Basic Mathematics

Lectures 11, 12, and 13

Solutions to Assignment 6 (20 Points)

1. Given the data set; 15, 18, 25, 32, 36, 40 and 45, find the following measures;

a) Mode $\mathbf{M_o}$ (1 Point)

$$m_0 - none$$

b) Median $\mathbf{M_d}$ (1 Point)

Rearranging in ascending order we have;

15, 18, 25, 32, 36, 40, 45

Clearly the median (middlemost score) is 32

c) Arithmetic mean $\overline{x}$ (1 Point)

$$\overline{x} = \frac{\sum x}{N} = \frac{15 + 18 + 25 + 32 + 36 + 40 + 45}{7} = \frac{211}{7} \approx 30.143$$

d) Range (1 Point)

Range = highest value – lowest value = 45 – 15 = 30

e) Quartile deviation (1 Point)

Arrange the data set in ascending order and divide it into four quarters i.e.

15, 18, 25, 32, 36, 40, 45

Q_1 Q_2 Q_3

Then the lower quartile (First Quartile is) Q_1 is 18

The upper quartile (Third Quartile) Q_3 is 40

The Interquartile Range IQR is Q_3 - Q_1 = 40 – 18 = 22

Therefore;

$$Quartile\ deviation\ QD = \frac{IQR}{2} = \frac{22}{2} = 11$$

f) Standard deviation σ (3 Points)

x	**x^2**
15	225
18	324
25	625

32	1094
36	1296
40	1600
45	2025
	7119

$$\sigma = \sqrt{\frac{\sum x^2}{N} - \bar{x}^2} = \sqrt{\frac{7119}{7} - \left(\frac{211}{7}\right)^2} \approx 10.412$$

2. Consider the data of number of magazines sold by 21 vendors;

No. of magazines x	9	18	20	24	12	9	
No of Vendors f	1	2	3	4	5	6	21
$\boldsymbol{fx}$	9	36	60	96	60	54	315
$\boldsymbol{x^2}$	81	324	400	576	144	81	
$\boldsymbol{fx^2}$	81	648	1200	2304	720	486	5439

(i) Determine the mode (1 Point)

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(ii) Determine the mean number of magazines sold per day. (3 Points)

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{315}{21} = 15$$

(iii) Find the variance. (3 Points)

$$Variance, \sigma^2 = \frac{\sum fx^2}{\sum f} - \bar{x}^2 = \frac{5439}{21} - 15^2 = 34$$

(iv) Find the standard deviation. (1 Point)

$$standard\ deviation, \sigma = \sqrt{Variance, \sigma^2} = \sqrt{34} = 5.831$$

3. The table below shows the distribution of sales by 50 salesmen of a commodity during a promotion.

Sales	10 – 14	15 – 19	20 – 24	25 – 29	30 – 34	35 – 39	**Total**
No. of salesmen f	1	3	14	19	10	3	50
$Midpoint\ x$	12	17	22	27	32	37	
fx	12	51	308	513	320	111	1315
Cf	1	4	18	37	47	50	
x^2	144	289	484	729	1024	1369	
fx^2	144	867	6776	13851	10240	4107	35985

a) Calculate the mean sale (2 Points)

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{1315}{50} = 26.3$$

b) Mode sale (3 Points)

$$M_0 = l + \left(\frac{f_0 - f_1}{2f_0 - f_1 - f_2}\right) \times i$$

Our modal class is class 25 – 29

Our **adjusted lower-class limit** of the modal class $l = 24.5$

Our **modal frequency** $f_0 = 19$

The frequency **before** modal frequency $f_1 = 14$

The frequency **after** the modal frequency $f_2 = 10$

The **class size or class interval** $i = 5$

Therefore, the mode M_0 is given by;

$$M_0 = 24.5 + \frac{19 - 14}{38 - 14 - 10} \times 5 = 24.5 + 1.7857 \approx 26.286$$

c) Median sale (3 Points)

$$M_d = l + \frac{\frac{N}{2} - cf}{f} \times i$$

Our **median position** is $\frac{N+1}{2} = \frac{50+1}{2} = 25.5^{th}$ position *(the 25.5^{th} position lies at cf 37)*

