

CONTINUOUS ASSESSMENT TEST - ANSWERS

Solutions.

Problem 1.

a) For first-order reaction kinetics

$$X_A = \frac{Da}{1 + Da} = \frac{k\tau}{1 + k\tau} = \frac{2.5 \times 10^{-3} \text{ min}^{-1} \times \frac{10000L}{0.3L/\text{sec}}}{1 + 2.5 \times 10^{-3} \text{ min}^{-1} \times \frac{10000L}{0.3L/\text{sec}}} = 0.58$$

b) Consider the non-steady state design equation for CSTR, where we can have

$$F_{A0} - F_A + r_A V = \frac{dN_A}{dt}$$

Since in liquid phase with constant density, we have

$$C_{A0} - C_A - kC_A \tau = \tau \frac{dC_A}{dt}$$

or equivalently

$$\frac{dC_A}{dt} + \frac{1 + k\tau}{\tau} C_A = \frac{C_{A0}}{\tau}$$

with the initial conditions $C_{A,t=0} = (1 - 0.58) \times 0.12 \text{ mol/L} = 0.05 \text{ mol/L}$, new $\tau = 10000L / (0.7 \times 0.3 \times 60 \text{ L/min}) = 793.7 \text{ min}$

Therefore, integrate this equation we can have

$$C_A(t) = \frac{C_{A0}}{1 + k\tau} \left\{ 1 - \exp\left[-\frac{1 + k\tau}{\tau} t\right] \right\} + C_{A,t=0} \exp\left(-\frac{1 + k\tau}{\tau} t\right)$$

Therefore the conversion at time t is

$$X_A(t) = 1 - \frac{1}{1 + k\tau} \left\{ 1 - \exp\left[-\frac{1 + k\tau}{\tau} t\right] \right\} - \frac{C_{A,t=0}}{C_{A0}} \exp\left[-\frac{1 + k\tau}{\tau} t\right]$$

After 60 min of changing flow rate,

$$X_A(60 \text{ min}) = 1 - \frac{1}{1 + k\tau} \left\{ 1 - \exp\left[-\frac{1 + k\tau}{\tau} t\right] \right\} - \frac{C_{A,t=0}}{C_{A0}} \exp\left[-\frac{1 + k\tau}{\tau} t\right] = 0.598$$

The new steady state fractional conversion is

$$X_A = \frac{Da}{1 + Da} = \frac{k\tau}{1 + k\tau} = \frac{2.5 \times 10^{-3} \text{ min}^{-1} \times \frac{10000L}{0.3L/\text{sec} \times 0.7}}{1 + 2.5 \times 10^{-3} \text{ min}^{-1} \times \frac{10000L}{0.3L/\text{sec} \times 0.7}} = 0.665$$

c) The steady state productivity (moles/time) of B is F_B , and the ratio of that in b) vs. that in a) is

$$\frac{F_{B,b}}{F_{B,a}} = \frac{0.3L/\text{sec} \times 0.7 \times 0.665}{0.3L/\text{sec} \times 0.58} = 0.803$$

Therefore, the productivity will be decreased by lowering the flow rate even though a higher conversion is to be achieved.

Problem 2.

We know for first order reaction, conversion X_A has the following relation with rate constant k , CSTR reactor volume V and volumetric flow rate v

$$X_A = \frac{k \frac{V}{v}}{1 + k \frac{V}{v}}$$

When we have a dead volume (denoted as subscript 1) which does not interact with the input and output streams, we can deduce this amount of volume from the overall reactor volume.

$$X_{A1} = \frac{k \frac{V_1}{v}}{1 + k \frac{V_1}{v}}$$

We are told $V_1=600L-400L=200L$, $X_{A1}=0.75$, therefore we can calculate

$$\frac{k}{v} = \frac{X_{A1}}{V_1(1-X_{A1})} = 0.015.$$

Then for the well-stirred reactor, $V=600L$

$$X_A = \frac{k \frac{V}{v}}{1 + k \frac{V}{v}} = 0.9$$

Problem 3.

For a PFR reactor, the design equation is

$$\frac{dX_{\text{cellulose}}}{dz} = S \frac{-r_{\text{cellulose}}}{F_{\text{cellulose, in}}}$$

Now $S=10\text{cm}^2$, length=2m, $F_{\text{cellulose, in}}=v_{\text{in}}[\text{cellulose}]_{\text{in}}$, in order to know v , we have to convert 60 grams of slurry into total moles of cellulose and water per second, that is

$$\begin{aligned} \dot{n}_{\text{total, in}} &= \frac{60 \text{ gram/sec} \times 20\%}{\text{MW}_{\text{cellulose}}} + \frac{60 \text{ gram/sec} \times (1 - 20\%)}{\text{MW}_{\text{water}}} = \\ &= \frac{60 \text{ gram/sec} \times 20\%}{720 \text{ gram/mol}} + \frac{60 \text{ gram/sec} \times (1 - 20\%)}{18 \text{ gram/mol}} = 2.683 \text{ mol/sec} \end{aligned}$$

Using ideal gas law to get the volumetric flow rate:

$$v_{\text{in}} = \frac{\dot{n}_{\text{total, in}} RT}{P} = \frac{2.683 \text{ mol/sec} \times 8.314 \text{ J/mol/K} \times 600\text{K}}{20 \text{ atm}} = 6.61 \text{ liter/sec}$$

Also

$$[\text{cellulose}]_{\text{in}} = \frac{y_{\text{cellulose,0}} P}{RT} = \frac{\frac{20\%}{720 \text{ gram/mol}}}{\left(\frac{20\%}{720 \text{ gram/mol}} + \frac{1 - 20\%}{18 \text{ gram/mol}}\right)} \frac{20 \text{ atm}}{8.314 \text{ J/(mol K)} \times 600\text{K}} = 2.52 \text{ mol/m}^3$$

where $y_{\text{cellulose,0}}$ is the molar fraction of cellulose in the inlet.

In this problem, since the gas phase reaction creates more molecules, the volumetric flow rate is not a constant.

$$-r_{\text{cellulose}} = k \frac{[\text{cellulose}]}{1 + a[\text{CO}]}$$

where the concentrations should be expressed (constant pressure and temperature are assumed)

$$[\text{cellulose}] = [\text{cellulose}]_{\text{in}} \frac{(1 - X)}{(1 + \varepsilon X)}$$

and

$$[\text{CO}] = [\text{H}_2] = [\text{cellulose}]_{\text{in}} \frac{24X}{(1 + \varepsilon X)}$$

Here $\varepsilon=(24+24-1)y_{\text{cellulose,0}}=47*0.0062=0.292$.

Now the final molar flow rate of H_2 is

$$v_{\text{final}}[\text{H}_2]=24v_{\text{in}}[\text{cellulose}]_{\text{in}}X_f$$