Basic Mathematics

Lectures 6, 7, and 8

Solutions to Assignment 4 (40 Points)

1) Expand the following expressions; (6 Points)
   a) $(x^2 + y^3)(2x + z^2 - y^3)$
   $= x^2(2x + z^2 - y^3) + y^3(2x + z^2 - y^3)$
   $= 2x^2 + x^2z^2 - x^2y^3 + 2xy^3 + z^2y^3 - y^6$
   b) $(8x^2 - 3y + 2)(x - y)$
   $= x(8x^2 - 3y + 2) - y(8x^2 - 3y + 2)$
   $= 8x^3 - 3xy + 2x - 8yx^2 + 3y^2 - 2y$
   c) $(y + 7)(y - 3)$
   $= y(y - 3) + 7(y - 3)$
   $= y^2 - 3y + 7y - 21$
   $= y^2 + 4y - 21$
2) Determine the corresponding quadratic equation of the form $ax^2 + bx + c = 0$ given the following pair of roots; (6 Points)
   a) $x = 7, x = -11$
   $=> x - 7 = 0$ or $x + 11 = 0$
   $\therefore (x - 7)(x + 11) = 0$
   $x(x + 11) - 7(x + 11) = 0$
   $x^2 + 11x - 7x - 77 = 0$
   $x^2 + 4x - 77 = 0$

   b) $x = 2i, x = 5i$
   $=> x - 2i = 0$ or $x - 5i = 0$
   $\therefore (x - 2i)(x - 5i) = 0$
   $x(x - 5i) - 2i(x - 5i) = 0$
   $x^2 - 5ix - 2ix + 10i^2 = 0$ but $i^2 = -1$ (imaginary number $i \in$ Complex set)
   $x^2 - 7ix = 10 = 0$

   c) $x = \frac{1}{7}, x = \frac{2}{9}$
   $=> x - \frac{1}{7} = 0$ or $x - \frac{2}{9} = 0$
   $\left(x - \frac{1}{7}\right)\left(x - \frac{2}{9}\right) = 0$
   $x\left(x - \frac{2}{9}\right) - \frac{1}{7}\left(x - \frac{2}{9}\right) = 0$
   $x^2 - \frac{2}{9}x - \frac{1}{7}x + \frac{2}{63} = 0$
   $x^2 - \frac{5}{63}x + \frac{2}{63} = 0$ OR $63x^2 - 5x + 2 = 0$

3) Solving the following using factorization method;(6 Points)

a) $x^2 + 10x - 11 = 0$

$$\text{Sum} = 10, \text{Product} = -11 \therefore 11 \text{ and } -1$$

Since the coefficient of x^2 is 1, we can conclude that $x = 11 \text{ or } 1$

Alternatively

Replace the middle term 10x with (11x - x) to get;

$$x^2 - x + 11x - 11 = 0$$

Then carry out group factorization i.e.

$$x(x-1) + 11(x-1) = -$$

$$(x-1)(x+11) = 0$$

$$\text{either } (x-1) = 0 \text{ or } (x+11) = 0$$

$$\therefore x = 1 \text{ or } -11$$

b) $5x^2 - 12x + 7 = 0$

$\text{Sum} = -12, \text{Product} = 5 \times 7 = 35 \text{ i.e.} -5 \text{ and } -7$

Since the coefficient of x^2 is 5, we can conclude that $x = \frac{5}{5} = 1 \text{ or } \frac{7}{5}$

Alternatively

Replace the middle term -12x with (-5x - 7x) to get;

$$5x^2 - 5x - 7x + 7 = 0$$

Then carry out group factorization i.e.

$$5x(x-1) - 7(x-1) = 0$$

$$(x-1)(5x-7) = 0$$

$$\text{either } (x-1) = 0 \text{ or } (5x-7) = 0$$

$$\therefore x = 1 \text{ or } x = \frac{7}{5}$$

c) $x^2 - 7x + 12 = 0$

$\text{Sum} = -7, \text{Product} = 1 \times 12 = 12 \text{ i.e.} -3 \text{ and } -4$

Since the coefficient of x^2 is 1, we can conclude that $x = 3 \text{ or } x = 4$

Alternatively

Replace the middle term -7x with (-3x - 4x) to get;

$$x^2 - 3x - 4x + 12 = 0$$

Then carry out group factorization i.e.

