STATIC ANALYSIS OF BEAMS

STIFFNESS METHOD OF STRUCTURAL ANALYSIS

INTRODUCTION

Stiffness method or displacement method is an important approach to the analysis of structures .This is an important approach to the analysis of structures. This is used in its basic form for the analysis of structures that are linear and elastic although it can be adapted to non linear analysis. It is generally used for the analysis of statically determinate cases. This method in its basic form considers the nodal displacements of the structures as unknown.

The DIRECT STIFFNESS METHOD is a highly organized, conceptually simple approach for the analysis of all types of structures that is easily implemented in the form of a computer aided analysis procedure using a matrix formulation.

Amongst the most far- reaching developments in structural engineering has been the ability to analyze automatically almost all types of structures with a high degree of accuracy and at reasonable cost. The availability of digital computer has made this development possible Methods of analysis that could easily be computerized were quickly developed.

STEPS IN DIRECT STIFFNESS METHOD

STEP 1

The first step in the analysis is to define the structures coordinate system , the support and loading conditions and the assumptions of analysis . In this stage each of the degree of freedom is numbered and for convenience each of the members is also numbered . The member properties such as M.I. modulus of elasticity , etc. are interested.

STEP2

With the structures defined , the member coordinate systems are defined and the MEMBER STIFFNESS MATRIX for each of the members is computed in its own coordinate system. The coordinates are defined by arbitrarily choosing one end of the member as origin, and then imposing a coordinate system identical to that used in derivation of member stiffness matrix. With the coordinate system defined member stiffness matrix foe each member is evaluated.

STEP3

Now we have to assemble the structures stiffness matrix and solve for displacements and internal forces. The problem is to insert the elements from the structure stiffness matrix. For each of the member stiffness matrixes , the corresponding structures degree of freedom should be defined. This information is needed to place the elements in the structures stiffness matrix. After all the elements of the first member have been written into the structure stiffness matrix, those of the second are written .when writing into an already filled a lot , the new contribution is added to the existing value. While this process is tedious for even a small structure using hand calculations.

STIFFNESS METHODS MERITS AND DEMERITS

MERITS

One basic form of the stiffness method could be applied to a wide range of structures, with only minor adjustments to cope with each variant. The advantages of the method can be summarized as:

1) A general purpose program is easy to write.
2) It requires a minimum of input data.
3) It can be made entirely automatic .Its use requires no understanding of structural mechanics.

DEMERITS

The method has a major disadvantage in that no account is taken of the degree of indeterminacy and therefore there is little opportunity to benefit from the structural expertise of the operator. Equally this will be seen as an essential concomitant of the advantage listed in above. The time required to perform an analysis and the amount of computer storage depends almost entirely on the number of degree of freedom involved. Structures having many degrees of freedom but few degree of static indeterminacy should be much more economically analyzed by the flexibility rather than the stiffness method.

FLEXURE AND SHEAR DESIGN OF CORBEL

INTRODUCTION

Corbel or bracket is a reinforced concrete member is a short-haunched cantilever used to support the reinforced concrete beam element. Corbel is structural element to support the ***pre-cast structural system*** such as pre-cast beam and pre-stressed beam. The corbel is cast monolithic with the column element or wall element.

This chapter is describes the design procedure of corbel or bracket structure. Since the load from pre-cast structural element is large then it is very important to make a good detailing in corbel.

BEHAVIOR OF CORBEL

The followings are the major items show the behavior of the reinforced concrete corbel, as follows :

- The ***shear span/depth ratio is less than 1.0***, it makes the corbel behave in two-dimensional manner.
- ***Shear deformation*** is significant is the corbel.
- There is ***large horizontal force*** transmitted from the supported beam result from ***long-term shrinkage*** and ***creep deformation***.
- ***Bearing failure*** due to large concentrated load.
- The cracks are usually ***vertical*** or ***inclined pure shear cracks***.
- The mode of failure of corbel are : yielding of the tension tie, failure of the end anchorage of the tension tie, failure of concrete by compression or shearinga and bearing failure.

The followings figure shows the mode of failure of corbel.

