

OPERATIONS RESEARCH

LECTURE THIRTEEN

Network models

Lecturer: Dr. Emily Roche

INTRODUCTION

This lecture will focus on definition of terms and time estimates in network analysis

Intended learning outcomes

At the end of this lecture, you will be able to explain project management model, construct a project network and construct the Critical Path (CP) through the network.

References


These lecture notes should be supplemented with relevant topics from the books listed in the Bibliography at the end of the lecture.

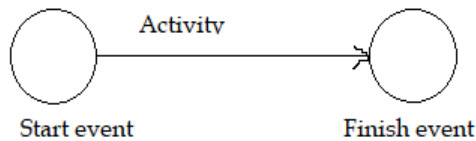
Network models

This refers to a family of related techniques that help management to plan and control projects. For example Building and construction, Budgeting and auditing, Assembly line scheduling, Missile development, Installation of complex equipment, Planning, Advertising programmes, Finding the best traffic flow pattern in a city, Research and development.

Definitions

1. **Activity** - A task or job which takes time and resources denoted by (\rightarrow).
2. **Predecessor activity** - An activity that must be completed immediately prior to the start of another activity.
3. **Successor activity** - An activity that succeeds other activities.

4. **Concurrent activities** - Activities that can be accomplished together.
5. **Critical activity** - An activity whose delay causes delay in the entire project.
6. **Dummy activity** - An activity that does not consume time or resource. It is denoted by (\dashrightarrow).
7. **Event** - a point in time that indicates the start or finish of an activity denoted by 

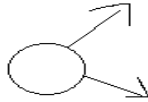


There are three types of events

- i. **Merge event** - more than one activity join an event.



- ii. **Burst event** - more than one activity leave an event.



- iii. **Merge and Burst event** - a combination of the two events.



8. **Network** - a combination of activities and events in a logical sequence according to the rules of constructing networks.

Rules for construction:

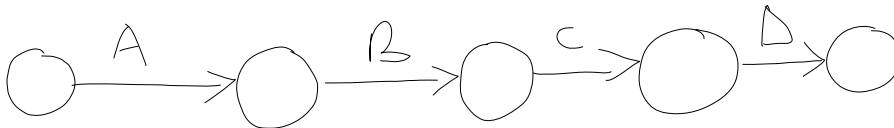
- a. Each activity is represented by one and only one arrow in the network.
- b. No two activities can be identified by the same start and finish event. In such cases a dummy activity is used.
- c. The flow of the network diagram should be from the left to right.
- d. Each activity must have a unique head and tail event.
- e. A complete network should have only one start event and only one finish event.

Illustrations:

Construction for the illustrations below are found in lecture thirteen recordings.

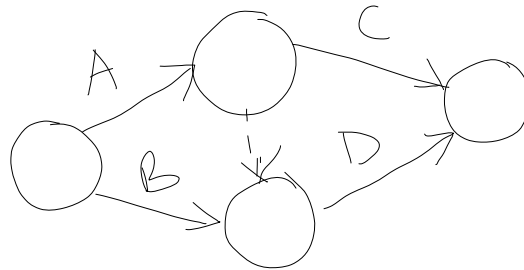
1)

Activity	Preceding activity
A. Buying chewing gum	-
B. Removing the wrapper	A
C. Putting the gum in the mouth	B
D. Chewing the gum	C



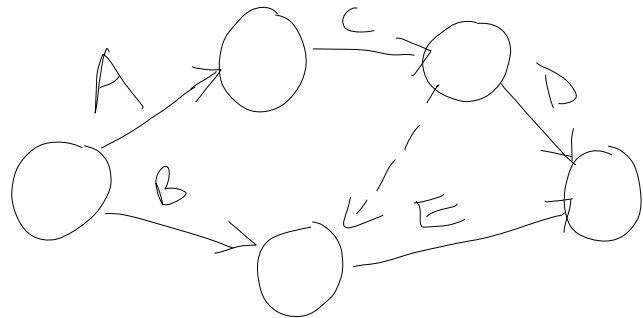
2)

Activity	Preceding activity
A	-
B	-
C	A
D	A,B



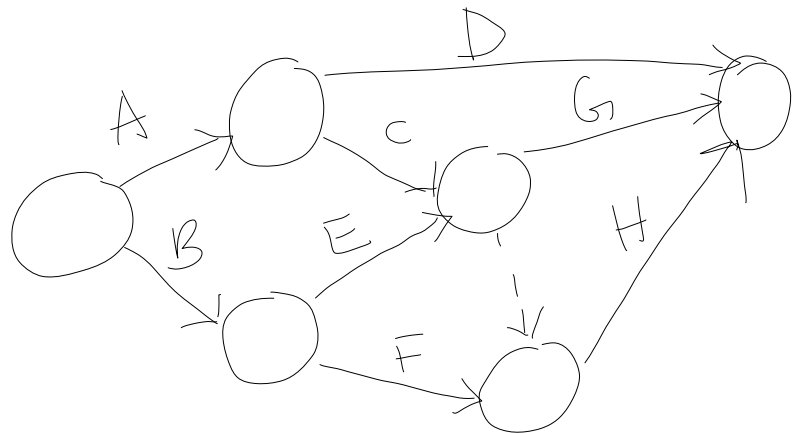
3)

