Lecture 08: Superposition

The Superposition Principle:

Consider a multi-input multi-output general circuit.

The inputs $S_1, S_2, S_3, ..., S_N$ represent either *independent* voltage or current sources

The outputs $O_1, O_2, O_3, ..., O_M$ represent the remaining voltages and currents

For instance, O_1 may be current through a resistor, and O_2 may be voltage across a current source

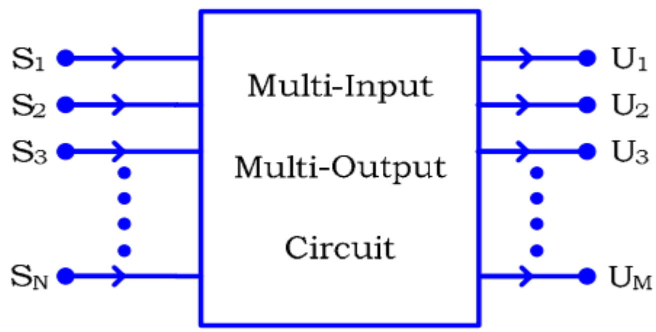

Figure 1

For simplicity, let us consider a *single-output* circuit, with one output quantity, U

All the inputs $S_1, S_2, S_3, ..., S_N$ affect the output U

In other words, U has *some contribution* from *each* of the sources $S_1, S_2, S_3, ..., S_N$

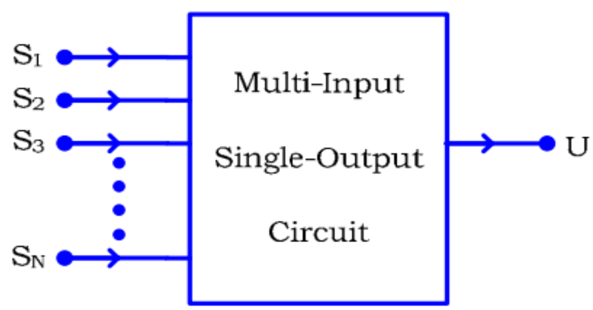

Figure 2

The contribution of S_1 to U is labeled U_1

The contribution of S_2 to U is labeled U_2

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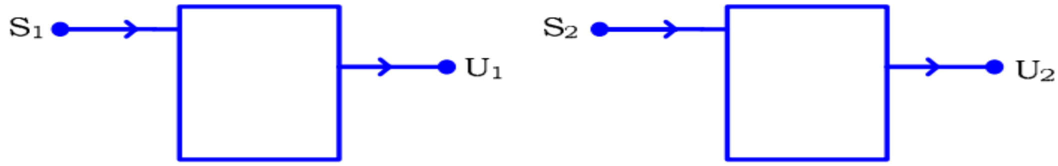

Figure 3

In general, the contribution of S_i to U is labeled U_i

$$\Downarrow$$

$$U = U_1 + U_2 + U_3 + \ldots\ldots + U_N$$

This is called the *Superposition Principle*.

This principle is valid for *linear circuits only*.

All the circuits covered in this course are linear circuits.

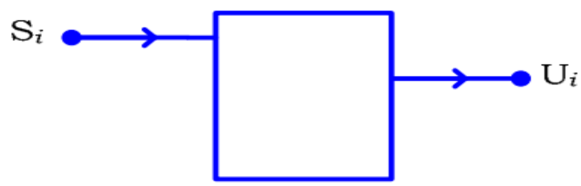

Figure 4

The output U may be *current or voltage*, but it *cannot be power or energy*.

Thus, the SP principle *applies* to currents and voltages, but it *does not* apply to power or energy.

To calculate U_1 $\Rightarrow$ set *all independent* sources to zero *except* S_1

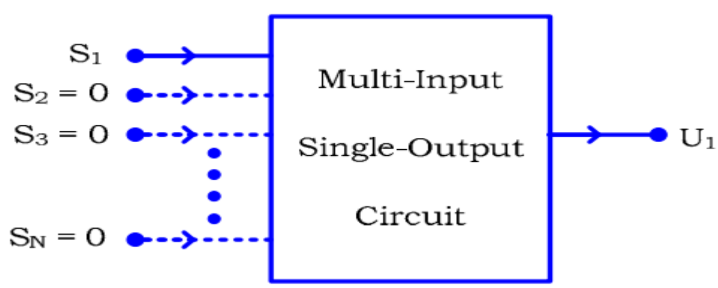

Figure 5

To calculate U_2 $\Rightarrow$ set *all independent* sources to zero *except* S_2

