

## DESIGN OF EARTH QUAKE ENGINEERING CONT'D

### Design lateral forces

*Design Seismic Base Shear:* The total design lateral force or design seismic base shear ( $V_B$ ) along any principal direction shall be determined by the following expression: [Clause 7.5.3, IS-1893 (2002)]

$$V_B = A_h W \quad (6)$$

Where

$A_h$  = Design horizontal acceleration spectrum value as per equation (5), using the fundamental natural period  $T_a$  as per equation (7) or (8) in the considered direction of vibration; and

$W$  = Seismic weight of the building is computed as given below [Clauses 7.4.2 & 7.4.3, IS-1893 (2002)]

☞ *Seismic Weight of floors:* The seismic weight of each floor is its full dead load plus appropriate amount of imposed load. While computing the seismic weight of each floor, the weight of columns and walls in any storey shall be equally distributed to the floors above and below the storey.

☞ *Seismic Weight of Building:*

- The seismic weight of the whole building is the sum of the seismic weights of all the floors.
- Any weight supported in between storeys shall be distributed to the floors above and below in inverse proportion to its distance from the floors.

### Fundamental period

➤ The approximate fundamental natural period of vibration ( $T_a$ ), in seconds, of a moment-resisting frame building without brick infill panels may be estimated by the empirical expression: [Clause 7.6.1, IS-1893 (2002)]

$$\begin{aligned} T_a &= 0.075h^{0.75} \dots(\text{for RC frame building}) \\ T_a &= 0.085h^{0.75} \dots(\text{for steel frame building}) \end{aligned} \quad (7)$$

➤ The approximate fundamental natural period of vibration ( $T_a$ ), in seconds, of all other buildings, including moment-resisting frame buildings with brick infill panels, may be estimated by the empirical expression: [Clause 7.6.2, IS-1893 (2002)]

$$T_a = \frac{0.09h}{\sqrt{d}} \quad (8)$$

where,

$h$  = Height of building, in m. This excludes the basement storeys, where basement walls are connected with the ground floor deck or fitted between the building columns. But, it includes the basement storeys, when they are not so connected.

$d$  = Base dimension of the building at the plinth level, in m, along the considered direction of the lateral force.

### **Earthquake Lateral Force Analysis**

The design lateral force shall first be computed for the building as a whole. This design lateral force shall then be distributed to the various floor levels. The overall design seismic force thus obtained at each floor level shall then be distributed to individual lateral load resisting elements depending on the floor diaphragm action. There are two commonly used procedures for specifying seismic design lateral forces:

1. Equivalent static force analysis
2. Dynamic analysis

### **Equivalent static force analysis**

The equivalent lateral force for an earthquake is a unique concept used in earthquake engineering. The concept is attractive because it converts a dynamic analysis into partly dynamic and partly static analyses for finding the maximum displacement (or stresses) induced in the structure due to earthquake excitation. For seismic resistant design of structures, only these maximum stresses are of interest, not the time history of stresses. The equivalent lateral force for an earthquake is defined as a set of lateral static forces which will produce the same peak response of the structure as that obtained by the dynamic analysis of the structure under the same earthquake. This equivalence is restricted only to a single mode of vibration of the structure. Inherently, equivalent static lateral force analysis is based on the following assumptions,

- Assume that structure is rigid.
- Assume perfect fixity between structure and foundation.
- During ground motion every point on the structure experience same accelerations
- Dominant effect of earthquake is equivalent to horizontal force of varying magnitude over the height.
- Approximately determines the total horizontal force (Base shear) on the structure

However, during an earthquake structure does not remain rigid, it deflects, and thus base shear is disturbed along the height.

The limitations of equivalent static lateral force analysis may be summarised as follows,

- In the equivalent static force procedure, empirical relationships are used to specify dynamic inertial forces as static forces.
- These empirical formulas do not explicitly account for the dynamic characteristics of the particular structure being designed or analyzed.
- These formulas were developed to approximately represent the dynamic behavior of what are called *regular* structures (Structures which have a reasonably uniform distribution of mass and stiffness). For such structures, the equivalent static force procedure is most often adequate.
- Structures that are classified as *irregular* violate the assumptions on which the empirical formulas, used in the equivalent static force procedure, are developed. Common types of irregularities in a structure include large floor-to-floor variation in mass or center of mass and soft stories etc. Therefore in such cases, use of equivalent static force procedure may lead to erroneous results. In these cases, a dynamic analysis should be used to specify and distribute the seismic design forces.

