Lecture 06: VDR, CDR & Circuit Solution by KVL and KCL

The Voltage Divider Rule (VDR)

The total voltage across the *series* resistors R_1 , R_2 , ... , R_N is V.

$$i = \frac{V}{R_{eq}} = \frac{V}{\sum_{i=1}^{N} R_i}$$

$$v_x = iR_x = \frac{V}{\sum_{i=1}^{N} R_i} R_x = \left(\frac{R_x}{\sum_{i=1}^{N} R_i} \right) V$$

$$\text{VDR} \quad \Rightarrow \quad v_x = \left(\frac{R_x}{\sum_{i=1}^{N} R_i} \right) V$$

$$\text{VDR} \quad \Rightarrow \quad v_{resistor} = \frac{resistor}{sum} \times (total\ voltage)$$

The VDR is valid for *any* number of resistors in *series*.

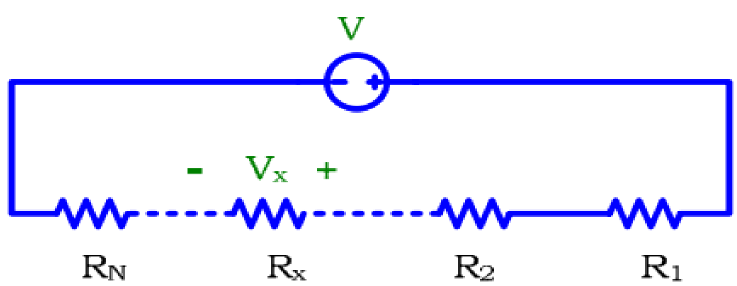


Figure 10

Example 3:

Calculate v_1 & v_2

$$\text{VDR} \quad \Rightarrow \quad v_1 = \frac{4}{4+6} \times 30 = 12V$$

VDR $\Rightarrow$ $v_2 = \frac{6}{4+6} \times 30 = 18V$

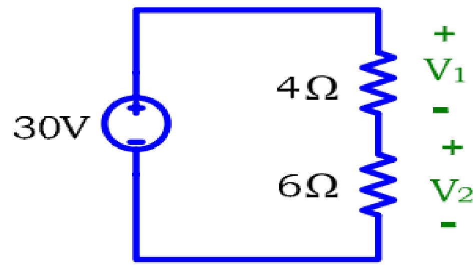


Figure 11

VDR $\Rightarrow$ *Higher* voltage drop across the *higher* resistance.

Example 4:

Calculate the unknown voltages.

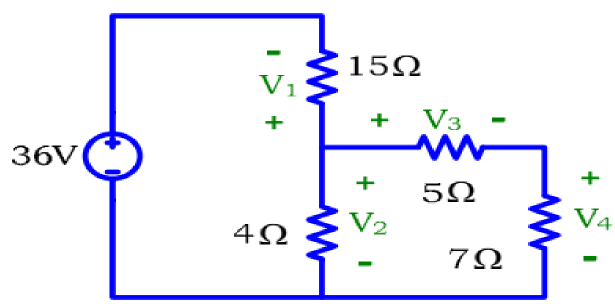


Figure 12

Solution:

$5 + 7 = 12\Omega \quad \Rightarrow \quad R_1 = \frac{4 \times 12}{4+12} = 3\Omega$

VDR $\Rightarrow$ $v_1 = -\frac{15}{15+3} \times 36$ (a *minus* sign is required here. Why?) $\Rightarrow$ $v_1 = -30V$

VDR $\Rightarrow$ $v_2 = \frac{3}{15+3} \times 36 \quad \Rightarrow \quad v_2 = 6V$

Check: KVL $\Rightarrow$ $-36 - v_1 + v_2 = -36 - (-30) + (6) = -36 + 30 + 6 = 0$

VDR $\Rightarrow$ $v_3 = \frac{5}{5+7} \times v_2 = \frac{5}{12} \times 6 \quad \Rightarrow \quad v_3 = 2.5V$

VDR $\Rightarrow$ $v_4 = \dfrac{7}{5+7} \times v_2 = \dfrac{7}{12} \times 6$ $\Rightarrow$ $v_4 = 3.5V$