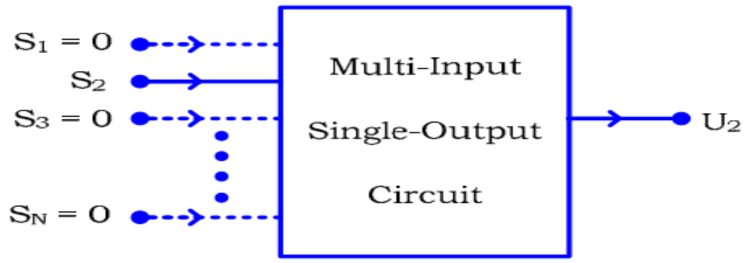

Figure 6

To calculate U_i $\Rightarrow$ set *all independent* sources to zero *except* S_i

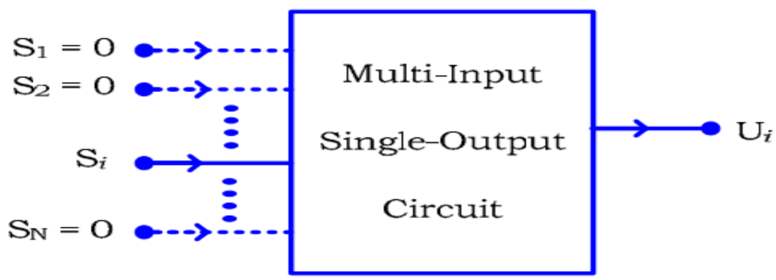

Figure 7

To set a voltage source to zero $\Rightarrow$ replace it with a short circuit

To set a current source to zero $\Rightarrow$ replace it with an open circuit

Extension of SP to multi-output circuits is straightforward.

Example 1:

Calculate I using SP.

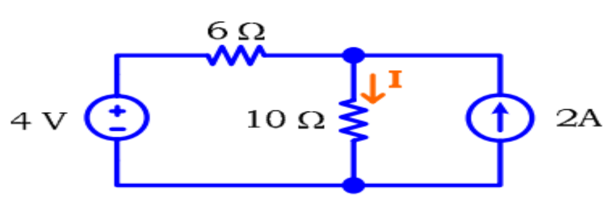

Figure 8

Solution:

First calculate $I^{'} = I\big|_{4V}$ (current I due to only the $4V$ source)

Set the remaining independent sources to zero $\Rightarrow$ replace $2A$ with an open circuit

$$I^{'} = \frac{4}{6+10} = 0.25A$$

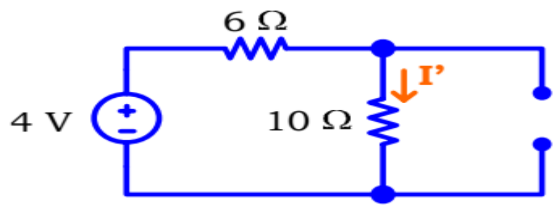

Figure 9

Next calculate $I^{''} = I|_{2A}$ (current I due to only the $2A$ source)

Set the remaining independent sources to zero $\Rightarrow$ replace $4V$ with a short circuit

CDR $\Rightarrow$ $I^{''} = \frac{6}{6+10} \times 2 = \frac{12}{16} = 0.75A$

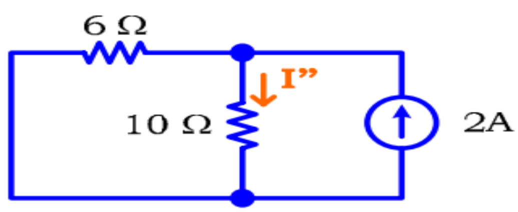

Figure 10

$\therefore I = I^{'} + I^{''} = 0.25 + 0.75 = 1.00A$

Example 2:

Calculate I using SP.

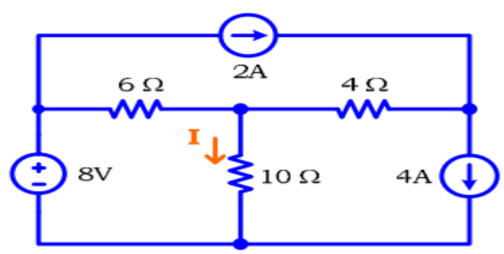

Figure 11

Solution:

Calculate: $I^{'} = I|_{8V}$

$2A$ & $4A$ $\Rightarrow$ replaced by open circuits

Current through 4Ω is zero (why?)