### **Step by step procedure for Equivalent static force analysis**

*Step-1:* Depending on the location of the building site, identify the seismic zone and assign

Zone factor ( $Z$ )

- Use Table 2 along with Seismic zones map or Annex of IS-1893 (2002)

*Step-2:* Compute the seismic weight of the building ( $W$ )

- As per Clause 7.4.2, IS-1893 (2002) – Seismic weight of floors
- As per Clause 7.4.3, IS-1893 (2002) – Seismic weight of the building

*Step-3:* Compute the natural period of the building ( $T_a$ )

- As per Clause 7.6.1 or Clause 7.6.2, IS-1893 (2002), as the case may be.

*Step-4:* Obtain the data pertaining to type of soil conditions of foundation of the building

- Assign type, I for hard soil, II for medium soil & III for soft soil

*Step-5:* Using  $T_a$  and soil type (I / II / III), compute the average spectral acceleration  $\left(\frac{S_a}{g}\right)$

- Use Figure 2 or corresponding table of IS-1893 (2002), to compute  $\frac{S_a}{g}$

*Step-6:* Assign the value of importance factor ( $I$ ) depending on occupancy and/or functionality of structure

- As per Clause 7.2 and Table 6 of IS-1893 (2002),

*Step-7:* Assign the values of response reduction factor ( $R$ ) depending on type of structure

- As per Clause 7.2 and Table 7 of IS-1893 (2002)

*Step-8:* Knowing  $Z$ ,  $S_a/g$ ,  $R$  and  $I$  compute design horizontal acceleration coefficient ( $A_h$ )

using the relationship,  $A_h = \frac{Z S_a I}{2 g R}$  [Clause 6.4.2, IS-1893 (2002)]

*Step-9:* Using  $A_h$  and  $W$  compute design seismic base shear ( $V_B$ ), from  $V_B = A_h W$  [Clause 7.5.3, IS-1893 (2002)]

*Step-10:* Compute design lateral force ( $Q_i$ ) of  $i^{\text{th}}$  floor by distributing the design seismic base

shear ( $V_B$ ) as per the expression,  $Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^n W_j h_j^2}$  [Clause 7.7.1, IS-1893 (2002)]

### **Dynamic Analysis**

- Dynamic analysis is classified into two types, namely, Response spectrum method and Time history method
- Dynamic analysis shall be performed to obtain the design seismic force, and its distribution to different levels along the height of the building and to the various lateral load resisting elements, for the following buildings:

- a) *Regular buildings* — Those greater than 40 m in height in Zones IV and V, and those greater than 90 m in height in Zones II and III.
- b) *Irregular buildings* — All framed buildings higher than 12 m in Zones IV and V, and those greater than 40 m in height in Zones II and III.
- *Time History Method:* Time history method of analysis, when used, shall be based on an appropriate ground motion and shall be performed using accepted principles of dynamics.
  - *Response Spectrum Method:* Response spectrum method of analysis shall be performed using the design spectrum specified in Clause 6.4.2 or by a site specific design, spectrum mentioned in Clause 6.4.6 of IS 1893 (2002)
  - When dynamic analysis is carried out either by the Time History Method or by the Response Spectrum Method, the design base shear computed from dynamic analysis ( $V_B$ ) shall be compared with a base shear calculated using a fundamental period  $T_a$  ( $\bar{V}_B$ ), where  $T_a$  is as per Clause 7.6. If base shear obtained from dynamic analysis ( $V_B$ ) is less than base shear computed from equivalent static load method ( $\bar{V}_B$  i.e., using  $T_a$  as per Clause 7.6), then as per Clause 7.8.2, all the response quantities (for example member forces, displacements, storey forces, storey shears and base reactions) shall be multiplied by ratio  $\frac{\bar{V}_B}{V_B}$ .
  - *Free Vibration Analysis:* Undamped free vibration analysis of the entire building shall be performed as per established methods of mechanics using the appropriate masses and elastic stiffness of the structural system, to obtain natural periods ( $T$ ) and mode shapes ( $\phi$ ) of those of its modes of vibration that need to be considered.
  - *Modes to be considered:* The number of modes to be used in the analysis should be such that the sum total of modal masses of all modes considered is at least 90%. If modes with natural frequency beyond 33 Hz are to be considered, modal combination shall be carried out only for modes up to 33 Hz. The effect of modes with natural frequency beyond 33 Hz be included by considering missing mass correction following well established procedures.