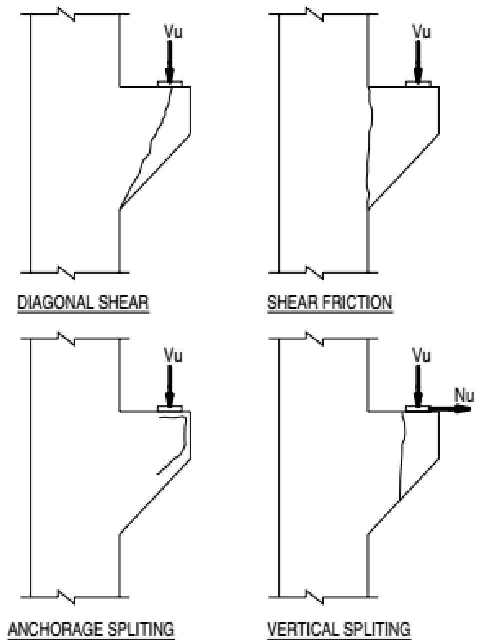


FIGURE 1 MODES OF FAILURE OF CORBEL

SHEAR DESIGN OF CORBEL

GENERAL

Since the corbel is cast at different time with the column element then the cracks occurs in the interface of the corbel and the column. To avoid the cracks we must provide the ***shear friction reinforcement*** perpendicular with the cracks direction.

ACI code uses the ***shear friction theory*** to design the interface area.

SHEAR FRICTION THEORY

In shear friction theory we use ***coefficient of friction*** μ to transform the ***horizontal resisting force*** into ***vertical resisting force***.

The basic design equation for shear reinforcement design is :

$$\phi V_n \geq V_u \qquad [1]$$

where :

V_n = nominal shear strength of shear friction reinforcement

V_u = ultimate shear force

ϕ = strength reduction factor (ϕ = 0.85)

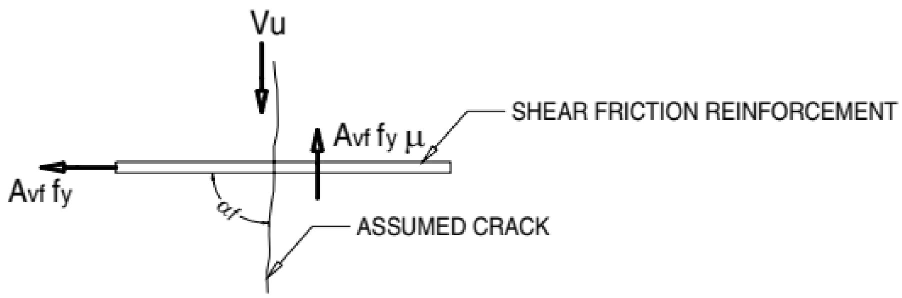


FIGURE 1 SHEAR FRICTION THEORY

The nominal shear strength of shear friction reinforcement is :

TABLE 1 SHEAR FRICTION REINFORCEMENT STRENGTH

VERTICAL SHEAR FRICTION REINFORCEMENT		INCLINED SHEAR FRICTION REINFORCEMENT	
V_n	A_{vf}	V_n	A_{vf}
$V_n = A_{vf} f_y \mu$	$A_{vf} = \dfrac{V_n}{f_y \mu}$ $A_{vf} = \dfrac{V_u / \phi}{f_y \mu}$	$V_n = A_{vf} f_y (\mu \sin\alpha_f + \cos\alpha_f)$	$A_{vf} = \dfrac{V_n}{f_y(\mu \sin\alpha_f + \cos\alpha_f)}$ $A_{vf} = \dfrac{V_u / \phi}{f_y(\mu \sin\alpha_f + \cos\alpha_f)}$

where :

V_n = nominal shear strength of shear friction reinforcement

A_{vf} = area of shear friction reinforcement

Fy = yield strength of shear friction reinforcement

μ = coefficient of friction

TABLE 2 COEFFICIENT OF FRICTION

METHOD	COEFFICIENT OF FRICTION μ
Concrete Cast Monolithic	1.4λ
Concrete Placed Against Roughened Hardened Concrete	1.0λ
Concrete Placed Against unroughened Hardened Concrete	0.6λ
Concrete Anchored to Structural Steel	0.7λ

