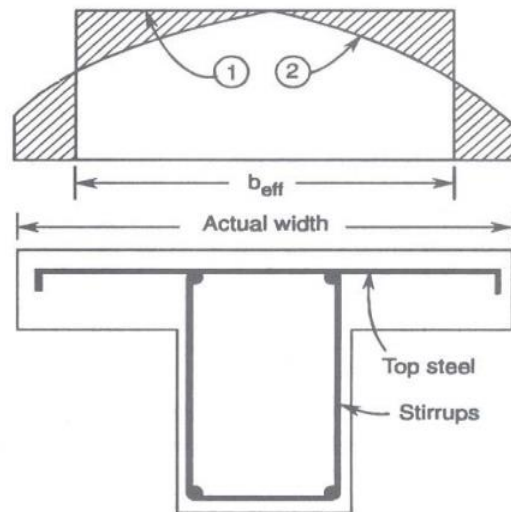
In reinforced concrete construction, slab is supported over beams. Simple concrete slabs of moderate depth and weight are limited to spans of 3m to 5m. If it is desired for long spans without excessive weight and material, slab is built monolithically with RC beams and beams are considered as flanged beams. At the interior portions of floor, slab with beam acts as a T-beam and at an end the portion acts as an L-beam. Shear reinforcement of beams and bent bars extend into slab and Complete construction is cast integrally. A part of slab acts with upper part in bending compressive stresses.



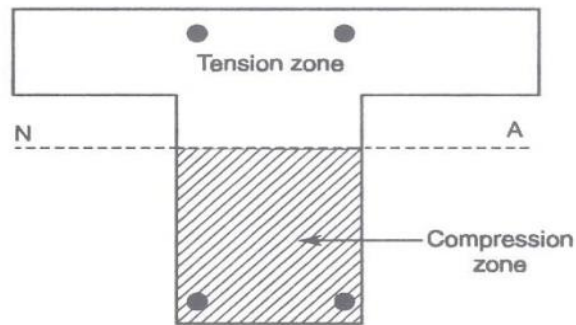
**Fig. 8.1** Flanged beams: (a) T beam—1-Compression in concrete; 2-Tension steel; 3-Transverse steel; 4-Stirrups for shear; 5-Anchorage of stirrups; (b) L beam.



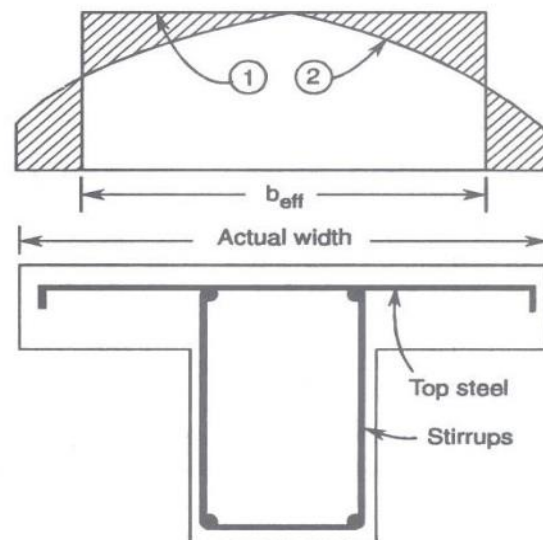
**Fig. 8.2** Flanged beams over supports with negative moments.



**Fig. 8.3** Effective width of T beams: 1. Actual stress distribution in compression flange; 2. Assumed stress distribution in compression flange.



**Fig. 8.2 Flanged beams over supports with negative moments.**



**Fig. 8.3 Effective width of T beams: 1. Actual stress distribution in compression flange; 2. Assumed stress distribution in compression flange.**

### Effective Width of Flange

Theoretically width of flange is supposed to act as top flange of beam. Elements of flange midway between webs of two adjacent beams are less highly stressed in longitudinal compression than those elements directly over webs of beams. An effective width of flange,  $b_f$  is used in the design of flanged beam and is treated to be uniformly stressed at the maximum value, which is smaller than actual width of flange. Effective width of flange primarily depends on span of the beam, breadth of web,  $b_w$  and thickness of flange,  $D_f$ .

IS: 456-2000 recommends for effective width of flanges of T- and L-beams.

- For symmetrical T-beams

$$b_f = \left[ \left( \frac{l_0}{6} \right) + b_w + 6D_f \right]$$

- For beams with slab on one side only

$$b_f = [(l_0/12) + b_w + 3D_f]$$

- For isolated T-beams

$$b_f = [(l_0/((l_0/b)+4)) + b_w]$$

- For Isolated L-beams

$$b_f = [(0.5l_0/((l_0/b)+4)) + b_w]$$

Calculated effective flange width,  $b_f$  shall be not greater than the breadth of web plus half the sum of clear distances to the adjacent beams on either side

- $b_f < 0.5 [l_1 + l_2] + b_w$
- $b_f < 0.5 [l_2 + l_3] + b_w$

### Location of Neutral Axis

Depending upon proportions of cross-section, area of steel reinforcement in tension, strength of materials

1. Neutral axis of a T-beam in one case may lie in the flange i.e. depth of NA,  $x_u$  is less than or equal to thickness of flange or depth of slab,  $D_f$  (Neutral axis lies within flange ( $x_u < D_f$ ))
2. NA may lie in web i.e. depth of neutral axis,  $x_u$  is more than thickness of slab,  $D_f$ .