- *Modal combination:* The peak response quantities (for example, member forces, displacements, storey forces, storey shears and base reactions) shall be combined as per Complete Quadratic Combination (CQC) method or alternatively, when building does not have closely spaced modes then Square Root of Square Sum (SRSS) method may be employed.

$$\text{CQC method: } \lambda = \sqrt{\sum_{i=1}^r \sum_{j=1}^r \lambda_i \rho_{ij} \lambda_j} \quad (9)$$

Where,

$r$  = Number of modes being considered,

$$\rho_{ij} = \frac{8\zeta^2(1 + \beta_{ij})\beta_{ij}^{1.5}}{(1 - \beta_{ij}^2)^2 + 4\zeta^2\beta_{ij}(1 + \beta_{ij})^2} = \text{Cross-modal coefficient,}$$

$\lambda_i$  = Response quantity in mode  $i$  (including sign),

$\lambda_j$  = Response quantity in mode  $j$  (including sign),

$\beta_{ij}$  = frequency ratio between  $i^{\text{th}}$  and the  $j^{\text{th}}$  mode is,  $\beta_{ij} = \frac{\omega_j}{\omega_i} = \frac{T_j}{T_i}$

$$\text{SRSS method: } \lambda = \sqrt{\sum_{k=1}^r \lambda_k^2} \quad (10)$$

$\lambda_k$  = Absolute value of response quantity in mode  $k$

### **Step by step procedure for Response spectrum method**

*Step-1:* Depending on the location of the building site, identify the seismic zone and assign Zone factor ( $Z$ )

- Use Table 2 along with Seismic zones map or Annex of IS-1893 (2002)

*Step-2:* Compute the seismic weight of the building ( $W$ )

- As per Clause 7.4.2, IS-1893 (2002) – Seismic weight of floors ( $W_i$ )

*Step-3:* Establish mass [ $M$ ] and stiffness [ $K$ ] matrices of the building using system of masses lumped at the floor levels with each mass having one degree of freedom, that

of lateral displacement in the direction under consideration. Accordingly, to develop stiffness matrix effective stiffness of each floor is computed using the lateral stiffness coefficients of columns and infill walls. Usually floor slab is assumed to be infinitely stiff.

*Step-4:* Using  $[M]$  and  $[K]$  of previous step and employing the principles of dynamics compute the modal frequencies,  $\{\omega\}$  and corresponding mode shapes,  $[\phi]$ .

*Step-5:* Compute modal mass  $M_k$  of mode  $k$  using the following relationship with  $n$  being number of modes considered

$$M_k = \frac{\left[ \sum_{i=1}^n W_i \phi_{ik} \right]^2}{g \sum_{i=1}^n W_i \phi_{ik}^2} \quad [\text{Clause 7.8.4.5a of IS 1893 (2002)}] \quad (11)$$

*Step-6:* Compute modal participation factors  $P_k$  of mode  $k$  using the following relationship with  $n$  being number of modes considered

$$P_k = \frac{\sum_{i=1}^n W_i \phi_{ik}}{\sum_{i=1}^n W_i \phi_{ik}^2} \quad [\text{Clause 7.8.4.5b of IS 1893 (2002)}] \quad (12)$$

*Step-7:* Compute design lateral force ( $Q_{ik}$ ) at each floor in each mode (i.e., for  $i^{\text{th}}$  floor in mode  $k$ ) using the following relationship,

$$Q_{ik} = A_{h(k)} \phi_{ik} P_k W_i \quad [\text{Clause 7.8.4.5c of IS 1893 (2002)}] \quad (13)$$

$A_{h(k)}$  = Design horizontal acceleration spectrum value as per Clause 6.4.2 of IS 1893

using the natural period  $\left( T_k = \frac{2\pi}{\omega_k} \right)$  of vibration of mode  $k$ .

*Step-8:* Compute storey shear forces in each mode ( $V_{ik}$ ) acting in storey  $i$  in mode  $k$  as given by,

$$V_{ik} = \sum_{i=1}^n Q_{ik} \quad [\text{Clause 7.8.4.5d of IS 1893 (2002)}] \quad (14)$$

*Step-9:* Compute storey shear forces due to all modes considered,  $V_i$  in storey  $i$ , by combining shear forces due to each mode in accordance with Clause 7.8.4.4 of IS 1893 (2002). i.e., either CQC or SRSS modal combination methods are used.