$$x(x-3) - 4(x-3) = 0$$

$$(x-3)(x-4) = 0$$

$$\text{either } (x-3) = 0 \text{ or } (x-4) = 0$$

$$\therefore x = 3 \text{ or } x = 4$$

d) $x^2 + x + 7 = 0$

x is not a Real number, hence cannot be factorize in the set of Reals

4) Solve the following using quadratic formula; (6 Points)

a) $x^2 + 70x + 851 = 0$

By definition given a quadratic equation $ax^2 + bx + C = 0$ then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

in our case a = 1, b = 70 and c = 851 then;

$$x = \frac{-70 \pm \sqrt{70^2 - 4(1)(851)}}{2} = \frac{-70 \pm \sqrt{1496}}{2} = -35 \pm \sqrt{374} \approx -54.34 \text{ or} - 15.66$$

b) $7x^2 + 5x - 2 = 0$

In our case a = 7, b = 5 and c = - 2 then we have;

$$x = \frac{-5 \pm \sqrt{5^2 - 4(7)(-2)}}{2(7)} = \frac{-5 \pm \sqrt{81}}{14} = \frac{-5 \pm 9}{14} = -1 \; or \; \frac{2}{7}$$

c) $5x^2 + x + 13 = 0$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(5)(13)}}{2(5)} = \frac{-1 \pm \sqrt{-259}}{10} = \frac{-1 \pm i\sqrt{259}}{10}$$

5) Use completing the square method to solve the following; (8 Points)

a) $5(x - 11)^2 + 1 = 721$

Subtract 1 from either side to get;

$$5(x - 11)^2 = 720$$

Divide either side by 5 to get;

$$(x - 11)^2 = 144$$

Find the square of both sides to get;

$$x - 11 = \pm 12$$

Add 11 to both sides to get;

$$x = 11 \pm 12$$

$$=> x = 23 \text{ or } x = -1$$

b) $x^2 + x + 11 = 0$

Subtract 11 from either side to get;

$$x^2 + x = -11$$

Add a constant k to either side so as to complete the square on the LHS;

$$x^2 + x + k = k - 11$$

But;

$$k = \left(\frac{b}{2}\right)^2 = \left(\frac{1}{2}\right)^2$$

Hence, we get;

$$x^2 + x + \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^2 - 11$$

$$\left(x + \frac{1}{2}\right)^2 = \frac{1}{4} - 11 = \frac{1 - 44}{4} = \frac{-43}{4}$$

Find the square root of both sides to get;

$$x + \frac{1}{2} = \pm\sqrt{-\frac{43}{4}} = \pm\frac{i}{2}\sqrt{43}$$

Hence we have;

$$x = -\frac{1}{2} \pm \frac{i}{2}\sqrt{43} \quad or - 0.5 \pm 0.5i\sqrt{43}$$

c) $7x^2 + 4x - 3 = 0$

Divide every term by 7 so as to make the coefficient of $x^2 = 1$ i.e.

$$x^2 + \frac{4}{7}x - \frac{3}{7} = 0$$

Add $\frac{3}{7}$ to either side to get;

$$x^2 + \frac{4}{7}x = \frac{3}{7}$$

Add a constant k to either side so as to complete the square on the LHS;

$$x^2 + \frac{4}{7}x + k = \frac{3}{7} + k$$

Where;

$$k = \left(\frac{b}{2}\right)^2 = \left(\frac{\frac{4}{7}}{2}\right)^2 = \left(\frac{2}{7}\right)^2$$

Hence we have;

$$x^2 + \frac{4}{7}x + \left(\frac{2}{7}\right)^2 = \left(\frac{2}{7}\right)^2 + \frac{3}{7} = \frac{4}{49} + \frac{3}{7} = \frac{25}{49}$$

$$\left(x + \frac{2}{7}\right)^2 = \frac{25}{49}$$

Square root of both sides to get;

$$x + \frac{2}{7} = \pm\frac{5}{7}$$

$$x = -\frac{2}{7} \pm \frac{5}{7}$$

$$\therefore x = \frac{3}{7} \text{ or } x = -1$$

d) $3x^2 + 2x - 1 = 0$

$$x^2 + \frac{2}{3}x - \frac{1}{3} = 0$$
$$x^2 + \frac{2}{3}x + k = k + \frac{1}{3}$$
$$x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2 = \left(\frac{1}{3}\right)^2 + \frac{1}{3} = \frac{1}{9} + \frac{1}{3} = \frac{4}{9}$$
$$\left(x + \frac{1}{3}\right)^2 = \frac{4}{9}$$
$$x + \frac{1}{3} = \pm\frac{2}{3}$$
$$x = -\frac{1}{3} \pm \frac{2}{3}$$
$$x = -1 \text{ or } x = \frac{1}{3}$$

