

Basic Mathematics

Lectures 2 and 3

Solutions to Assignment 2 (30 Points)

- 1) Solve for x in the following; $4|x - 2| = 24$ (2 Points)

$$|x - 2| = 6$$

$$\begin{aligned} \text{either } x - 2 &= 6 & \text{or } x - 2 &= -6 \\ x &= 6 + 2 = 8 & \text{or } x &= 2 - 6 = -4 \\ \therefore x &= \{-4, 8\} \end{aligned}$$

- 2) Solve; $|x - 5| = |x + 7|$ (2 Points)

$$\text{Either } x - 5 = x + 7 \dots \text{(i) Or } x - 5 = -(x + 7) \dots \text{(ii)}$$

$$\text{For equation (i) we have: } x - x = 5 + 7$$

$$0 = 12 - \text{invalid}$$

$$\text{For equation (ii) we have: } x - 5 = -x - 7$$

$$2x = 5 - 7$$

$$2x = -2 \therefore x = -1$$

i.e. the inequality is only valid when $x = -1$

- 3) Solve: $|x - 7| < 12$ (2 Points)

Either;

$$\text{(i) } x - 7 < 12 \Rightarrow x < 19 \text{ or}$$

$$\text{(ii) } x - 7 > -12 \Rightarrow x > -5$$

Therefore, we can say that;

$$-5 < x < 19$$

- 4) Solve; $|x + 2| \geq 17$ (2 Points)

Either

$$\text{(i) } x + 2 \geq 17 \Rightarrow x \geq 15 \text{ or}$$

$$\text{(ii) } x + 2 \leq -17 \Rightarrow x \leq -19$$

- 5) Factorize: $x^2 - 8x + 12$ (2 Points)

We find two terms such that;

$$\text{(i) Their SUM is } -8$$

$$\text{(ii) Their product is } 12 \text{ i.e. } -2 \text{ and } -6$$

Hence we have;

$$\begin{aligned} x^2 - 8x + 12 &= x^2 - 2x - 6x + 12 \\ &= x(x - 2) - 6(x - 2) \\ &= (x - 6)(x - 2) \end{aligned}$$

- 6) Factorize: $36x^2 - 49$ (2 Points)

This is a case of difference of two squares i.e. $a^2 - b^2 = (a - b)(a + b)$

$$\text{Therefore we have; } 36x^2 - 49 = (6x - 7)(6x + 7)$$

- 7) $3x^2 + 10x + 8$ (2 Points)

$$\begin{aligned} 3x^2 + 10x + 8 &= 3x^2 + 6x + 4x + 8 \\ &= 3x(x + 2) + 4(x + 2) \\ &= (x + 2)(3x + 4) \end{aligned}$$

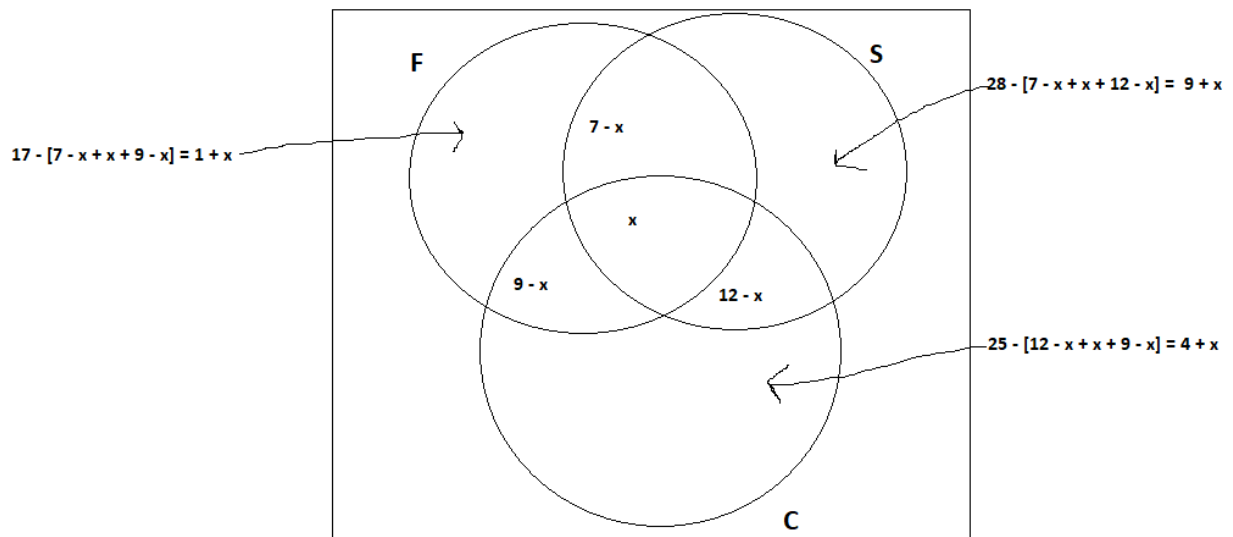
- 8) A survey was carried in a local hotel. It was noted that 17 people took Fanta F, 25 took coke C, and 28 took sprite S. 9 people took a Fanta and coke, 12 took a coke and sprite, and 7 took a Fanta and sprite. Suppose 51 people were in the survey and 5 people never took any of these three drinks. Use a Venn diagram to determine how many took all the three types of drinks. (4 points)

Solution

The above information can be represented using set connectives as shown below;