*Step-10:* Finally compute design lateral forces at each storey as,

$$F_{roof} = V_{roof} \text{ and} \quad [ \text{Clause 7.8.4.5f of IS 1893 (2002)} ] \quad (15)$$

$$F_i = V_i - V_{i+1}$$

### **EXAMPLE: 1**

Plan and elevation of a four-storey reinforced concrete office building is shown in Fig. 1.1. The details of the building are as follows.

Number of Storey = 4

Zone = III

Live Load = 3 kN/m<sup>2</sup>

Columns = 450 x 450 mm

Beams = 250 x 400 mm

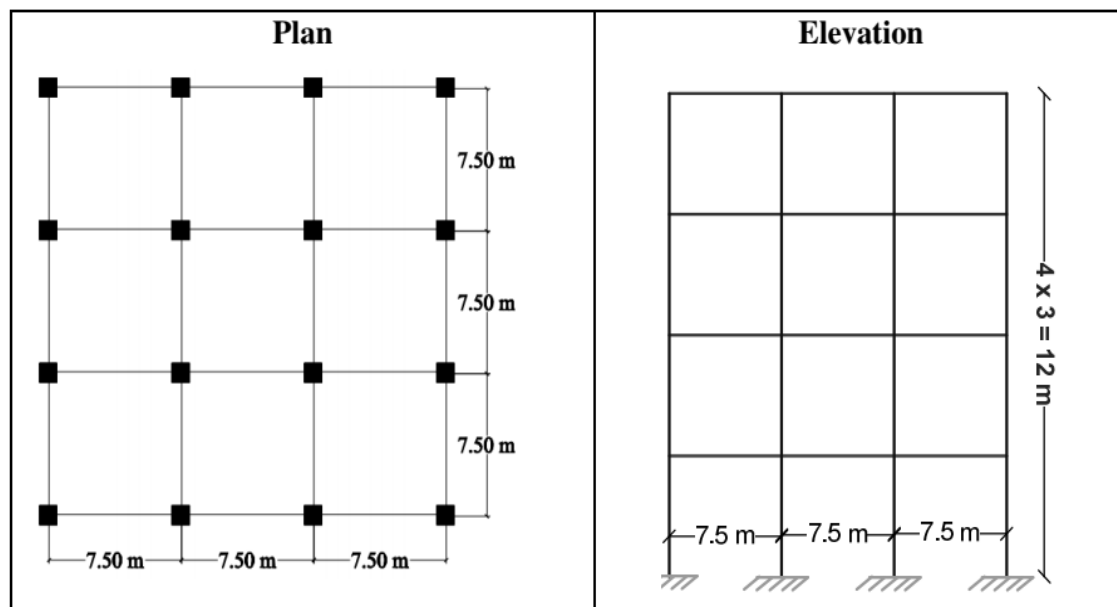
Thickness of Slab = 150 mm

Thickness of Wall = 120 mm

Importance factor = 1.0

Structure type = OMRF Building

Determine design seismic lateral load and storey shear force distribution.



**Solution: Analysis considering stiffness of infill masonry****1. Computation of Seismic weights**

(Assuming unit weight of concrete as  $25 \text{ kN/m}^3$  &  $22.5 \text{ kN/m}^3$  for masonry)

## 1) Slab:

$$\text{DL due to self weight of slab} = (22.5 \times 22.5 \times 0.15) \times 25 = 1898.40 \text{ kN}$$

## 2) Beams:

$$\text{Self weight of beam per unit length} = 0.25 \times 0.4 \times 25 = 2.5 \text{ kN/m}$$

$$\text{Total length} = 4 \times 22.5 \times 2 = 180 \text{ m}$$

$$\text{DL due to self weight of beams} = (2.5 \times 180) \times 4 \times 2 = 450 \text{ kN}$$

## 3) Columns:

$$\text{Self weight of column per unit length} = 0.45 \times 0.45 \times 25 = 5.0625 \text{ kN/m}$$

$$\text{DL due to self weight of columns (16 No.s)} = 16 \times 5.0625 \times 3.0 = 243 \text{ kN}$$

