Basic Mathematics

Lectures 4 and 5

Solutions to Assignment 3 (35 Points)

1) Solve the following using Cramer's method. (4 Points)

$$x + 2y = 11$$
$$5x - y = 11$$

$$x = \frac{\begin{vmatrix} 11 & 2 \\ 11 & -1 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 5 & -1 \end{vmatrix}} = \frac{-11 - 22}{-1 - 10} = \frac{-33}{-11} = 3; \quad y = \frac{\begin{vmatrix} 1 & 11 \\ 5 & 11 \end{vmatrix}}{\begin{vmatrix} 1 & 2 \\ 5 & -1 \end{vmatrix}} = \frac{11 - 55}{-1 - 10} = \frac{-44}{-11} = 4$$

2) Solve the following system using Substitution method ; $\begin{matrix} 5x + 2y = 9 \\ 2x - 3y = -4 \end{matrix}$ (4 Points)

$$\text{let } 5x = 9 - 2y \Rightarrow x = \frac{9 - 2y}{5}$$

substitute for x in the second equation to get;

$$2\left(\frac{9 - 2y}{5}\right) - 3y = -4$$

$$\frac{(18 - 4y)}{5} - 3y = -4$$

$$18 - 4y - 15y = -20$$

$$-19y = -38 \therefore y = 2$$

Hence

$$x = \frac{9 - 2y}{5} = \frac{9 - 4}{5} = \frac{5}{5} = 1$$

3) Solve the following linear system using elimination method; $\begin{matrix} 3x + 2y = 17 \\ 5x - 7y = -13 \end{matrix}$ (4 Points)

To eliminate x multiply the first equation by 5 and the second by 3 and then subtract to get;

$$15x + 10y = 85$$
$$\underline{15x - 21y = -39}$$
$$31y = 124$$
$$\therefore y = 4$$

To eliminate y multiply the first equation by 7 and the second by 2 and then add to get;

$$21x + 14y = 119$$
$$\underline{10x - 14y = -26}$$
$$31x = 93$$
$$\therefore x = 3$$

4) Anita bought 7 mangoes and 4 pears at a total cost of 285/-. Chris bought 11 similar mangoes and 5 similar pears at a total cost 390/-. Find how much Kemboi paid for 12 similar mangoes and 8 similar pears. (5 Points)

$$7\mathrm{m} + 4\mathrm{p} = 285$$
$$11\mathrm{m} + 5\mathrm{p} = 390$$

Solving for m and p to get;

$$11\left(\frac{285 - 4p}{7}\right) + 5p = 390$$
$$3135 - 44p + 35p = 2730$$
$$-9p = -405 \therefore p = 45\ shs$$

Hence;

$$\mathrm{m} = \frac{285 - 4\mathrm{p}}{7} = \frac{285 - 180}{7} = 15\ \mathrm{shs}$$

5) Solve the following system using Cramer's method; $x - 2y - 3z = -52$ (6 Points)
$x + y - z = 10$
$x + 3y + z = 60$

$$\mathrm{x} = \frac{\begin{vmatrix} 10 & 1 & -1 \\ -52 & -2 & -3 \\ 60 & 3 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & -3 \\ 1 & 3 & 1 \end{vmatrix}} = \frac{-22}{-2} = 11;\ \mathrm{y} = \frac{\begin{vmatrix} 1 & 10 & -1 \\ 1 & -52 & -3 \\ 1 & 60 & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & -3 \\ 1 & 3 & 1 \end{vmatrix}} = \frac{-24}{-2} = 12$$

$$\mathrm{z} = \frac{\begin{vmatrix} 1 & 1 & 10 \\ 1 & -2 & -52 \\ 1 & 3 & 60 \end{vmatrix}}{\begin{vmatrix} 1 & 1 & -1 \\ 1 & -2 & -3 \\ 1 & 3 & 1 \end{vmatrix}} = \frac{-26}{-2} = 13$$

6) Solve the following system using substitution rule; $x + y + z = 18$
$2x - y - 2z = -10$ (6 Points)
$x + 3y - z = 16$

From equation (i) we have

$$\mathrm{x} = 18 - \mathrm{y} - \mathrm{z} \cdots (\mathrm{iv})$$

From equation (iii) we have;

$$\mathrm{x} = 16 - 3\mathrm{y} + \mathrm{Z} \cdots (\mathrm{v})$$

from equations (iv) and (v) we have;

$$18 - \mathrm{y} - \mathrm{z} = 16 - 3\mathrm{y} + \mathrm{z}$$
$$2 = -2\mathrm{y} + 2\mathrm{z} \Rightarrow 1 = \mathrm{z} - \mathrm{y} \cdots (\mathrm{vi})$$

Note that equation two can be written as;

$$2(18 - y - z) - y - 2z = -10$$
$$36 - 2y - 2z - y - 2z = -10$$
$$46 = 3y + 4z \cdots \text{(vii)}$$

Next we work we equations (vi) and (vii)

$$46 = 3y + 4(1 + y)$$
$$46 = 3y + 4 + 4y$$
$$42 = 7y \therefore y = 6$$

Hence;

$$z = 1 + y = 7$$

And

$$x = 18 - y - z = 18 - 6 - 7 = 5$$

7) Three times the tens digit of a certain 2-digit number is 2 more than four times the units digit. The difference between the given number and the number obtained by reversing the digits is two less than twice the sum of the digits. Find the number. (6 Points)

Let the two digits be x (tens digit) and y (the ones digit) hence we have;

$$3x = 2 + 4y \cdots \text{(i)}$$

We also have;

$$(10x + y) - (10y + x) = 2(x + y) - 2$$
$$7x - 11y = -2 \cdots \text{(ii)}$$
$$\text{from (i); } x = \frac{2 + 4y}{3}$$
$$7\left(\frac{2 + 4y}{3}\right) - 11y = -2$$
$$14 + 28y - 33y = -6$$
$$-5y = -20$$
$$\therefore y = 4$$
$$\text{hence; } x = \frac{2 + 16}{3} = 6$$