Lateral load calculation by modal analysis – SRSS method

Storey level	Weight Wi (kN)	**Mode – 1** [$Q_{i1} = 0.03015(\phi_{i1}\ W_i)$]		
		ϕ_{i1}	Q_{i1}	V_{i1}
4	2793.5	2.77	233.30	**233.30**
3	3619	2.48	270.60	**503.90**
2	3619	1.87	204.04	**707.94**
1	3619	1.00	109.11	**817.05**

Storey level	Weight Wi (kN)	**Mode – 2** [$Q_{i2} = 0.0223(\phi_{i2}\ W_i)$]		
		ϕ_{i2}	Q_{i2}	V_{i2}
4	2793.5	-1.07	-66.66	**-66.66**
3	3619	-0.17	-13.72	**-80.38**
2	3619	0.91	73.44	**-6.93**
1	3619	1.00	80.70	**73.77**

Storey level	Weight Wi (kN)	**Mode – 3** [$Q_{i3} = 0.01368(\phi_{i3}\ W_i)$]		
		ϕ_{i3}	Q_{i3}	V_{i3}
4	2793.5	0.85	32.48	**32.48**
3	3619	-0.77	-38.12	**-5.64**
2	3619	-0.48	-23.76	**-29.40**
1	3619	1.00	49.51	**20.11**

Storey level	Weight Wi (kN)	**Mode – 4** [$Q_{i4} = 0.0034(\phi_{i4}\ W_i)$]		
		ϕ_{i4}	Q_{i4}	V_{i4}
4	2793.5	-0.87	-8.26	**-8.26**
3	3619	1.55	19.07	**10.81**
2	3619	-1.6	-19.69	**-8.88**
1	3619	1.00	12.30	**3.43**

SRSS method

The contribution of different modes are combined by Square Root of the Sum of the Squares (SRSS) using the following relationship, $V_i = \sqrt{V_{i1}^2 + V_{i2}^2 + V_{i3}^2 + V_{i4}^2}$

Then, storey lateral forces are calculated by, $F_i = V_i - V_{i+1}$.

The results obtained are tabulated in the following table.

Storey level	V_{i1}	V_{i2}	V_{i3}	V_{i4}	Combined shear force (SRSS) V_i (kN)	Combined lateral force (SRSS) F_i (kN)
4	**233.30**	**-66.66**	**32.48**	**-8.26**	**244.94**	**244.94**
3	**503.90**	**-80.38**	**-5.64**	**10.81**	**510.42**	**265.48**
2	**707.94**	**-6.93**	**-29.40**	**-8.88**	**708.64**	**198.22**
1	**817.05**	**73.77**	**20.11**	**3.43**	**820.63**	**111.99**

CQC method

(**Important note:** *Since modal frequencies are well separated in this example, the SRSS modal combination method is sufficient to combine contribution of each mode. For the purpose of demonstration CQC method of modal combination and to compare SRSS and CQC methods following calculations are carried out. However, CQC method is preferred when modal frequencies are closely spaced*)

The contributions of different modes are combined by Complete Quadratic Combination (CQC) method as demonstrated in the following calculations. Shear force quantities in each of the four modes can be expressed as,

$$\lambda_4 = \{V_{41} \quad V_{42} \quad V_{43} \quad V_{44}\} = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}$$
$$\lambda_3 = \{V_{31} \quad V_{32} \quad V_{33} \quad V_{34}\} = \{503.90 \quad -80.38 \quad -5.64 \quad 10.81\}$$
$$\lambda_2 = \{V_{21} \quad V_{22} \quad V_{23} \quad V_{24}\} = \{707.94 \quad -6.93 \quad -29.40 \quad -8.88\}$$
$$\lambda_1 = \{V_{11} \quad V_{12} \quad V_{13} \quad V_{14}\} = \{817.05 \quad 73.77 \quad 20.11 \quad 3.43\}$$

Where λ_i is the shear force in the i^{th} mode.