## 4) Walls:

$$\text{Self weight of wall per unit length} = 0.12 \times 3 \times 20 = 7.2 \text{ kN/m}$$

$$\text{Total length} = 4 \times 22.5 \times 2 = 180 \text{ m}$$

$$\text{DL due to self weight of Walls} = 7.2 \times 180 \times 4 = 648 \text{ kN}$$

5) Live Load [Imposed load] (25 %) =  $(0.25 \times 3) \times 22.5 \times 22.5 = 380 \text{ kN}$ **Load on all floors:**

$$W_1 = W_2 = W_3 = 1898 + 380 + 450 + 243 + 648 = \mathbf{3619 \text{ kN}}$$

**Load on roof slab (Live load on slab is zero)**

$$W_4 = 1898 + 0 + 450 + (243/2) + (648/2) = \mathbf{2793.5 \text{ kN}}$$

$$\text{Total Seismic weight, } W = (3619 \times 3) + 2793.5 = \mathbf{13650.5 \text{ kN}}$$

**Fundamental period:**

$$\text{Natural period, } T_a = 0.09 \frac{h}{\sqrt{d}} = 0.09 \frac{12}{\sqrt{22.5}} = 0.2277$$

(Moment resisting frame with in-fill walls)

**Spectral acceleration:**

Type of soil: Medium Soil

For  $T_a=0.2277$  s

$$S_a/g = 2.5$$

**Zone factor:** For Zone III,  $Z = 0.16$

**Importance Factor:**  $I = 1.0$

**Response Reduction Factor:**  $R = 3.0$  (OMRF)

**Horizontal acceleration coefficient ( $A_h$ ):**

$$A_h = \frac{Z S_a I}{2 g R} = \frac{0.16}{2} (2.5) \left( \frac{1}{3} \right)$$

$$A_h = 0.0667$$

**Base shear ( $V_B$ ):**

$$V_B = A_h W = 0.0667 \times 13650.50$$

$$V_B = 910.0333 \text{ kN}$$

Storey lateral forces and shear forces are calculated and tabulated in the following table.

Floor level (i)	$W_i$ (kN)	$h_i$ (m)	$W_i h_i^2$ (kN-m <sup>2</sup> )	Storey forces $Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^n W_j h_j^2}$	Storey shear forces [ $V_i$ ] (Cumulative sum) (kN)
4	2793.5	12.0	402264	426.53	426.53
3	3619	9.0	293139	310.83	737.35
2	3619	6.0	130284	138.14	875.50
1	3619	3.0	32571	34.54	910.03

Storey shear forces are calculated as follows (last column of the table),

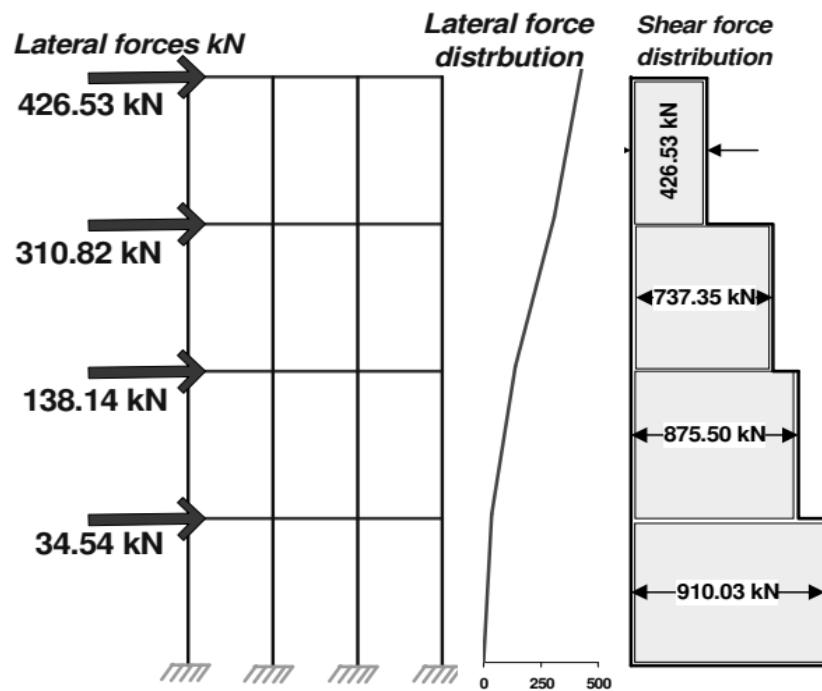
$$V_4 = Q_4 = 426.53 \text{ kN}$$

$$V_3 = V_4 + Q_3 = 426.53 + 310.82 = 737.35 \text{ kN}$$

$$V_2 = V_3 + Q_2 = 737.35 + 138.14 = 875.50 \text{ kN}$$

$$V_1 = V_2 + Q_1 = 875.50 + 34.54 = 910.03 \text{ kN} = V_B$$

Lateral force and shear force distribution is shown in the Figure-EX1.