Activity	Preceding activity
A	-
B	-
C	A
D	C
E	B,C



4)

Activity	Preceding activity
A	-
B	-
C	A
D	A
E	B
F	B
G	C,E
H	C,E,F



Time estimates in Network analysis

This is useful for planning the various activities of a project. It requires an estimation of time to complete an activity. An activity time is the forecast of time expected of each activity from the starting time to its completion under normal conditions.

The time analysis plan includes:

1. Total completion time for the project.
2. Earliest time when each activity can begin.
3. The latest time when each activity can be started without delaying the total project.
4. Float for each activity – the amount of time by which the completion of an activity can be delayed without delaying the total project completion.
5. Identification of the critical activities and critical path.

There are two approaches to the basic time analysis:

1. Critical path method

Critical path is the path of a network which gives the shortest time in which the whole project can be completed. It is the chain of activities with the longest duration times.

- **Earliest Start Times (EST)** – this is the earliest possible time at which a succeeding activity can start.

Where two or more activities arrive at an event, the longest activity duration is taken as EST.

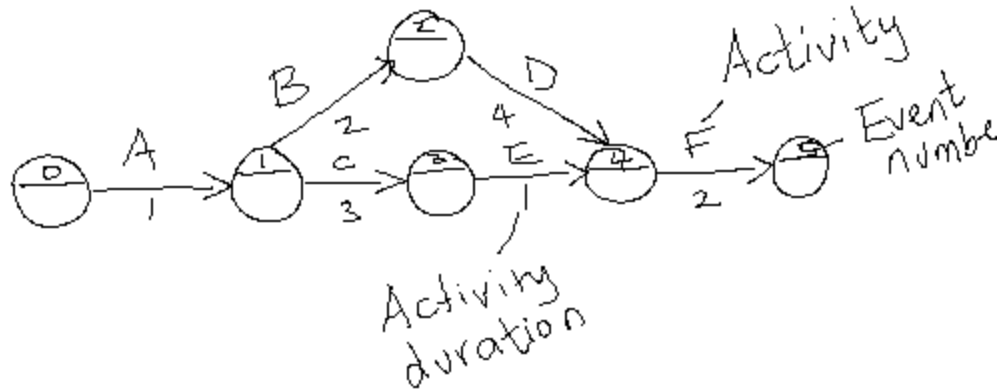
The EST of the finish event is the project duration and is the shortest time in which the whole project can be completed.

- **Latest Start Times (LST)** – this is the latest possible time at which a preceding activity can finish without increasing the project duration.

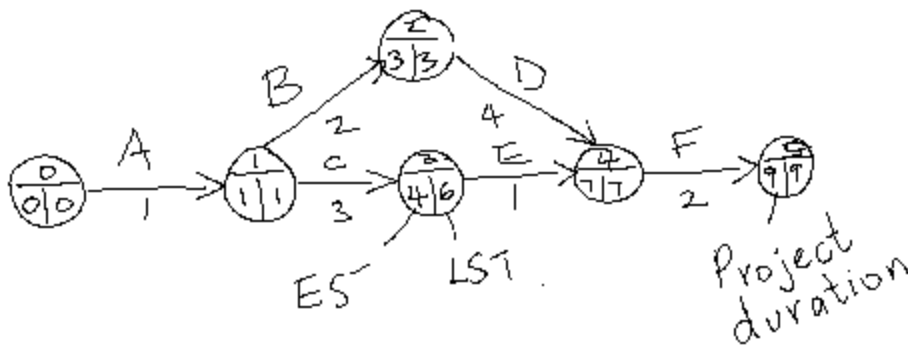
LST's are inserted starting at the finish event working backwards through the network deducting each activity duration from the previously calculated LST.

Where the tails of two or more activities join an event, the lowest number is taken as LST for the event.

From the network below



The EST's and LST's can be inserted as



Critical activity - an activity is critical when $EST = LST$ of the tail event and $EST = LST$ of the head event, and the EST of the head event minus EST of the tail event equals the activity duration.

Implications

Activities along the critical path are vital activities which must be completed by their EST s and LST s otherwise the project will be delayed.

Non critical activities have spare time (float) available meaning they can take up additional time in total without delaying the project duration.