β_{ij} is the frequency ratio between i^{th} and the j^{th} mode is, $\beta_{ij} = \frac{\omega_j}{\omega_i} = \frac{T_j}{T_i}$, hence considering all the four modes, β_{ij} may be expressed in matrix form as,

$$\beta_{ij} = \begin{bmatrix} T_1/T_1 & T_2/T_1 & T_3/T_1 & T_4/T_1 \\ T_1/T_2 & T_2/T_2 & T_3/T_2 & T_4/T_2 \\ T_1/T_3 & T_2/T_3 & T_3/T_3 & T_4/T_3 \\ T_1/T_4 & T_2/T_4 & T_3/T_4 & T_4/T_4 \end{bmatrix} = \begin{bmatrix} 1.00 & 0.35 & 0.23 & 0.19 \\ 2.86 & 1.00 & 0.66 & 0.55 \\ 4.33 & 1.51 & 1.00 & 0.84 \\ 5.17 & 1.80 & 1.20 & 1.00 \end{bmatrix}$$

Where natural periods of different modes are, $T = \begin{Bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{Bmatrix} = \begin{Bmatrix} 0.424 \\ 0.148 \\ 0.098 \\ 0.082 \end{Bmatrix}$ sec

Now calculate cross modal coefficient ρ_{ij},

$$\rho_{ij} = \frac{8\varsigma^2(1+\beta_{ij})\beta_{ij}^{1.5}}{(1-\beta_{ij}^2)^2 + 4\varsigma^2\beta_{ij}(1+\beta_{ij})^2}$$

Taking damping ratio, $\varsigma = 0.05$ and β_{ij} values computed above, cross modal coefficient ρ_{ij} may be computed and expressed in matrix form as,

$$\rho_{ij} = \begin{bmatrix} 1.0000 & 0.01294 & 0.00460 & 0.00311 \\ 0.09597 & 1.0000 & 0.13526 & 0.06039 \\ 0.07799 & 0.26212 & 1.0000 & 0.51229 \\ 0.07494 & 0.17222 & 0.59962 & 1.0000 \end{bmatrix}$$

For example in the above matrix ρ_{12} & ρ_{34} are computed as,

$$\rho_{12} = \frac{8(0.05)^2(1+0.35)0.35^{1.5}}{(1-0.35^2)^2 + 4(0.05)^2(0.35)(1+0.35)^2} = 0.01294$$

$$\rho_{34} = \frac{8(0.05)^2(1+0.84)0.84^{1.5}}{(1-0.84^2)^2 + 4(0.05)^2(0.35)(1+0.84)^2} = 0.51229$$

Storey shear forces are computed by combining shear forces of different modes as follows,

$$V_4 = \sqrt{\{\lambda_4\}\left[\rho_{ij}\right]\{\lambda_4\}^T}$$

$$\lambda_4 = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}$$

$$\{\lambda_4\}^T = \{233.30 \quad -66.66 \quad 32.48 \quad -8.26\}^T$$

$$\therefore V_4 = \sqrt{\{\lambda_4\}\begin{bmatrix} 1.0000 & 0.01294 & 0.00460 & 0.00311 \\ 0.09597 & 1.0000 & 0.13526 & 0.06039 \\ 0.07799 & 0.26212 & 1.0000 & 0.51229 \\ 0.07494 & 0.17222 & 0.59962 & 1.0000 \end{bmatrix}\{\lambda_4\}^T}$$

$$V_4 = \sqrt{57747}$$

$$V_4 = 240.31 \ kN$$

Simillarly,

$$\text{V}_3 = 532.85 \ kN$$

$$\text{V}_2 = 706.97 \ kN$$

$$\text{V}_1 = V_{Base} = 826.01 \ kN$$

Now, storey lateral forces are computed from storey shear forces

$$Q_4 = V_4 = 240.31 \ kN$$

$$Q_3 = V_3 - V_4 = 532.85 - 240.31 = 292.31 \ kN$$

$$Q_2 = V_2 - V_3 = 706.97 - 532.85 = 174.12 \ kN$$

$$Q_1 = V_1 - V_2 = 826.01 - 706.97 = 119.04 \ kN$$

Table: Summary of results from different methods of analyses

Storey level	Equivalent static Method		CQC Method		SRSS Method	
	Shear force V_i (kN)	**Lateral force F_i (kN)**	**Shear force V_i (kN)**	**Lateral force F_i (kN)**	**Shear force V_i (kN)**	**Lateral force F_i (kN)**
4	**426.53**	**426.53**	**240.31**	**240.31**	**244.94**	**244.94**
3	**737.35**	**310.83**	**532.85**	**292.31**	**510.42**	**265.48**
2	**875.50**	**138.14**	**706.97**	**174.12**	**708.64**	**198.22**
1	**910.03**	**34.54**	**826.01**	**119.04**	**820.63**	**111.99**