**Figure – EX1: Lateral and Shear Force distribution along the height of the structure**

**Solution: Analysis without considering stiffness of infill masonry**

**Fundamental period:**

$$\text{Natural period, } T_a = 0.075h^{0.75} = 0.075 \times 12^{0.75} = 0.4836$$

(Moment resisting frame without in-fill walls)

**Spectral acceleration:**

Type of soil: Medium Soil

For  $T_a = 0.4836 \text{ s}$

$S_a/g = 2.5$  (because,  $T_a = 0.4836 \text{ s}$ , i.e.,  $0.10 \leq T_a \leq 0.55$ )

**Zone factor:** For Zone III,  $Z = 0.16$

**Importance Factor:**  $I = 1.0$

**Response Reduction Factor:**  $R = 3.0$  (OMRF)

**Horizontal acceleration coefficient ( $A_h$ ):**

$$A_h = \frac{Z S_a I}{2 g R} = \frac{0.16}{2} (2.5) \left( \frac{1}{3} \right)$$

$$A_h = 0.0667$$

**Base shear ( $V_B$ ):**

$$V_B = A_h W = 0.0667 \times 13650.50$$

$$V_B = 910.0333 \text{ kN}$$

Since, base shear  $V_B$  is same as in case of considering stiffness of infill walls, the storey lateral forces and shear forces are same as in the previous case. Therefore, Lateral and Shear Force distribution along the height of the structure shown in Figure-EX1 is valid. That is, for the structure under consideration, the lateral force and shear force distribution is unaltered irrespective of stiffness of infill walls is included or not in the analysis.

### **EXAMPLE: 2**

Analyse the building frame considered in Example-1 using response spectrum method (Dynamic analysis) with all other data being same.

**Solution:**

Note: In plan structure is symmetrical about both X and Y directions)

1) Seismic weights:

$$W_1 = W_2 = W_3 = 1898 + 380 + 450 + 243 + 648 = \mathbf{3619 \text{ kN}}$$

$$W_4 = 1898 + 0 + 450 + (243/2) + (648/2) = \mathbf{2793.5 \text{ kN}}$$

Therefore, seismic masses are,

$$M_1 = M_2 = M_3 = 368.91 \times 10^3 \text{ kg.}$$

$$M_4 = 284.76 \times 10^3 \text{ kg}$$

2) Floor stiffness (Without considering stiffness of infill wall):

$$MI \text{ of columns, } I_C = (0.45)^4 / 12 = 3.1417875 \times 10^{-3} \text{ m}^4$$

$$\text{Young's Modulus, } E_C = 5000(f_{ck})^{0.5} = 25000 \text{ MPa} = 25 \times 10^9 \text{ N/m}^2$$

(Assuming M25 concrete for columns)

$$K1 = K2 = K3 = K4 = 16 \times (12 \times 25 \times 10^9 \times 3.1417875 \times 10^{-3}) / (3^3) = 0.6075 \times 10^9 \text{ N/m}$$

3) Natural frequencies and Mode shapes:

Mass matrix,

$$M = \begin{bmatrix} M1 & 0 & 0 & 0 \\ 0 & M2 & 0 & 0 \\ 0 & 0 & M3 & 0 \\ 0 & 0 & 0 & M4 \end{bmatrix} = \begin{bmatrix} 368.91 & 0 & 0 & 0 \\ 0 & 368.91 & 0 & 0 \\ 0 & 0 & 368.91 & 0 \\ 0 & 0 & 0 & 284.76 \end{bmatrix} \times 10^3 \text{ kg}$$

Stiffness Matrix,

$$K = \begin{bmatrix} K1+K2 & -K2 & 0 & 0 \\ -K2 & K2+K3 & -K3 & 0 \\ 0 & -K3 & K3+K4 & -K4 \\ 0 & 0 & -K4 & -K4 \end{bmatrix} = \begin{bmatrix} 1.215 & -0.6075 & 0 & 0 \\ -0.6075 & 1.215 & -0.6075 & 0 \\ 0 & -0.6075 & 1.215 & -0.6075 \\ 0 & 0 & -0.6075 & 0.6075 \end{bmatrix} \times 10^9 \text{ N/m}$$

