

OPERATIONS RESEARCH

LECTURE ELEVEN

Transportation problems (2)

Lecturer: Dr. Emily Roche

INTRODUCTION

This lecture will focus on the Least Cost Method (LCM) and Vogel's Approximation Method (VAM) of the Transportation problem as methods of solving transportation problems.

Intended learning outcomes

At the end of this lecture, you will be able to apply the LCM and VAM to solve transportation problem.

References

These lecture notes should be supplemented with relevant topics from the books listed in the Bibliography at the end of the lecture

1. Least Cost Method (LCM)

The procedure of finding the initial feasible solution is

- Select the cell with the lowest transportation cost among all the rows and columns of the transportation table. If the minimum cost is not unique, then select any cell with this minimum cost arbitrarily.
- Allocate as many units as possible to the cell identified and eliminate the row or column in which either supply or demand is satisfied.
- Repeat the process until all the possible cells are fully allocated.

Example

Use LCM to determine the starting solution to the transportation problem below

To \ From	1	2	3	4	5	Supply
1	4	1	2	6	9	100
2	6	4	3	5	7	120
3	5	2	6	4	8	120
Demand	40	50	70	90	90	

Solution

Identify the cell with the least transportation cost among all the cells in the transportation tableau and allocate the maximum possible units. The least cost in this tableau is in row one column two, whose demand is 50 and supply 100. The maximum possible allocation is the least between the two units.

To \ From	1	2	3	4	5	Supply
1	4	1 / 50	2 / 50	6	9	50
2	6	4	3	5	7	120
3	5	2	6	4	8	120
Demand	40	50	70	90	90	

Since the demand in column two has been met, we delete column two in the next tableau, adjust the row supplies and repeat the process. The next least cost is in row one column 3 (current column two)

To \ From	1	3	4	5	Supply
1	4	2	6	9	50
2	6	3	5	7	120
3	5	6	4	8	120
Demand	40	70	90	90	

The supply in row one has now been satisfied. This then means that row one is deleted in the next tableau and the column supply adjusted for the next allocation

To \ From	1	3	4	5	Supply
2	6	3	5	7	120
3	5	6	4	8	120
Demand	40	20	90	90	

The next tableau then is

To \ From	1	4	5	Supply
2	6	5	7	100
3	5	4	8	120
Demand	40	90	90	

The remaining allocations for the last four cells can be done on a single tableau as

To \ From	1	5	Supply
2	6	7	100
3	5	8	30
Demand	40	90	

The final allocations consolidated in a single tableau is:

To \ From	1	2	3	4	5	Supply
1	4	1 50	2 50	6	9	100
2	6 10	4	3 20	5	7 90	120
3	5 30	2	6	4 90	8	120
Demand	40	50	70	90	90	

The initial feasible solution will then be

$$6(10) + 5(30) + 1(50) + 2(50) + 3(20) + 4(90) + 7(90) = 1410$$

2. Vogel's Approximation Method (VAM)

This is a more preferred method because the initial basic feasible solution obtained is either optimum or very close to the optimum. The procedure is

- Compute a penalty (difference between the smallest and the next smallest) for each row and column.
- Identify the row or column with the largest penalty, choose the cell with the least cost and allocate the maximum possible units.
- Reduce the row supply or column demand by the amount assigned to the cell. If the supply or demand is zero, eliminate the row or column or both.
- Repeat the process for the reduced transportation table until the cells are fully allocated.

Example

Use VAM to determine the starting solution to the transportation problem below

To \ From	1	2	3	4	Supply
1	21	16	25	13	11
2	17	18	14	23	13
3	32	27	18	41	19
Demand	6	10	12	15	90

Solution

Compute the row and column penalties and make allocations. The first allocation is made in the cell on row one column four, since this is the cell with the least cost in the column with the highest penalty.

To \ From	1	2	3	4	Supply	Row penalty
1	21	16	25	13	11	3
2	17	18	14	23	13	3
3	32	27	18	41	19	9
Demand	6	10	12	15	90	
Column penalty	4	2	4	10		

Delete row one whose supply has then been satisfied. Adjust the demand requirements for column four and repeat the procedure. The highest penalty is in column four with the least cost in row two (current row one) thus:

To \ From	1	2	3	4	Supply	Row penalty
2	17/6	18/10	14/12	23/4	13	3
3	32/6	27/10	18/12	41/4	19	9
Demand	6	10	12	4	90	
Column penalty	15	9	4	18		

Column four is then deleted since its demand has been met and the procedure repeated

To \ From	1	2	3	Supply	Row penalty
2	17/6	18/10	14/12	9	3
3	32/6	27/10	18/12	19	9
Demand	6	10	12	90	
Column penalty	15	9	4		

The next allocation has resulted to the deletion of column one. The next tableau has no unique high penalty, therefore the next allocation is done in a cell chosen arbitrarily. With the arbitrary allocation done, there will only be two cells left whose allocations can also be done within the same tableau as below

To \ From	2	3	Supply	Row penalty
2	18/3	14/12	3	4
3	27/7	18/12	19	9
Demand	10	12	90	
Column penalty	9	4		

The allocations are then consolidated in a single tableau to help check on the row and column totals. The final allocations from the VAM procedure then is

To \ From	1	2	3	4	Supply
1	21/ 11	16/ 11	25/ 11	13/ 11	11
2	17/ 6	18/ 3	14/ 14	23/ 4	13
3	32/ 6	27/ 7	18/ 12	41/ 15	19
Demand	6	10	12	15	90

The initial feasible starting solution will be

$$13(11) + 17(6) + 18(3) + 23(4) + 27(7) + 18(12) = 796.$$

From these starting feasible solutions form both LCM and VAM, we determine the optimum solution using the **stepping stone** method as we had learnt in the previous lesson.

Bibliography

Lucey, T. (2002). *Quantitative Techniques* (6th ed.). Cengage Learning.

Taha, H. A. (2017). *Operation Research An introduction* (10th ed.). Prentice-Hall, Inc.