The storey lateral forces and shear forces computed from equivalent static method and response spectrum method (dynamic analysis) are compared in the above table. In case of dynamic analysis the responses computed from both CQC and SRSS are tabulated for purpose of comparing these two methods of combining the individual modal response contributions. Comparison clearly indicates that both CQC and SRSS techniques consistently yield comparable shear and lateral force distribution with insignificantly small variation. It is important to note that in this particular example the modal frequencies are well separated. In case of such a situation, it sufficient to implement SRSS combination rule because CQC is a comparatively tedious when calculations are carried out manually.

Note: In the previous example, the building considered herein was analysed using equivalent static load method, wherein the fundamental period (T_a) of the structure is obtained using equation given under Clause 7.6 of IS-1893 (2002). When dynamic analysis is carried out either by the Time History Method or by the Response Spectrum Method, the design base shear computed from dynamic analysis (V_B) shall be compared with a base shear calculated using a fundamental period T_a ($\bar{V}_B$), where T_a is as per Clause 7.6. If base shear obtained from dynamic analysis (V_B) is less than base shear computed from equivalent static load method ($\bar{V}_B$ *i.e., using* T_a *as per Clause 7.6*), then as per Clause 7.8.2, all the response quantities (for example member forces, displacements, storey forces, storey shears and base reactions) shall be multiplied by ratio $\frac{\bar{V}_B}{V_B}$.

In the above example, $\bar{V}_B = 910.03\ kN$

Base shear as calculated by response spectrum method (SRSS) is, $V_B = 820.63\ kN$

$$\therefore\ \frac{\bar{V}_B}{V_B} = \frac{910.03}{820.63} = 1.109$$

Thus, the seismic forces obtained above by dynamic analysis should be scaled up as follows:

$$Q_4 = 244.94 \times 1.109 = 271.64\ kN$$
$$Q_3 = 265.48 \times 1.109 = 294.42\ kN$$
$$Q_2 = 198.22 \times 1.109 = 219.83\ kN$$
$$Q_1 = 111.99 \times 1.109 = 124.20\ kN$$

EXAMPLE: 3

For a four storeyed RCC office building located in zone V and resting on hard rock, compute the seismic forces as per IS-1893-2002 equivalent static procedure. Height of first is 4.2 m and the remaining three stories are of height 3.2 m each. Plan dimensions (length and width) of the structure are 15 m x 20 m. The RCC frames are infilled with brick masonry.

Dead load on floor 12 kN/m^2 on floors and 10 kN/m^2 on roof. Live = 4 kN/m^2 on floors and 1.5 kN/m^2 on roof.

Also compute the base shear, neglecting the stiffness of infill walls. Compare the base shears for the two cases and comment on the result.

Solution

Given data

Floor area = 15×20=300 m^2

Dead load: on floor = 12 kN/m^2

On roof = 10 kN/m^2

Live load: on floor = 4 kN/m^2

On roof = 1.5 kN/m^2

Note: Only 50% of the live load is lumped at the floors. At roof, no live load is to be lumped

Zone V, Z = 0.36

Assume SMRF thus, R = 5, Soil type = Hard Rock (Type-I)

Load at floor levels:

$$\text{Floors}: W1 = W2 = W3 = [12 + (0.5\times 4)]\times 300 = 4200 \text{ kN}$$
$$\text{Roof}: W4 = 10\times 300 = 3000 \text{ kN}$$

Total seismic weight:

$$W = W1 + W2 + W3 + W4 = (3\times 4200) + 3000 = \mathbf{15600\ kN}$$

Total height of the building:

$$h = (3.2\times 3) + 4.2 = \mathbf{13.8\ m}$$

CASE – 1: With infill walls

Fundamental natural period :