The value of λ is :

$\lambda = 1.0$ normal weight concrete

$\lambda = 0.85$ sand light weight concrete

$\lambda = 0.75$ all light weight concrete

The ultimate shear force must follows the following condiitons :

$$V_u \le \phi(0.2f'_c)b_w d \qquad [\quad 1]$$
$$V_u \le \phi(5.50)b_w d$$

where :

V_u	= ultimate shear force	(N)
f'_c	= concrete cylinder strength	(MPa)
b_w	= width of corbel section	(mm)
d	= effective depth of corbel	(mm)

STEP – BY – STEP PROCEDURE

The followings are the step – by – step procedure used in the shear design for ***corbel (bracket)***, as follows :

- Calculate the ultimate shear force $\mathbf{V_u}$.
- Check the ultimate shear force for the following condition, if the following condition is not achieved then ***enlarge the section***.

$$V_u \le \phi(0.2f'_c)b_w d$$
$$V_u \le \phi(5.50)b_w d$$

- Calculate the area of shear friction reinforcement $\mathbf{A_{vf}}$.

VERTICAL SHEAR FRICTION REINFORCEMENT		INCLINED SHEAR FRICTION REINFORCEMENT	
$\mathbf{V_n}$	$\mathbf{A_{vf}}$	$\mathbf{V_n}$	$\mathbf{A_{vf}}$
$V_n = A_{vf} f_y \mu$	$A_{vf} = \frac{V_n}{f_y \mu}$ $A_{vf} = \frac{V_u/\phi}{f_y \mu}$	$V_n = A_{vf} f_y(\mu \sin\alpha_f + \cos\alpha_f)$	$A_{vf} = \frac{V_n}{f_y(\mu \sin\alpha_f + \cos\alpha_f)}$ $A_{vf} = \frac{V_u/\phi}{f_y(\mu \sin\alpha_f + \cos\alpha_f)}$

- The design must be follows the basic design equation as follows :

$$\phi V_n \ge V_u$$

FLEXURAL DESIGN OF CORBEL

GENERAL

The corbel is design due to ultimate flexure moment result from the supported beam reaction $\mathbf{V_u}$ and horizontal force from creep and shrinkage effect $\mathbf{N_u}$.

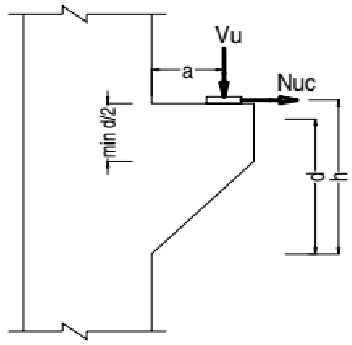


FIGURE 2 DESIGN FORCE OF CORBEL

TENSION REINFORCEMENT

The ultimate horizontal force acts in the corbel $\mathbf{N_{uc}}$ is result from the creep and shrinkage effect of the pre-cast or pre-stressed beam supported by the corbel.

This ultimate horizontal force must be resisted by the tension reinforcement as follows :

$$A_n = \frac{N_{uc}}{\phi f_y} \qquad [\quad 2]$$

where :

A_n = area of tension reinforcement

N_{uc} = ultimate horizontal force at corbel

f_y = yield strength of the tension reinforcement

ϕ = strength reduction factor (ϕ = 0.85)

Minimum value of $\mathbf{N_{uc}}$ is $\mathbf{0.2V_{uc}}$.

The strength reduction factor is taken ***0.85*** because the major action in corbel is ***dominated by shear***.

