Basic Mathematics

Examination

Solutions

Objective Section

1. C (1 Point)
2. B (1 Point)
3. A (1 Point)
4. A (1 Point)
5. C (1 Point)
6. C (1 Point)
7. B (1 Point)
8. D (1 Point)
9. C (1 Point)
10. D (1 Point)
11. D (1 Point)
12. State without proof the De Morgan's laws of set (2 Points)
    Union and intersections of sets interchanges under complementation
13. Differentiate between absolute complement and relative complement (2 Points) – *Given a set A then the absolute complement of set A is the set of elements in the universal set not in set A while given sets A and B then the relative complement of set A on B is the elements in set A and not in set B.*
14. Differentiate between a series and a sequence. Illustrate with a relevant example in each case. (4 Points)
    Sequence is an ordered arrangement of elements while a series is the sum of the terms of a sequence. Sequence: 1,2,3,4,5 Series: 1+2+3+4+5
15. Define the following terms: (5 Points)
    a) Median -*it is the middlemost element in a data set.*
    b) Arithmetic series – *it is the progression or series that proceeds with a common difference.*
    c) Absolute value of a real number x – *it is the distance of x from the origin.*
    d) Discrete probability space – *it is a sample space that is countable.*
    e) A linear equation- *it is a first-degree polynomial.*
16. The ages of boys in a certain class were recorded as follows: 18yrs – 2 boys; 19 years – 8 boys; 20 years – 11 boys; 21 years – 5 boys and 22 years – 2 boys. What is the probability of selecting a boy who is either 19 years or 21 years old? (2 Points)

$$p(19\ or\ 21) = \frac{8}{28} + \frac{5}{28} = \frac{13}{28}$$

17. Determine the roots of 3x^2-8x-11=0 (2 Points)
    Sum = -8 and product = -33. We have {-11, 3} then $x = \{\frac{11}{3}, -1\}$
18. Solve for x in the following: 2|2x-5|> 12 (2 Points)

$$2x - 5 > 6 \Rightarrow 2x > 11 \therefore x > 5.5$$
$$2x - 5 < -6 \Rightarrow 2x < -1 \therefore x < -0.5$$

Subjective Section

1) Determine a quadratic equation whose roots are $3i$ and $2 - i$ (2 Points)

$$(x - 3i)(x - 2 + i) = x^2 - 2x + xi - 3xi + 6i + 3 = 0$$
$$= x^2 - 2(1 + i)x + (6i + 3) = 0$$

2) Determine the probability of getting a head and an even number when a fair coin is tossed, and a die rolled once. (2 Points)

$$p(EH) = p(2H) or\ p(4H) or\ p(6H) = \frac{1}{12} + \frac{1}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$$

3) Two fair dice are tossed.

(i) Draw a contingency table for the total possible outcomes (1 Points)

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

(ii) Find the probability of two number showing up whose product is a factor of 36.(2 Points)

$$= \frac{20}{36} = \frac{5}{9}$$

4) Show that $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots = 2$ (2 Points)

$$S_\infty = \frac{1}{1 - \frac{1}{2}} = 2$$

5) The following table represents the number of masks sold to motorists by some hawkers along Thika road.

No. of masks	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 - 39	40 – 44	45 – 49	50 – 54	Totals
No. of Hawkers f	4	5	7	18	28	16	11	8	3	100
x	12	17	22	27	32	37	42	47	52	
fx	48	85	154	486	896	592	462	376	156	3255
Cf	4	9	16	34	62	78	89	97	100	
x^2	144	289	484	729	1024	1369	1764	2209	2704	
fx^2	576	1445	3388	13122	28672	21904	19404	17672	8112	114295

(i) Calculate the mean number of masks sold (2 Points)

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{3255}{100} = 32.55$$

(ii) Estimate the median mask (2 Points)

$$md = 29.5 + \frac{50.5 - 34}{28} \times 5 = 32.45$$

(iii) Calculate the standard deviation (3 Points)

$$\sigma = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2} = \sqrt{\frac{114295}{100} - 32.55^2} = \sqrt{1142.95 - 1059.5025} = \sqrt{83.4475} \approx 9.135$$

6) The first and the last term of a GP are 3 and 729 respectively and the sum of all terms is 1092. Find.

(i) Find the common ratio and the number of terms (3 Points)

$$a = 3, T_n = 729 \therefore 729 = 3 \times r^{n-1} \cdots (i)$$
$$S_n = 1092 = \frac{3(r^n - 1)}{r - 1} \cdots (ii)$$
$$\text{from (i) } 729r = 3r^n \Rightarrow r^n = 243r$$

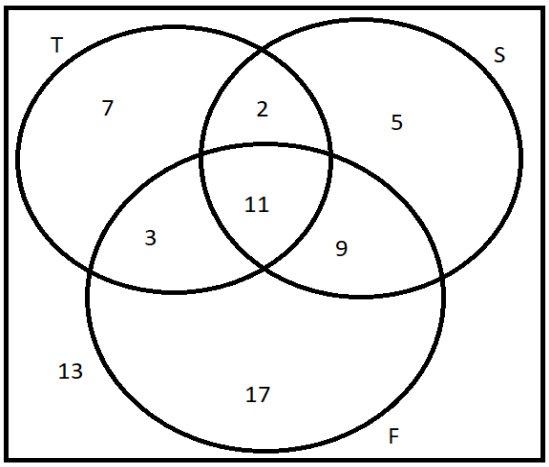
Hence (ii) becomes.

$$364(r - 1) = r^n - 1$$
$$364r - 364 = 243r - 1$$
$$121r = 363 \therefore r = 3$$
$$\text{but } T_n = ar^{n-1} = 3 \times 3^{n-1} \Rightarrow 729 = 3^n \therefore 3^6 = 3^n \Rightarrow n = 6 \text{ terms}$$

7) Apply the Cramer's rule to solve the linear system; $3x + 2y - z = 12$, $5x + 2z = 17$, $x - y + 7z = 8$ (4 Points)

$$x = \frac{\begin{vmatrix} 12 & 2 & -1 \\ 17 & 0 & 2 \\ 8 & -1 & 7 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & -1 \\ 5 & 0 & 2 \\ 1 & -1 & 7 \end{vmatrix}} = \frac{-165}{-55} = 3; y = \frac{\begin{vmatrix} 3 & 12 & -1 \\ 5 & 17 & 2 \\ 1 & 8 & 7 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & -1 \\ 5 & 0 & 2 \\ 1 & -1 & 7 \end{vmatrix}} = \frac{-110}{-55} = 2; z = \frac{\begin{vmatrix} 3 & 2 & 12 \\ 5 & 0 & 17 \\ 1 & -1 & 8 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & -1 \\ 5 & 0 & 2 \\ 1 & -1 & 7 \end{vmatrix}} = \frac{-55}{-55} = 1$$

8) Study the Venn diagram below that represent the number of people in a party who consumed certain beverages i.e. Tea T, Soda S, Fresh juice F, and others. Use the information to attempt the following questions.



(i) Supposed 5 people did not honor the invitation to attend the party, find how many people were invited. *72* (1 Point)

(ii) How many people drunk all the drinks i.e. tea, soda, and fresh juice: 11 (1 Point)

(iii) How many people took at least one of the three drinks i.e. tea, soda, and fresh juice: 54 (1 Point)

(iv) How many too exactly one of the drinks i.e. tea, soda, and fresh juice: 29 (1 Point)

(v) $n(T \cup S) = 37$ (1 Point)

(vi) $|(F \cap S) \cap T^c| = 9$ (1 Point)

(vii) How many took both fresh juice and tea but not soda: 3 (1 Point)