Stress diagram consists of a rectangular portion of depth  $0.43x_u$  and a parabolic portion of depth  $0.57x_u$ .

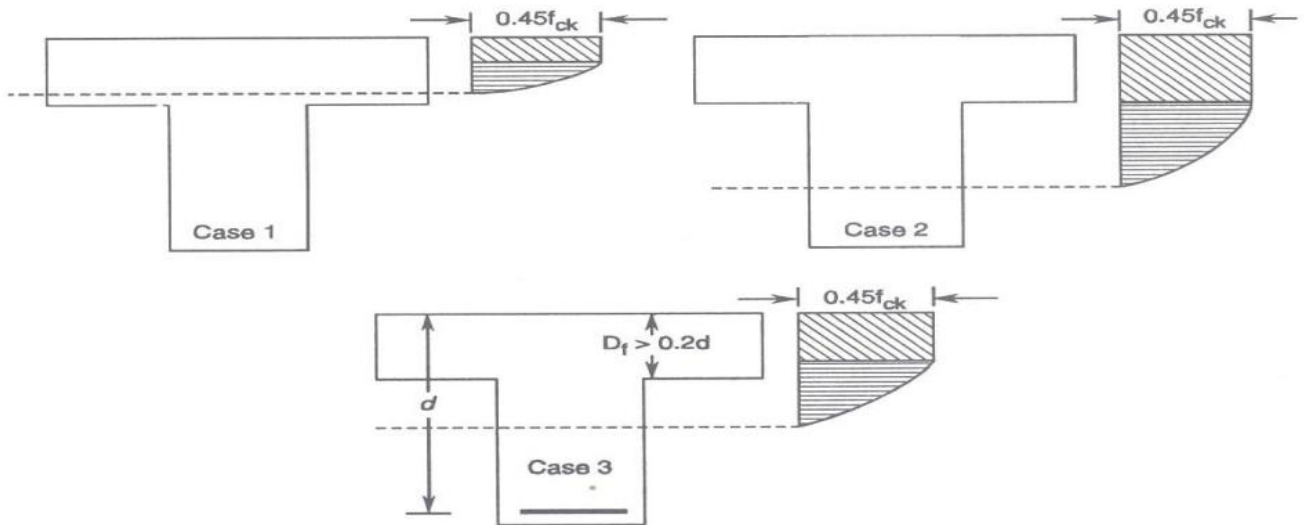


Fig. 8.4 Three possible positions of neutral axis in T beams.

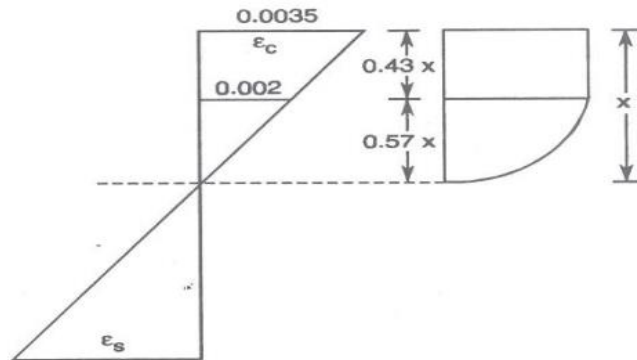


Fig. 8.5 Compression stress block in T beams.

**IF NEUTRAL AXIS LIES OUT SIDE FLANGE [i.e.  $x_u > D_f$ ]**

- When NA of T-section lies outside flange, it lies in web of T-beam. However, there are two possibilities depending upon whether depth of flange  $D_f$  is less than or equal to  $0.43x_u$  or  $D_f$  is more than  $0.43x_u$ .
- Comparison of  $D_f$  with  $0.43x_u$  (i.e.  $3/7x_u$ ) is more rational as  $0.43x_u$  is actual depth of rectangular portion of stress block.
- In IS:456-2000, if  $(D_f/d)$  is less than 0.2, the flange of T-beam is considered as small.

$D_f$  is less than  $0.43x_u$

- Total area in compression consists of sum of compressive force in concrete in web of width,  $b_w$ ,  $C_{w,cu}$  and compressive force in concrete in the flange excluding web,  $C_{f,cu}$ .

**IF NEUTRAL AXIS LIES OUT SIDE FLANGE [i.e.  $x_u > D_f$ ]**

Depth of flange  $D_f$  is more than  $0.43x_u$  [ $D_f > 0.43 x_u$  or ( $D_f > 0.2d$ )], some portion is subjected to uniform stress equal to  $0.446f_{ck}$  ( $0.43x_u$ ) and remaining portion is subjected to parabolic stress.