The 4Ω has no effect $\Rightarrow$ 6Ω & 10Ω are *in series*

$$\therefore I' = \frac{8}{6+10} = 0.5A$$

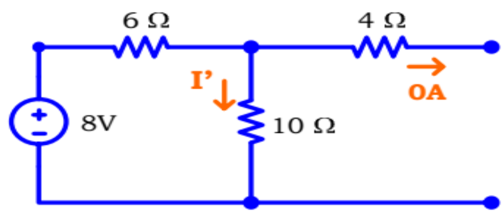

Figure 12

Next calculate: $I' = I|_{4A}$

$8V$ $\Rightarrow$ replaced by a short circuit

$2A$ $\Rightarrow$ replaced by an open circuit

4Ω *in series* with $4A$ $\Rightarrow$ equivalent to $4A$

CDR $\Rightarrow$ $I'' = -\frac{6}{6+10} \times 4 = -1.5A$

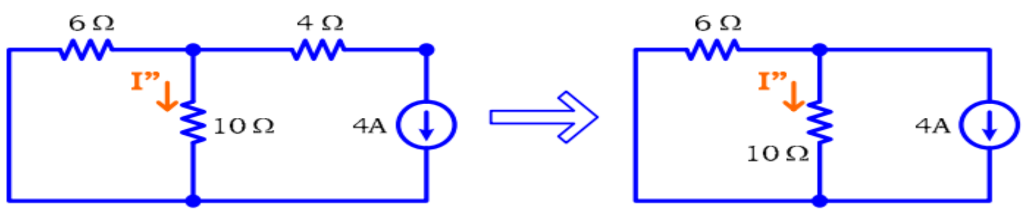

Figure 13

Finally calculate: $I''' = I|_{2A}$

$8V$ $\Rightarrow$ replaced by a short circuit

$4A$ $\Rightarrow$ replaced by an open circuit

4Ω *in series* with $2A$ $\Rightarrow$ equivalent to $2A$

CDR $\Rightarrow$ $I''' = \frac{6}{6+10} \times 2 = 0.75A$

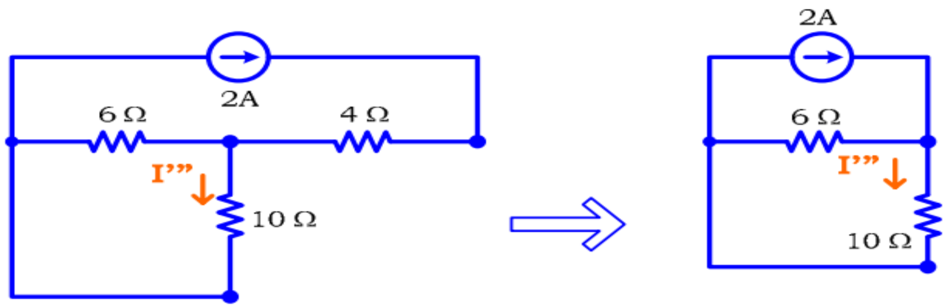

Figure 14

$\therefore I = I' + I'' + I''' = (0.5) + (-1.5) + (0.75) = -0.25A$

Example 3:

Calculate:

a) $P' = P_{5\Omega}|_{8V}$ (Power absorbed by the 5Ω resistor due only the $8V$ source)

b) $P'' = P_{5\Omega}|_{10V}$ (Power absorbed by the 5Ω resistor due only the $10V$ source)

c) Show that $P \neq P' + P''$

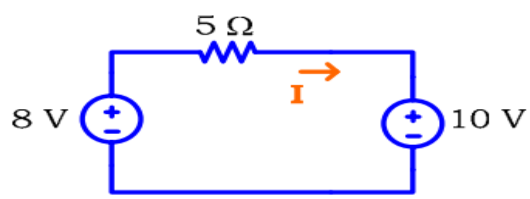

Figure 20

Solution:

a) $I' = \frac{8}{5} = 1.6A \quad \Rightarrow \quad P' = (1.6)^2 5 = 12.8W$

b) $I'' = -\frac{10}{5} = -2A \quad \Rightarrow \quad P' = (-2)^2 5 = 20W$

c) $I = I' + I'' = 1.6 - 2 = -0.4A \quad \Rightarrow \quad P = (-0.4)^2 5 = 0.8W$

$P' + P'' = 12.8 + 20 = 32.8$

$\therefore P \neq P' + P''$

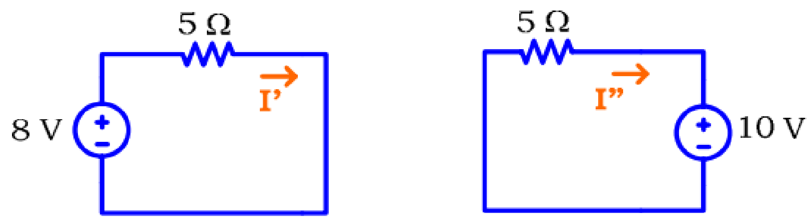

Figure 21

Therefore, for power calculation, we can use SP to calculate *total* currents and voltages, from which we can calculate the power.

From the previous examples we can draw the following conclusions:

1- The number of partial-circuits *equals* the number of *independent* sources.

2- The *algebraic sign* of the unknown *must be* accounted for.

3- The voltage *polarity* and the current *direction* remain the *same* in *all* partial-circuits.

4- *Dependent* sources are *never* set to zero.

5- SP is *not* applicable to Power (or to energy).