Solving the Eigen equation,  $|K - M\omega^2| = 0$ , we get Eigen value and corresponding Eigen vectors as,

$$\text{Eigen values, } \omega^2 = \begin{Bmatrix} 219.9 \\ 1793.2 \\ 4079.8 \\ 5920.9 \end{Bmatrix} \therefore \text{the natural frequencies are, } \omega = \begin{Bmatrix} 14.83 \\ 42.35 \\ 63.87 \\ 76.95 \end{Bmatrix} \text{ rad/s}$$

The mode shapes are,

$$\phi_1 = \begin{Bmatrix} 1.00 \\ 1.87 \\ 2.48 \\ 2.77 \end{Bmatrix}, \phi_2 = \begin{Bmatrix} 1.00 \\ 0.91 \\ -0.17 \\ -1.07 \end{Bmatrix}, \phi_3 = \begin{Bmatrix} 1.00 \\ -0.48 \\ -0.77 \\ 0.85 \end{Bmatrix}, \& \phi_4 = \begin{Bmatrix} 1.00 \\ -1.60 \\ 1.55 \\ -0.87 \end{Bmatrix}$$

The natural periods are,  $T = 2\pi / \{\omega\} = \begin{Bmatrix} 0.424 \\ 0.148 \\ 0.098 \\ 0.082 \end{Bmatrix}$  seconds

Calculation of modal participation factor

Storey Level	Seismic weight ( $W_i$ ), kN	MODE-1		
		$\phi_{i1}$	$W_i \phi_{i1}$	$W_i \phi_{i1}^2$
4	2793.5	2.77	7737.995	21434.25
3	3619	2.48	8975.12	22258.3
2	3619	1.87	6767.53	12655.28
1	3619	1.00	3619.00	3619.00
<b><math>\Sigma</math></b>	<b>13650.5</b>		<b>27099.65</b>	<b>59966.82</b>
Modal mass $M_1 = \frac{[\sum W_i \phi_{i1}]^2}{g \sum W_i \phi_{i1}^2}$		$= \frac{27099.65^2}{59966.82g} = 12246.62 \text{ kN/g}$		
% of Total weight		89.72 %		
Modal participation factor, $P_1 = \frac{\sum W_i \phi_{i1}}{\sum W_i \phi_{i1}^2}$		$= \frac{27099.65}{59966.82} = \mathbf{0.452}$		

Storey Level	Seismic weight ( $W_i$ ), kN	MODE-2		
		$\phi_{i2}$	$W_i \phi_{i2}$	$W_i \phi_{i2}^2$
4	2793.5	-1.07	-2989.05	3198.278
3	3619	-0.17	-615.23	104.5891
2	3619	0.91	3293.29	2996.894
1	3619	1.00	3619.00	3619.00
<b><math>\Sigma</math></b>	<b>13650.5</b>		<b>3308.015</b>	<b>9918.761</b>
Modal mass, $M_2 = \frac{[\sum W_i \phi_{i2}]^2}{g \sum W_i \phi_{i2}^2}$		$= \frac{3308.015^2}{9918.761g} = 1103.26 \text{ kN/g}$		
% of Total weight		8.08 %		
Modal participation factor, $P_2 = \frac{\sum W_i \phi_{i2}}{\sum W_i \phi_{i2}^2}$		$= \frac{3308.015}{9918.761} = \mathbf{0.334}$		

Storey Level	Seismic weight ( $W_i$ ), kN	MODE-3		
		$\phi_{i3}$	$W_i \phi_{i3}$	$W_i \phi_{i3}^2$
4	2793.5	0.85	2374.475	2018.304
3	3619	-0.77	-2786.63	2145.705
2	3619	-0.48	-1737.12	833.8176
1	3619	1.00	3619.00	3619.00
<b><math>\Sigma</math></b>	<b>13650.5</b>		<b>1469.725</b>	<b>8616.826</b>
Modal mass, $M_3 = \frac{[\sum W_i \phi_{i3}]^2}{g \sum W_i \phi_{i3}^2}$		$= \frac{1469.725^2}{8616.826g} = 250.683 \text{ kN/g}$		
% of Total weight		1.84 %		
Modal participation factor, $P_3 = \frac{\sum W_i \phi_{i3}}{\sum W_i \phi_{i3}^2}$		$= \frac{1469.725}{8616.826} = 0.171$		