- To obtain compressive force in portion of flange, concept of modified thickness of flange equal to  $y_f = (0.15x_u + 0.65D_f)$  is recommended by IS456-2000 Average stress is assumed to be  $0.446f_{ck}$

### Assignment problems

#### Analysis type problems

1. A reinforced concrete beam of rectangular section 300 mm wide  $\times$  650 mm deep is reinforced with 4 bars of 25 mm diameter at an effective depth of 600 mm. Calculate the neutral axis depth and estimate the safe moment of resistance of the section adopting M25 grade concrete and Fe415 HYSD bars.
2. A reinforced concrete beam of rectangular section 350 mm wide  $\times$  750 mm overall depth is reinforced with 3 bars of 20 mm diameter at an effective depth of 700 mm. Adopting M30 grade concrete and FE500 grade steel reinforcement, calculate the safe moment of resistance of the section. If the beam spans over 5 m, estimate the safe permissible live load on the beam.
3. A reinforced concrete beam of rectangular section having a width of 400 mm and overall depth 850 mm is reinforced with 4 bars of 25 mm diameter both on the compression and tension sides at an effective cover of 50 mm. Using M20 grade concrete and Fe415 HYSD bars, compute (a) the actual neutral axis; (b) the critical neutral axis; and (c) the safe moment of resistance of the section.
4. A reinforced concrete rectangular section 300 mm wide  $\times$  600 mm overall depth is reinforced with 4 bars of 25 diameter at an effective cover of 50 mm on the tension side. Assuming M20 grade concrete and FE415 HYSD bars, determine the allowable bending moment and the stresses in steel and concrete corresponding to this moment.

#### Design problems

1. A single reinforced rectangular beam is 400mm wide. The effective depth of the beam section is 560mm and its effective cover is 40mm. The steel reinforcement consists of 4 MS 18mm diameter bars in the beam section. The grade of concrete is M20. Locate the neutral axis of the beam section.
2. In example 1, the bending moment at a transverse section of beam is 105 kN-m. Determine the strains at the extreme fibre of concrete in compression and steel bars provided as reinforcement in tension. Also determine the stress in steel bars.
3. In example 2, the strain in concrete at the extreme fibre in compression  $\epsilon_{cu}$  is 0.00069 and the tensile stress in bending in steel is  $199.55 \text{ N/mm}^2$ . Determine the depth of neutral axis and the moment of resistance of the beam section.

4. Determine the moment of resistance of a section 300mm wide and 450mm deep up to the centre of reinforcement. If it is reinforced with (i) 4-12mm fe415 grade bars, (ii) 6-18mm fe415 grade bars.
5. A rectangular beam section is 200mm wide and 400mm deep up to the centre of reinforcement. Determine the reinforcement required at the bottom if it has to resist a factored moment of 40kN-m. Use M20 grade concrete and fe415 grade steel.
6. A rectangular beam section is 250mm wide and 500mm deep up to the centre of tension steel which consists of 4-22mm dia. bars. Find the position of the neutral axis, lever arm, forces of compression and tension and safe moment of resistance if concrete is M20 grade and steel is Fe500 grade.
7. A rectangular beam is 200mm wide and 450 mm overall depth with an effective cover of 40mm. Find the reinforcement required if it has to resist a moment of 35 kN.m. Assume M20 concrete and Fe250 grade steel.
8. A singly reinforced slab 120mm thick is supported by T-beam spaced at 3.5m c/c has an effective depth,  $d = 550\text{mm}$ , width,  $b_w = 400\text{mm}$ . The beam is provided with steel reinforcement consisting of 5 bars of 20mm diameter in one layer,  $d' = 50\text{mm}$ .  $l_e = 3.7\text{m}$ . Use M20 grade concrete and Fe415 steel. Determine the depth of neutral axis and the moment of resistance of the beam, MR?
9. Calculate the moment of resistance of a T-beam for M20 and Fe415,  $D_f = 120\text{ mm}$ ,  $b_f = 750\text{mm}$ ,  $b_w = 250\text{mm}$ ,  $d' = 50\text{mm}$ ,  $D = 500\text{mm}$
10. T-beam floor,  $D_f = 150\text{mm}$ ,  $b_w = 250\text{mm}$  spacing = 3.5m c/c,  $l_e = 8.0\text{m}$ . LL = 6.5 kN/m. Design an intermediate beam using M20 and Fe415 steel.
11. T-beam  $d = 750\text{mm}$ ,  $b_f = 1400\text{mm}$ ,  $D_f = 100\text{mm}$ ,  $b_w = 300\text{mm}$ ,  $A_{st} = ?$   $M = 100\text{kN-m}$ . Use M20 and Fe 415 HYSD bars.