To reduce the overall project duration, the time of one or more of the activities on the critical path must be reduced by using more labour, better machinery, improved technology and so on.

Exercise

Find the critical path of the following network using ESTs and LSTs.

Activity	Preceding activity	Activity duration
A	-	9
B	-	3
C	A	8
D	A	2
E	A	3
F	C	2
G	C	6
H	C	1
J	B,D	4
K	F,J	1
L	E,H,G,K	2
M	E,H	3
N	L,M	4

Solution - found in the lecture thirteen recording.

Float

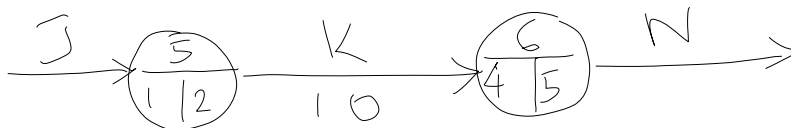
Float or spare time can only be associated with activities which are non-critical. By definition, activities on the critical path cannot have float. There are three types of float:

- Total float** - this is the total amount of time a path of activities could be delayed without affecting the overall project duration.

A negative value indicates inadequate resources, hence the activity may not finish on time, a zero value shows that there are just enough resources to complete the activity so no delay and a positive value depicts excess resources thus they can be relocated to other activities.

$$\text{Total float} = \text{Latest head time} - \text{Earliest Tail time} - \text{Activity duration}$$

For example, consider the part of network below



$$\text{Total float} = 50 - 10 - 10 = 30 \text{ days}$$

- b) **Free float** - this is the amount of time an activity can be delayed without affecting the commencement of a subsequent activity at its earliest start time, but may affect float of previous activity.

$$\text{Free float} = \text{Earliest head time} - \text{Earliest Tail time} - \text{Activity duration}$$

That is

$$\text{Free float} = 40 - 10 - 10 = 20 \text{ days}$$

- c) **Independent float** - this is the amount of time an activity can be delayed when all preceding activities are completed as late as possible and all succeeding activities completed as early as possible. It does not the float of either preceding or subsequent activities.

Independent float

$$= \text{Earliest head time} - \text{Latest Tail time} - \text{Activity duration}$$

$$\text{Independent float} = 40 - 20 - 10 = 10 \text{ days}$$

Alternative names for Earliest Head Time and Latest Head Time are Earliest Finishing Times (EFT) and Latest Finishing Time (LFT). Float calculations for the previous example is as below

Activity	EST	LST	EFT	LFT	D	TF	FF	IF
A	0	0	9	9	9	0	0	0
B	0	0	11	18	3	15	8	8
C	9	9	17	17	8	0	0	0
D	9	9	11	18	2	7	0	0
E	9	9	18	22	3	10	6	6
F	17	17	19	22	2	3	0	0
G	17	17	23	23	6	0	0	0
H	17	17	18	22	1	4	0	0
I	11	18	19	22	4	7	4	0
J	19	22	23	23	1	3	3	0
K	23	23	25	25	2	0	0	0
L	18	22	25	25	3	4	4	0
M	25	25	29	29	4	0	0	0

2. Project Evaluation and Review Technique (PERT)

This requires:

a. **Optimistic time (O, a or t_0)**

Estimate of the shortest minimum possible time an activity can take to be completed. This is usually under ideal conditions.

b. **Most likely time (M, t_m or M_L)**

The time which the activity will take most frequently if performed a number of times.

c. **Pessimistic time (P, b or t_p).**

The longest possible time that an activity can take.

The three times O, M and P are combined to develop the expected time (t_e) for an activity.

The probability of $O:M:P = 1:4:1$

The expected time duration = $\frac{O}{6} + \frac{4M}{6} + \frac{P}{6}$

$$\therefore t_e = \frac{O+4M+P}{6} \text{ or } \frac{a+4m+b}{6}$$

$$\text{Variance} = \left(\frac{P-O}{6}\right)^2 \Rightarrow \text{Standard deviation} = \sqrt{\left(\frac{P-O}{6}\right)^2}$$

Standard deviation of the project = $\sqrt{\text{Sum of variances of the critical activities}}$

Procedure for calculating time estimates using PERT

1. Prepare a table of expected duration and the variance for each of the activities of the project.
2. Draw the project network diagram of activities based on the expected duration of activities.
3. Determine the critical path.
4. Find the total expected duration of the project based on the network.
5. Compute the standard deviation of the entire project.
6. Find the probability of completing the project within a particular time period.

Bibliography

Lucey, T. (2002). *Quantitative Techniques* (6th ed.). Cengage Learning.

Taha, H. A. (2017). *Operation Research An introduction* (10th ed.). Prentice-Hall, Inc.