FLEXURAL REINFORCEMENT

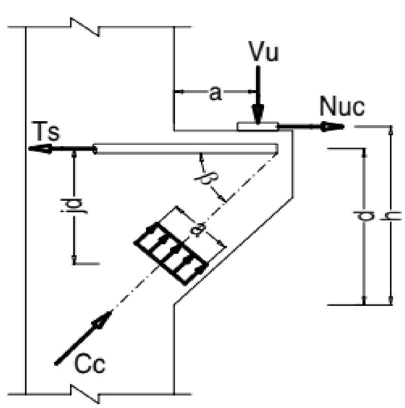


FIGURE 3 ULTIMATE FLEXURE MOMENT AT CORBEL

The ultimate flexure moment **M_u** result from the support reactions is :

$$M_u = V_u(a) + N_{uc}(h - d) \qquad [3]$$

where :

M_u = ultimate flexure moment

V_u = ultimate shear force

a = distance of V_u from face of column

N_{uc} = ultimate horizontal force at corbel

h = height of corbel

d = effective depth of corbel

The resultant of tensile force of tension reinforcement is :

$$T_f = A_f f_y \qquad [4]$$

where :

T_f = tensile force resultant of flexure reinforcement

A_f = area of flexure reinforcement

f_y = yield strength of the flexure reinforcement

The resultant of compressive force of the concrete is :

$$C_c = 0.85 f'_c\, ba(\cos\beta) \qquad [5]$$

where :

Cc = compressive force resultant of concrete

f'_c = concrete cylinder strength

b = width of corbel

a = depth of concrete compression zone

The horizontal equilibrium of corbel internal force is :

$$\sum H = 0 \Rightarrow C_c = T_s \qquad [6]$$
$$0.85 f'_c\, ba(\cos\beta) = A_f f_y$$
$$a = \frac{A_f f_y}{0.85 f'_c\, b(\cos\beta)}$$

The flexure reinforcement area is :

$$A_f = \frac{M_u}{\phi f_y\left(d - \frac{a}{2}\right)} \qquad [7]$$

$$A_f = \frac{M_u}{\phi f_y \left(d - \frac{\left(\frac{A_f f_y}{0.85 f'_c \, b(\cos\beta)} \right)}{2} \right)}$$

Cos β value can be calculated based on the Tan β value as follows :

$$\mathrm{Tan}\beta = \frac{jd}{a} \qquad [\ \ 8]$$

where :

a	= distance of V_u from face of column
jd	= lever arm

Based on the equation above we must trial and error to find the reinforcement area $\mathbf{A_f}$.
For practical reason the equation below can be used for preliminary :

$$A_f = \frac{M_u}{\phi f_y (jd)} \qquad [\ \ 9]$$

$$A_f = \frac{M_u}{\phi f_y (0.85d)}$$

where :

A_f	= area of flexural reinforcement
M_u	= ultimate flexure moment at corbel
f_y	= yield strength of the flexural reinforcement
ϕ	= strength reduction factor (ϕ = 0.9)
d	= effective depth of corbel

DISTRIBUTION OF CORBEL REINFORCEMENTS

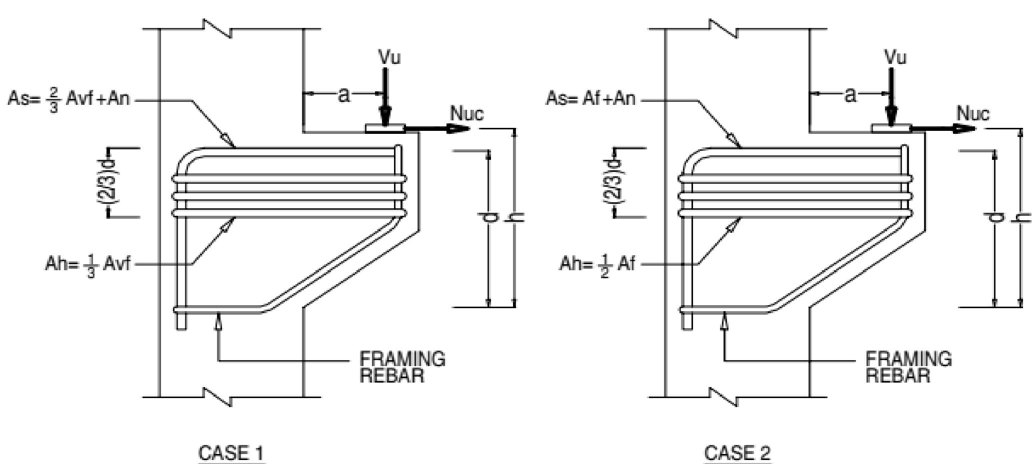


FIGURE 4 DISTRIBUTION OF CORBEL REINFORCEMENTS

From the last calculation we already find the ***shear friction reinforcement*** A_{vf}, ***tension reinforcement*** A_n and ***flexural reinforcement*** A_f. We must calculate the ***primary tension reinforcement*** A_s based on the above reinforcements.

