

Basic Mathematics

Lectures 9 and 10

Solutions to Assignment 5 (30 marks)

- 1) Define a sequence and a series and in each illustrate with a relevant example. (4 Points)
A sequence is an ordered arrangement of objects/elements/numbers, while a series is the sum of the terms of a sequence. For example; 1,2,3, 4, ... is a sequence and 1+2+3+4+....is a series.
- 2) Evaluate the following given $x_1 = 2, x_2 = -2, x_3 = -1, x_4 = 3, x_5 = 0$ (3 Points)

$$\sum_{i=1}^5 (-3x_i^3 - 2x_i - 9)$$

$$= (-3x_1^3 - 2x_1 - 9) + (-3x_2^3 - 2x_2 - 9) + (-3x_3^3 - 2x_3 - 9) + (-3x_4^3 - 2x_4 - 9) + (-3x_5^3 - 2x_5 - 9)$$

$$= (-24 - 4 - 9) + (24 + 4 - 9) + (3 + 2 - 9) + (-81 - 6 - 9) + (0 - 0 - 9)$$

$$= -37 + 19 - 4 - 96 - 9 = -127$$

- 3) Evaluate the series below using the values of x_i given in question (2) above; (3 Points)

$$\sum_{i=1}^3 \left(\frac{5x_i + x_i^2}{-7x_i} \right) = \left(\frac{5x_1 + x_1^2}{-7x_1} \right) + \left(\frac{5x_2 + x_2^2}{-7x_2} \right) + \left(\frac{5x_3 + x_3^2}{-7x_3} \right)$$

$$= \left(\frac{10 + 4}{-14} \right) + \left(\frac{-10 + 4}{14} \right) + \left(\frac{-5 + 1}{7} \right) = \frac{14}{-14} + \frac{-6}{14} - \frac{4}{7} = -2$$

- 4) Consider the following series; $17 + 22 + 27 + 32 + \dots$ Determine the; (6 Points)

- a) 200^{th} term

$$T_{200} = a + (n - 1)d = 17 + (200 - 1)5 = 17 + 995 = 1012$$

- b) S_{∞} i.e. sum to infinity

$$S_{\infty} = \infty \text{ (since the series is increasing)}$$

- c) S_{1000} i.e. sum of the first 1000 terms.

$$S_{1000} = \frac{n}{2} [2a + (n - 1)d] = \frac{1000}{2} [2 \times 17 + (1000 - 1)5] = 500[34 + 4995] \\ = 2514500$$

- 5) Consider the series $0.23232323 \dots$ Determine the; (6 Points)

- a) 10^{th} term

$$0.2323 \dots = 0.23 + 0.0023 + 0.000023 + \dots$$

$$T_{10} = ar^{n-1} = 0.23 \times 0.01^9$$

- b) S_{∞}

$$S_{\infty} = \frac{a}{1 - r} = \frac{0.23}{1 - 0.01} = \frac{0.23}{0.99} = \frac{23}{99} \text{ (Always leave your answer as a fraction)}$$

c) S_{10}

$$S_{10} = \frac{a(1 - r^n)}{1 - r} = \frac{0.23(1 - 0.01^{10})}{0.99} = \frac{23}{99}(1 - 0.01^{10})$$

- 6) The 3rd term of a GP is $\frac{18}{49}$ and the 5th term is $\frac{162}{2401}$. Find the 9th term. Leave your answer as a fraction. (4 Points)

$$T_3 = ar^2 = \frac{18}{49}; T_5 = ar^4 = \frac{162}{2401}$$

This means;

$$T_5 = T_3 r^2 \Rightarrow r^2 = \frac{T_5}{T_3} = \frac{162}{2401} \times \frac{49}{18} = \frac{9}{49} \therefore r = \frac{3}{7}$$

Since

$$T_3 = ar^2 = \frac{18}{49} \Rightarrow \frac{9}{49} a = \frac{18}{49} \Rightarrow a = 2$$

Hence;

$$T_9 = ar^8 = 2 \times \left(\frac{3}{7}\right)^8 = \frac{13122}{5764801}$$

- 7) Outline 4 areas where series and/or sequences are used or occur naturally. (4 Points)

Time

Patterns of flowers

Seasons

Growth pattern of population of a species