(Moment resisting frame with in - fill walls)

a) Along 20 m direction :

$$T_a = 0.09\frac{h}{\sqrt{d}} = 0.09\frac{13.8}{\sqrt{20.0}} = 0.2777$$

b) Along 15 m direction :

$$T_a = 0.09\frac{h}{\sqrt{d}} = 0.09\frac{13.8}{\sqrt{15.0}} = 0.3207$$

Along 20 m direction

(Hard Rock),

For Ta=0.2777 s Sa/g = 2.5

For Zone V, Z = 0.36

Importance Factor, I = 1.0

Response Reduction Factor, R = 5.0 (SMRF)

Along 15 m direction

(Hard Rock),

For Ta=0.3207 s Sa/g = 2.5

For Zone V, Z = 0.36

Importance Factor, I = 1.0

Response Reduction Factor, R = 5.0 (SMRF)

Note: Since, Sa/g, Z, I and R values are same for both principal directions, it is sufficient calculate lateral forces in any one the principal axis.

Calculation of Base shear

a) A_h & V_B Along 20 m direction:

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(2.5)\left(\frac{1}{5}\right) = 0.09$$

$$V_B = A_h W = 0.09 \times 15600 = \mathbf{1404.00\ kN}$$

b) A_h & V_B Along 15 m direction:

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(2.5)\left(\frac{1}{5}\right) = 0.09$$

$$V_B = A_h W = 0.09 \times 15600 = \mathbf{1404.00\ kN}$$

Storey lateral forces and shear forces are calculated and tabulated in the following table.

Floor level (i)	W_i (kN)	h_i (m)	$W_i h_i^2$ (kN-m^2)	Storey forces $Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$	Storey shear forces [V_i] (Cumulative sum) (kN)	
					Along 20 m Direction	Along 15 m Direction
4	3000	13.8	571320	595.36	595.36	595.36
3	4200	10.6	471912	491.77	1087.13	1087.13
2	4200	7.4	229992	239.67	1326.79	1326.79
1	4200	4.2	74088	77.21	**1404.00**	**1404.00**

CASE – 2: Without infill walls

Fundamental Natural period :

(Moment resisting frame without in-fill walls)

a) Along 20 m & 15 m directions:

$$T_a = 0.075h^{0.75} = 0.075 \times 13.8^{.75} = 0.537 \text{ sec}$$

Along 20 m & 15 m directions

(Hard Rock),

For Ta=0.537 s   Sa/g = 1/T = 1/0.537= 1.862

For Zone V,   Z = 0.36

Importance Factor,   I = 1.0

Response Reduction Factor,   R = 5.0 (SMRF)

Base shear:

a) A_h & V_B Along 20 m & 15 m directions:

$$A_h = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(1.862)\left(\frac{1}{5}\right) = 0.06704$$

$$V_B = A_h W = 0.06704 \times 15600 = \mathbf{1045.81\ kN}$$

Storey lateral forces and shear forces are calculated and tabulated in the following table.

Floor level (i)	W_i (kN)	h_i (m)	$W_i h_i^2$ (kN-m^2)	Storey forces $Q_i = V_B \frac{W_i h_i^2}{\sum_{j=1}^{n} W_j h_j^2}$	Storey shear forces [V_i] (Cumulative sum) (kN	
					Along 20 m Direction	Along 15 m Direction
4	3000	13.8	571320	443.47	443.47	443.47
3	4200	10.6	471912	366.31	809.78	809.78
2	4200	7.4	229992	178.52	988.30	988.30
1	4200	4.2	74088	57.51	**1045.81**	**1045.81**

EXAMPLE: 4

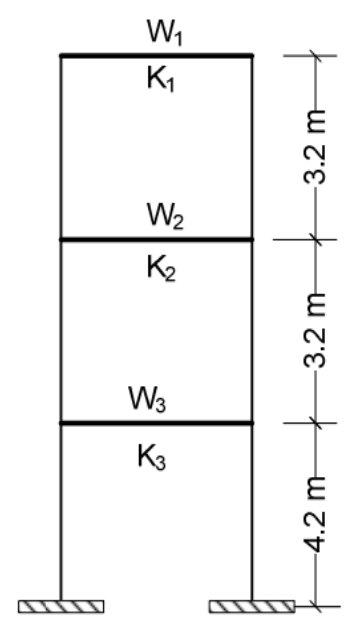
For the residential RCC (SMRF) building founded on soft soil and situated in zone V shown in figure. Compute the seismic forces for each storey using dynamic analysis procedure. Given the free vibration analysis results as follows,