TABLE 3 DISTRIBUTION OF CORBEL REINFORCEMENTS

CASE	A_s	PRIMARY REINFORCEMENT	CLOSED STIRRUP	
			A_h	LOCATION
1	$A_s \geq \frac{2}{3}A_{vf} + A_n$	$A_s = \frac{2}{3}A_{vf} + A_n$	$A_h = \frac{1}{3}A_{vf}$	$\frac{2}{3}d$
2	$A_s \geq A_f + A_n$	$A_s = A_f + A_n$	$A_h = \frac{1}{2}A_f$	$\frac{2}{3}d$

where :

A_s = area of primary tension reinforcement

A_{vf} = area of shear friction reinforcement

A_n = area of tension reinforcement

A_f = area of flexure reinforcement

A_h = horizontal closed stirrup

d = effective depth of corbel

The reinforcements is taken which is larger, case 1 or case 2, the distribution of the reinforcements is shown in the figure above.

LIMITS OF REINFORCEMENTS

The limits of primary steel reinforcement at corbel design is :

$$\rho = \frac{A_s}{bd} \geq 0.04\frac{f'_c}{f_y} \qquad [\quad 10]$$

where :

A_s = area of primary tension reinforcement

b = width of corbel

d = effective depth of corbel

The limits of horizontal closed stirrup reinforcement at corbel design is :

$$A_h \geq 0.5(A_s - A_n) \qquad [\quad 11]$$

where :

A_s = area of primary tension reinforcement

A_n = area of tension reinforcement

STEP – BY – STEP PROCEDURE

The followings are the step – by – step procedure used in the flexural design for ***corbel (bracket)***, as follows :

- Calculate ultimate flexure moment **M_u**.

$$M_u = V_u(a) + N_{uc}(h - d)$$

- Calculate the area of tension reinforcement **A_n**.

$$A_n = \frac{N_{uc}}{\phi f_y}$$

- Calculate the area of flexural reinforcement **A_f**.

$$A_f = \frac{M_u}{\phi f_y (0.85d)}$$

- Calculate the area of primary tension reinforcement **A_s**.

CASE	A_s	PRIMARY REINFORCEMENT	CLOSED STIRRUP	
			A_h	LOCATION
1	$A_s \geq \frac{2}{3}A_{vf} + A_n$	$A_s = \frac{2}{3}A_{vf} + A_n$	$A_h = \frac{1}{3}A_{vf}$	$\frac{2}{3}d$
2	$A_s \geq A_f + A_n$	$A_s = A_f + A_n$	$A_h = \frac{1}{2}A_f$	$\frac{2}{3}d$

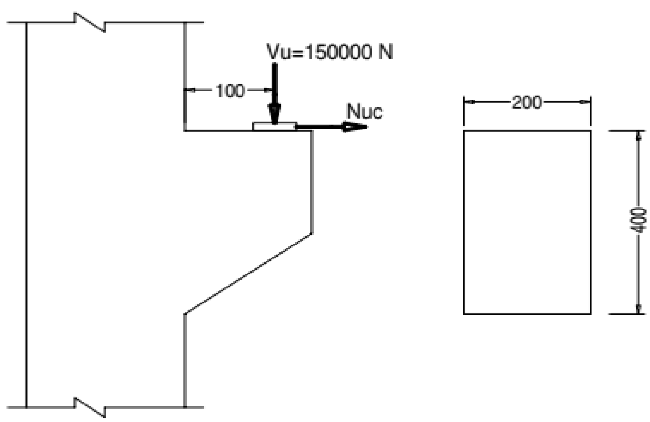
- Check the reinforcement for ***minimum reinforcement***.

$$\rho = \frac{A_s}{bd} \geq 0.04\frac{f'_c}{f_y}$$

$$A_h \geq 0.5(A_s - A_n)$$

APPLICATIONS

APPLICATION 01 – DESIGN OF CORBEL



PROBLEM

Design the flexural and shear friction reinforcement of corbel structure above.