In English	Using set connectives
Fanta F and Coke C	$F \cap C = 9$
Coke C and Sprite S	$C \cap S = 12$
Fanta F and Sprite S	$F \cap S = 7$
Those who took the three drinks were $51 - 5 = 46$	$F \cup C \cup S = 46$
Those who didn't took either of the drinks were 5	$(F \cup C \cup S)^c = 5$
Took all the three drinks x	$F \cap C \cap S = x$

Next, we represent the above in a Venn diagram as below;



Note that;

$$46 = (1 + x) + (7 - x) + x + (9 - x) + (12 - x) + (9 + x) + (4 + x)$$

$$46 = 42 + x$$

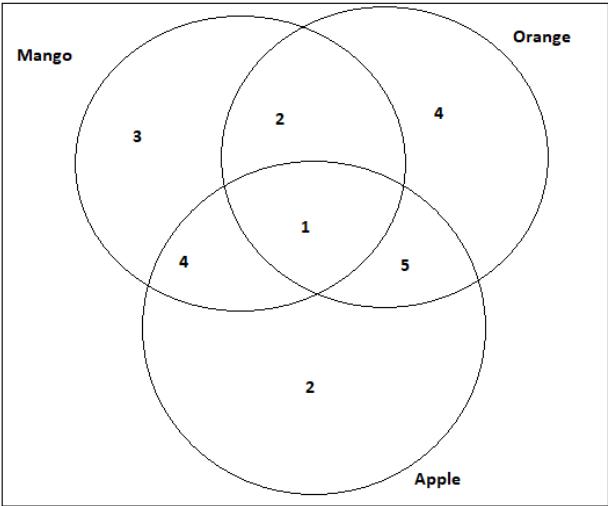
$$x = 4$$

i.e. 4 people took ALL the three drinks.

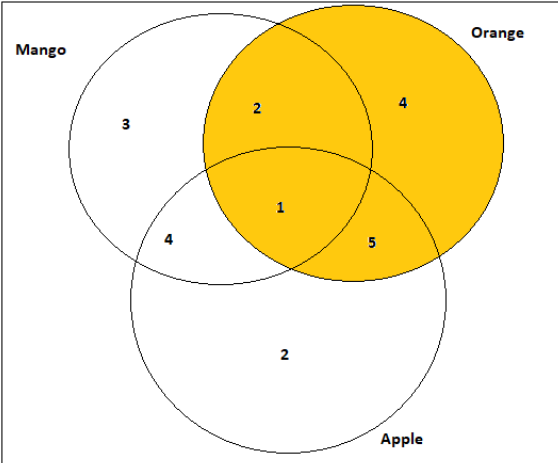
- 9) The following table represent the number of people who ate 3 types of fruits

Fruit	No. of people
Mango only	3
Mango but not orange	7
Mango and Apple	5
Mango	10
Orange and apple	6

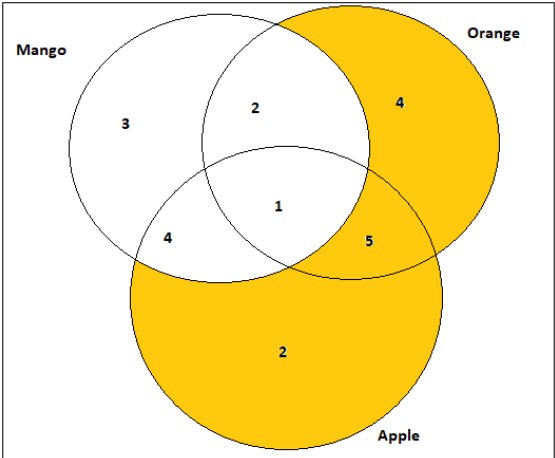
If the total number of fruits was 21, use a Venn diagram to determine the number of people that ate;



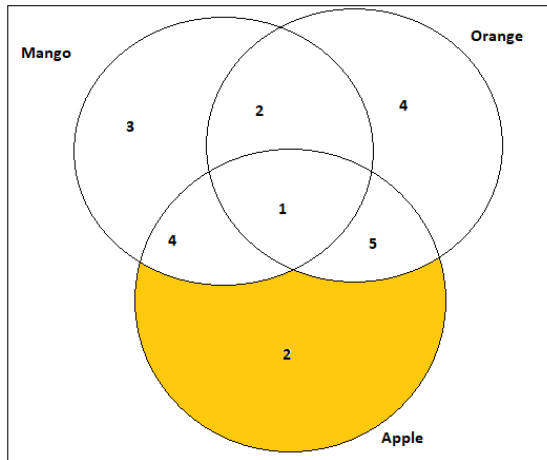
(i) Oranges (4 points) - 12



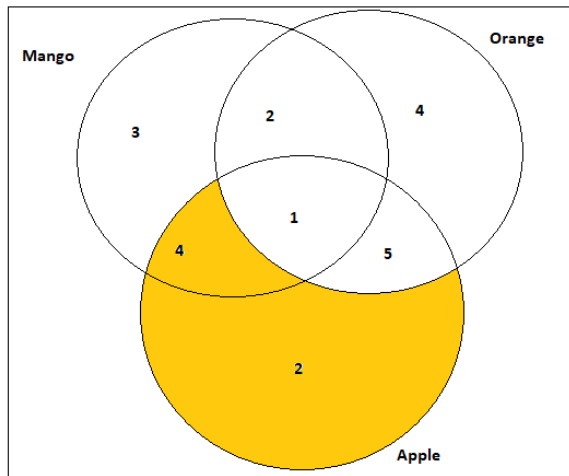
(ii) Orange or apple but not a mango (2 points) - $(O \cup A) \cap M^c = 11$



(iii) Apple only (2 points) = 2



(iv) Apple but not an orange (2 points) = 6



(v) All (2 points) = 1