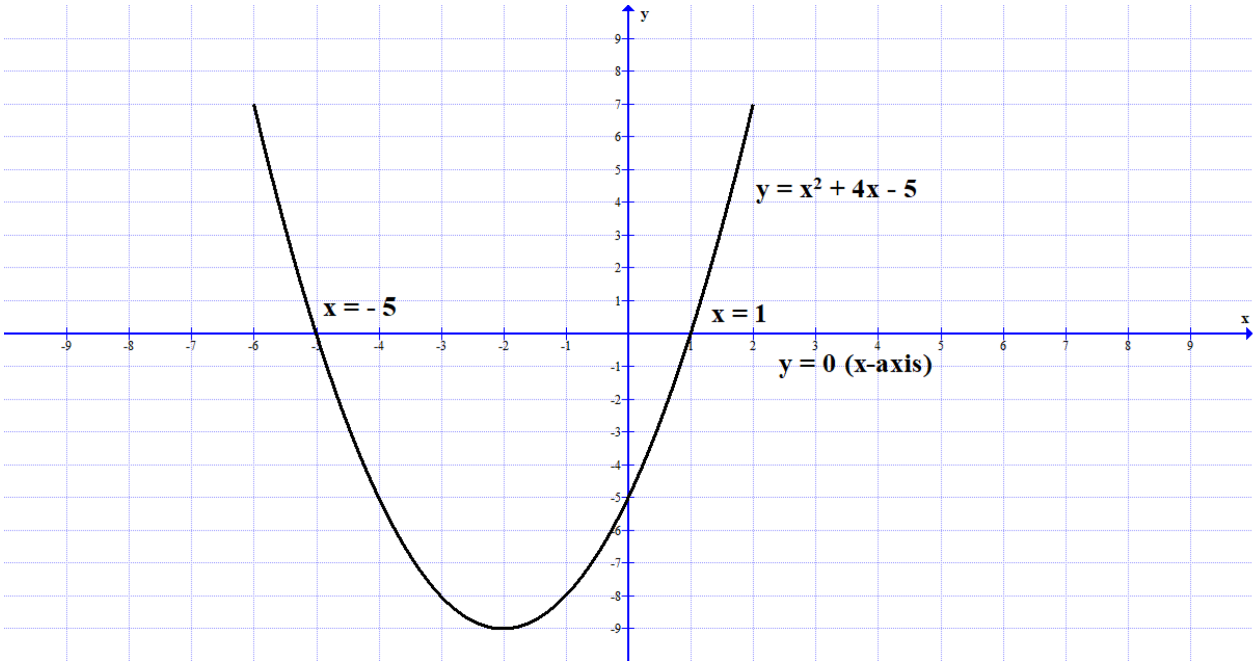
6) Use the Graph© software to plot the graph of $y = x^2 + 4x - 5$ over $-6 \leq x \leq 2$ and hence use it to solve the equation $x^2 + 4x - 5 = 0$ (8 Points)
To solve $x^2 + 4x - 5 = 0$ *we subtract (in some cases we may need to add) it from the graph function* $y = x^2 + 4x - 5$ *and then plot the resultant linear graph. The value of x at the point of intersection is the solutions to the quadratic equations (in situation where the resultant line doesn't intersect the graph, we say the equation has no Real roots).*

Thus in our case we have;

$$\begin{array}{c} y = x^2 + 4x - 5 \\ \underline{0 = x^2 + 4x - 5} \\ y = 0 \ \ (x - axis) \end{array}$$

Note in the graph below, the resultant line $y = 0$ and the graph function $y = x^2 + 4x - 5$ intersect at the point $x = -5$ and $x = 1$.

Hence the roots/solutions of $x^2 + 4x - 5 = 0$ are – 5 and 1.


y
x
$y = x^2 + 4x - 5$
x = - 5
x = 1
y = 0 (x-axis)