MATERIAL

Concrete strength = K – 300

Steel grade = Grade 400

Concrete cylinder strength = $f'_c = 0.83 \times 30 = 24.9$ MPa

$\beta_1 = 0.85$

DIMENSION

b	= 200	mm
h	= 400	mm
Concrete cover	= 30	mm
d	= 370	mm

DESIGN FORCE

$$V_u = 150000 \text{ N}$$

$$N_{uc} = 0.2V_u = 0.2 \times 150000 = 30000 \text{ N}$$

$$M_u = V_u(a) + N_{uc}(h-d) = 150000(100) + 30000(400-370) = 15900000 \text{ Nmm}$$

LIMITATION CHECKING

$$\phi(0.2f'_c)b_w d = 0.85(0.2 \times 24.9)200 \times 370 = 313242 \text{ N}$$

$$\phi(5.5)b_w d = 0.85 \times 5.5 \times 200 \times 370 = 345950 \text{ N}$$

$$V_u = 150000 < \phi(0.2f'_c)b_w d = 313242 < \phi(5.5)b_w d = 345950$$

SHEAR FRICTION REINFORCEMENT

$$\mu = 1.4\lambda = 1.4 \times 1.0 = 1.4$$

$$A_{vf} = \frac{V_u/\phi}{f_y \mu} = \frac{150000/0.85}{400 \times 1.4} = 315 \text{ mm}^2$$

TENSION REINFORCEMENT

$$A_n = \frac{N_{uc}}{\phi f_y} = \frac{30000}{0.85 \times 400} = 88 \text{ mm}^2$$

FLEXURAL REINFORCEMENT

$$A_f = \frac{M_u}{\phi f_y(0.85d)} = \frac{15900000}{0.9 \times 400(0.85 \times 370)} = 140 \text{ mm}^2$$

PRIMARY TENSION REINFORCEMENT

CASE	A_s (mm^2)	PRIMARY REINFORCEMENT (mm^2)	CLOSED STIRRUP	
			A_h (mm^2)	LOCATION (mm)
1	$A_s \geq \frac{2}{3}A_{vf} + A_n$ $A_s \geq \frac{2}{3}(315) + 88 \geq 298$	$A_s = 298$	$A_h = \frac{1}{3}A_{vf}$ $A_h = \frac{1}{3}(315) = 105$	$\frac{2}{3}d$ 247
2	$A_s \geq A_f + A_n$ $A_s \geq 140 + 88 \geq 228$	$A_s = 228$	–	–

The reinforcement of the corbel are :

$A_s = 298 \text{ mm}^2$

$A_h = 105 \text{ mm}^2$

CHECK FOR A_S MINIMUM AND A_S MAXIMUM

$$\rho_{min} = 0.04\frac{f'_c}{f_y} = 0.04\frac{24.9}{400} = 0.00249$$

$$\rho = \frac{A_s}{bd} = \frac{298}{200 \times 370} = 0.00402 > \rho_{min} = 0.00249$$ → **OK**

$$A_{h-min} = 0.5(A_s - A_n) = 0.5(298 - 88) = 210 \text{ mm}^2$$

$$A_h = 105 < A_{h-min} = 210 \Rightarrow A_h = 210 \text{ mm}^2$$

The final reinforcement of the corbel are :

$A_s = 298 \text{ mm}^2$

$A_h = 210 \text{ mm}^2$

CORBEL REINFORCEMENT

A_s (mm^2)	A_h (mm^2)
A_s=3D16 $A_s = 3\left(\frac{1}{4}\pi D^2\right) = 3\left(\frac{1}{4}\pi \times 16^2\right) = 603$	A_h=3(2 Legs D10) $A_s = 3\left(2 \times \frac{1}{4}\pi D^2\right) = 3\left(2 \times \frac{1}{4}\pi \times 10^2\right) = 471$

SKETCH OF REINFORCEMENT