(Note that for accuracy purpose we shall use this value in place of $\frac{N}{2}$ in our formula. If the data set was big enough, then we can work directly with $\frac{N}{2}$)

Hence our median class is class 25 – 29 (it coincides with the modal class)

Thus, the **median frequency** $f = 19$

The **adjusted lower-class limit** of the median class $l = 24.5$

The **cumulative frequency BEFORE** the median class is $cf = 18$

The **class size or class interval** $i = 5$

Therefore, the median M_d is given by;

$$M_d = l + \frac{\frac{N}{2} - cf}{f} \times i = 24.5 + \frac{25.5 - 18}{19} \times 5 = 24.5 + 0.3947 \approx 24.895$$

d) Standard deviation (4 Points)

$$Standard\ deviation\ \sigma = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \bar{x}^2} = \sqrt{\frac{35985}{50} - 26.3^2} = \sqrt{28.01} = 5.292$$

4. In a survey of 50 hawkers, it was observed that each sold between 40 and 100 sweets as shown below.

87	~~76~~	58	92	59	41	50	90	~~75~~	67
80	81	~~70~~	~~73~~	69	61	88	46	85	97
50	47	81	87	~~75~~	60	65	92	~~77~~	~~71~~
~~70~~	~~74~~	53	43	61	89	84	83	~~70~~	46
80	~~76~~	~~78~~	64	69	~~76~~	~~78~~	67	~~74~~	64

Use the data set above to;

a) Construct a frequency distribution table starting with class; 40 – 49, 50 – 59, 60 – 69, ...(2 Points)

Class	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89	90 – 99	**Total**
f	5	5	10	15	11	4	50

Hence use the table to;

Class	40 – 49	50 – 59	60 – 69	**70 – 79**	80 – 89	90 – 99	**Total**
$\boldsymbol{f}$	5	5	10	**15**	11	4	50
	5	10	20	**35**	46	50	
$\boldsymbol{x}$	44.5	54.5	64.5	**74.5**	84.5	94.5	
$\boldsymbol{fx}$	222.5	272.5	645	**1117.5**	929.5	378	3565
$\boldsymbol{x^2}$							
$\boldsymbol{fx^2}$	9901.25	14851.25	41602.5	**83253.75**	78542.75	35721	263872.5

$$Q_1 = 59.5 + \frac{12.5 - 10}{10} \times 10 = 59.5 + 2.5 = 62$$

$$Q_3 = 79.5 + \frac{37.5 - 35}{11} \times 10 = 79.5 + 2.27 = 81.77$$

$$IQR = 81.77 - 62 = 19{,}77$$

b) Calculate the mean for the data. (3 Points)

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{3565}{50} = 71.3$$

c) Calculate the variance of the data. (4 Points)

$$variance\ \sigma^2 = \frac{\sum fx^2}{\sum f} - \bar{x}^2 = \frac{263872.5}{50} - 71.3^2 = 193.76$$

d) Calculate the mode of the data. (4 Points)

$$M_0 = l + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times i = 69.5 + \frac{15 - 10}{30 - 10 - 11} \times 10 = 69.5 + \frac{5}{9} \times 10 \approx 75.06$$

3) What is the probability that the sun will set tomorrow? (1Point)

P(sun sets tomorrow) = 1

4) What is the probability that you will grow a horn tomorrow? (1 Point)

P(grow a horn tomorrow) = 0

5) A bag contains 11 red marbles and 6 green marbles of similar size. A marble is chosen at random. Find the probability of picking;

(i) A green one. (1 Point)

$$p(Green) = \frac{6}{17}$$

(ii) A red marble. (1 Point)

$$p(Red) = \frac{11}{17}$$

(iii) A yellow marble. (1 Point)

$$p(Yellow) = 0$$

(iv) A red or a green marble. (1 Point)

$$p(R) or\ p(G) = \frac{6}{17} + \frac{11}{17} = 1$$

6) Kama rolls two fair dice.

(i) Generate a contingency table for the total possible outcomes. (2 Point)

Dice B / Dice A	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

(ii) Two even numbers showing up. (2 Points)

$$p(Even\ Numbers) = \frac{9}{36} = \frac{1}{4}$$