Storey Level	Seismic weight ( $W_i$ ), kN	MODE-4		
		$\phi_{i4}$	$W_i \phi_{i4}$	$W_i \phi_{i4}^2$
4	2793.5	-0.87	-2430.35	2114.4
3	3619	1.55	5609.45	8694.648
2	3619	-1.60	-5790.4	9264.64
1	3619	1.00	3619.00	3619.00
<b><math>\Sigma</math></b>	<b>13650.5</b>		<b>1007.705</b>	<b>23692.69</b>
Modal mass, $M_4 = \frac{[\sum W_i \phi_{i4}]^2}{g \sum W_i \phi_{i4}^2}$		$= \frac{1007.705^2}{23692.69g} = 42.86 \text{ kN/g}$		
% of Total weight		0.314 %		
Modal participation factor, $P_4 = \frac{\sum W_i \phi_{i4}}{\sum W_i \phi_{i4}^2}$		$= \frac{1007.705}{23692.69} = 0.043$		

The lateral load  $Q_{ik}$  acting at  $i^{\text{th}}$  floor in the  $k^{\text{th}}$  mode is,

$$Q_{ik} = A_{h(k)} \phi_{ik} P_k W_i \dots\dots\dots \text{(Clause 7.8.4.5c of IS: 1893 Part 1)}$$

The value of  $A_{h(k)}$  for different modes is obtained from clause 6.4.2.

MODE-1:

$$T_1 = 0.424 \text{ s}$$

$$\frac{S_a}{g} = 2.5 \dots (0.10 \leq T_1 \leq 0.55 - \text{Medium soil})$$

$$A_{h(1)} = \frac{Z}{2} \frac{S_a}{g} \frac{I}{R} = \frac{0.16}{2} (2.5) \frac{1}{3} = 0.0667$$

$$Q_{i1} = A_{h(1)} \phi_{i1} P_1 W_i = 0.0667 \times 0.452 \times (\phi_{i1} W_i) = 0.03015 (\phi_{i1} W_i)$$

MODE-2:

$$T_2 = 0.148 \text{ s}$$

$$\frac{S_a}{g} = 2.5 \dots (0.10 \leq T_2 \leq 0.55 - \text{Medium soil})$$

$$A_{h(2)} = \frac{Z}{2} \frac{S_a}{g} \frac{I}{R} = \frac{0.16}{2} (2.5) \frac{1}{3} = 0.0667$$

$$Q_{i2} = A_{h(2)} \phi_{i2} P_2 W_i = 0.0667 \times 0.334 \times (\phi_{i2} W_i) = 0.0223 (\phi_{i2} W_i)$$

MODE-3:

$$T_3 = 0.098 \text{ s}$$

$$\frac{S_a}{g} = 1 + 15T_3 = 2.47 \dots (0.00 \leq T_3 \leq 0.10 - \text{Medium soil})$$

$$A_{h(3)} = \frac{Z}{2} \frac{S_a}{g} \frac{I}{R} = \frac{0.16}{2} (2.47) \frac{1}{3} = 0.0659, \text{ But, } T_3 \leq 0.10,$$

$$\therefore A_{h(3)} = \frac{Z}{2} = 0.08 > 0.0659$$

$$Q_{i3} = A_{h(3)} \phi_{i3} P_4 W_i = 0.08 \times 0.171 \times (\phi_{i3} W_i) = 0.01368 (\phi_{i3} W_i)$$

MODE-4:

$$T_4 = 0.082 \text{ s}$$

$$\frac{S_a}{g} = 1 + 15T_4 = 2.23 \dots (0.00 \leq T_4 \leq 0.10 - \text{Medium soil})$$

$$A_{h(4)} = \frac{Z}{2} \frac{S_a}{g} \frac{I}{R} = \frac{0.16}{2} (2.23) \frac{1}{3} = 0.0595, \text{ But, } T_3 \leq 0.10,$$

$$\therefore A_{h(4)} = \frac{Z}{2} = 0.08 > 0.0595$$

$$Q_{i4} = A_{h(4)} \phi_{i4} P_4 W_i = 0.08 \times 0.043 \times (\phi_{i4} W_i) = 0.00344 (\phi_{i4} W_i)$$