Frequency: $\{\omega\} = \{47.832\ 120.155\ 167.00\}\ rad/\sec$

Modes: $\{\phi_1\} = \begin{Bmatrix} 1.00 \\ 0.759 \\ 0.336 \end{Bmatrix}$ $\{\phi_2\} = \begin{Bmatrix} 1.00 \\ -0.805 \\ -1.157 \end{Bmatrix}$ $\{\phi_3\} = \begin{Bmatrix} 1.00 \\ -2.427 \\ 0.075 \end{Bmatrix}$

Seismic weights: $W_1 = W_2 = W_3 = 1962\ kN$

Stiffness: $k_1 = k_2 = 160 \times 10^3\ kN/m$ and $k_3 = 240 \times 10^3\ kN/m$



Solution

Given data

W3=W2=W1=1962 kN,

Frame: SMRF, R = 5,

Zone V: Z = 0.36

Soil type: Soft soil (Type-III)

Structure type: Residential, I=1

Free vibration characteristics:

Modes	Natural Period (sec)	Mode shapes		
		3rd Floor	2nd Floor	1st Floor
Mode 1	0.131	1.000	0.759	0.336
Mode 2	0.052	1.000	-0.805	-1.157
Mode 3	0.038	1.000	-2.427	0.075

Calculation of modal mass and modal participation factor

Storey Level	Seismic weight (W_i), kN	MODE-1		
		ϕ_{i1}	$W_i\ \phi_{i1}$	$W_i\ \phi_{i1}^{\ 2}$
3	1962.00	1.000	1962	1962.00
2	1962.00	0.759	1489.16	1130.27
1	1962.00	0.336	659.23	221.50
Σ	**5886.00**		4110.39	3313.77
Modal mass $M_1 = \frac{\left[\sum W_i \phi_{i1}\right]^2}{g \sum W_i \phi_{i1}^2}$		$= \frac{3313.77^2}{4110.39g} =$ 5098.512 kN/g		
% of Total weight		86.62 %		
Modal participation factor, $P_1 = \frac{\sum W_i \phi_{i1}}{\sum W_i \phi_{i1}^2}$		$= \frac{4110.39}{3313.77} =$ **1.2404**		

Storey Level	Seismic weight (W_i), kN	MODE-2		
		ϕ_{i2}	$W_i\ \phi_{i2}$	$W_i\ \phi_{i2}^{\ 2}$
3	1962.00	1.000	1962.00	1962.00
2	1962.00	-.805	-1579.41	1130.27
1	1962.00	-1.157	-2270.03	221.50
Σ	**5886.00**		-1887.44	5859.85
Modal mass, $M_2 = \dfrac{\left[\sum W_i\phi_{i2}\right]^2}{g\sum W_i\phi_{i2}^2}$		$= \dfrac{-1887.44^2}{5859.85g} =$ 607.94 kN/g		
% of Total weight		10.34 %		
Modal participation factor, $P_2 = \dfrac{\sum W_i\phi_{i2}}{\sum W_i\phi_{i2}^2}$		$= \dfrac{-1887.44}{5859.85} =$ **-0.322**		

Storey Level	Seismic weight (W_i), kN	MODE-3		
		ϕ_{i3}	$W_i\ \phi_{i3}$	$W_i\ \phi_{i3}^{\ 2}$
3	1962.00	1.000	1962	1962
2	1962.00	-2.247	-4761.77	11556.83
1	1962.00	0.075	147.15	11.03625
Σ	**5886.00**		-2652.62	13529.86
Modal mass, $M_3 = \dfrac{\left[\sum W_i\phi_{i3}\right]^2}{g\sum W_i\phi_{i3}^2}$		$= \dfrac{-2652.62^2}{13829.56g} =$ 520.066 kN/g		
% of Total weight		8.84 %		
Modal participation factor, $P_3 = \dfrac{\sum W_i\phi_{i3}}{\sum W_i\phi_{i3}^2}$		$= \dfrac{-2652.62}{13529.86} =$ **-0.196**		

<u>MODE-1:</u>

$$T_1 = 0.131\ s$$

$$\frac{S_a}{g} = 2.5....(0.10 \le T_1 \le 0.67 - \text{Soft soil})$$

$$A_{h(1)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(2.5)\frac{1}{5} = 0.09$$

$$Q_{i1} = A_{h(1)}\ \phi_{i1}\ P_1\ W_i = 0.09 \times 1.2404 \times (\phi_{i1}\ \ W_i) = 0.11164(\phi_{i1}\ \ W_i)$$

MODE-2:

$$T_2 = 0.052\ s\ (T_2 \le 0.10)$$

$$\frac{S_a}{g} = 1 + (15 \times 0.052) = 1.78\ \ .....\ (0 \le T_2 \le 0.10,\ Soft\ soil)$$

$$A_{h(2)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(1.78)\frac{1}{5} = 0.0641$$

$$\therefore A_{h(2)} = \frac{Z}{2} = \frac{0.36}{2} = 0.18 > 0.0641$$

$$Q_{i2} = A_{h(2)}\ \phi_{i2}\ P_2\ W_i = 0.18 \times -0.322 \times (\phi_{i2} W_i) = -0.05796(\phi_{i2}\ W_i)$$

MODE-3:

$$T_3 = 0.038\ s\ (T_2 \le 0.10)$$

$$\frac{S_a}{g} = 1 + (15 \times 0.052) = 1.57\ \ .....\ (0 \le T_3 \le 0.10,\ Soft\ soil)$$

$$A_{h(3)} = \frac{Z}{2}\frac{S_a}{g}\frac{I}{R} = \frac{0.36}{2}(1.57)\frac{1}{5} = 0.0565$$

$$\therefore A_{h(3)} = \frac{Z}{2} = \frac{0.36}{2} = 0.18 > 0.0565$$

$$Q_{i3} = A_{h(3)}\ \phi_{i3}\ P_3\ W_i = 0.18 \times -0.196 \times (\phi_{i3} W_i) = -0.03528(\phi_{i3}\ W_i)$$

Lateral load calculation by modal analysis – SRSS method

Storey level	Weight *Wi* (kN)	**Mode – 1** [$Q_{i1} = 0.11164(\phi_{i1}\ W_i)$]		
		ϕ_{i1}	Q_{i1}	V_{i1}
3	1962.00	1.000	219.04	**219.04**
2	1962.00	0.759	166.25	**385.29**
1	1962.00	0.336	73.60	**458.88**

Storey level	Weight *Wi* (kN)	**Mode – 2** [$Q_{i2} = -0.05796(\phi_{i2}\ W_i)$]		
		ϕ_{i2}	Q_{i2}	V_{i2}
3	1962.00	1.000	-113.72	**-113.72**
2	1962.00	-.805	91.54	**-22.18**
1	1962.00	-1.157	131.57	**109.39**

Storey level	Weight Wi (kN)	Mode – 3 [$Q_{i3} = -0.03528(\phi_{i3}\ W_i)$]		
		ϕ_{i3}	Q_{i3}	V_{i3}
3	1962.00	1.000	-69.22	**-69.22**
2	1962.00	-2.247	155.54	**86.32**
1	1962.00	0.075	-5.19	**81.13**

SRSS method

The contribution of different modes are combined by Square Root of the Sum of the Squares (SRSS) using the following relationship, $V_i = \sqrt{V_{i1}^2 + V_{i2}^2 + V_{i3}^2}$

Then, storey lateral forces are calculated by, $F_i = V_i - V_{i+1}$.

The results obtained are tabulated in the following table.

Storey level	V_{i1}	V_{i2}	V_{i3}	**Combined shear force (SRSS) V_i (kN)**	**Combined lateral force (SRSS) F_i (kN)**
3	**219.04**	**117.25**	**69.22**	**256.32**	**256.32**
2	**385.29**	**22.86**	**-86.32**	**401.13**	**144.81**
1	**458.88**	**-112.79**	**-81.13**	**478.66**	**77.53**