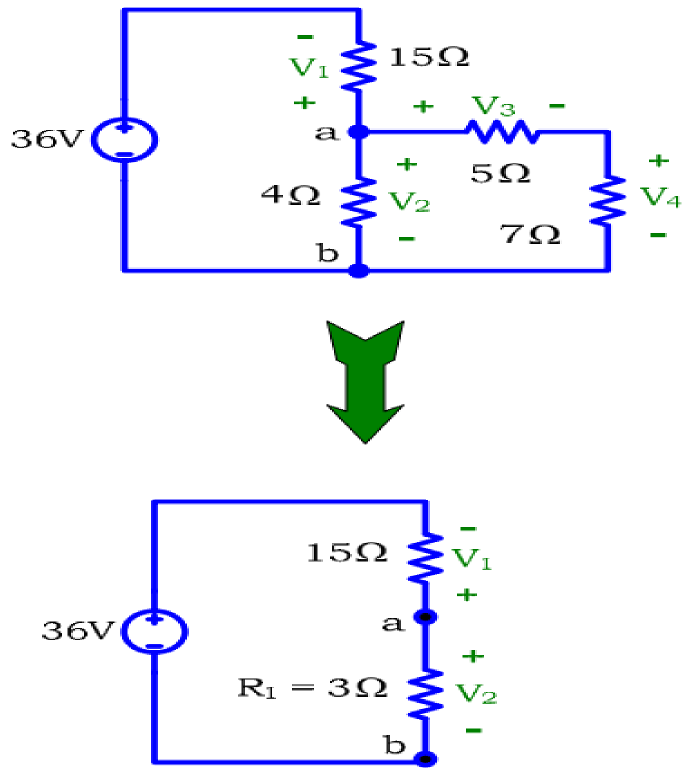


Figure 13

The Current Divider Rule (CDR)

The *total current* entering into the *parallel* combination of R_1 & R_2 is I

$$V = IR_{eq} = I\frac{R_1 R_2}{R_1 + R_2}$$

$$I_1 = \frac{V}{R_1} \qquad \& \qquad I_2 = \frac{V}{R_2}$$

$$I_1 = \frac{1}{R_1} \times \frac{R_1 R_2}{R_1 + R_2} I \quad \Rightarrow \quad I_1 = \frac{R_2}{R_1 + R_2} I \qquad (1)$$

Similarly $\Rightarrow$ $I_2 = \dfrac{R_1}{R_1 + R_2} I$ (2)

CDR $\Rightarrow$ $I = \dfrac{\text{other resistor}}{\text{sum}} \times \text{total current}$

CDR applies to only *two* resistors in *parallel*.

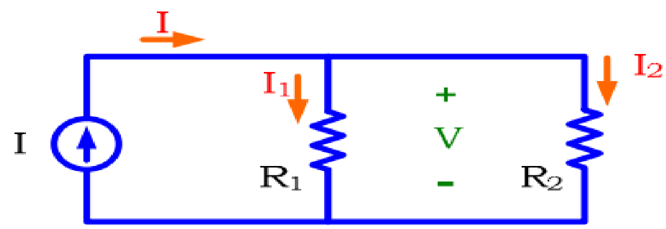


Figure 14

Example 5:

a) Use CDR to calculate I_1 & I_2 .

b) Verify your results by checking KCL.

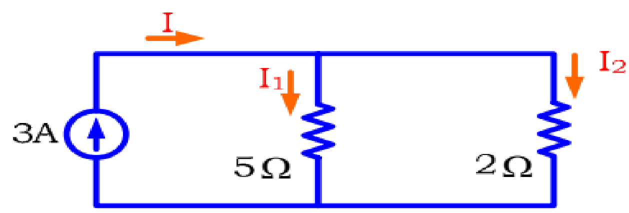


Figure 15

Solution:

a)

$$I_1 = \frac{2}{2+5} \times 3 \quad \Rightarrow \quad I_1 = \frac{6}{7} A$$

$$I_2 = \frac{5}{2+5} \times 3 \quad \Rightarrow \quad I_2 = \frac{15}{7} A$$

b)

KCL at node "a" $\Rightarrow$ $I_s - I_1 - I_2 = 3 - \frac{6}{7} - \frac{15}{7} = 3 - \frac{21}{7} = 0$ (KCL verified)

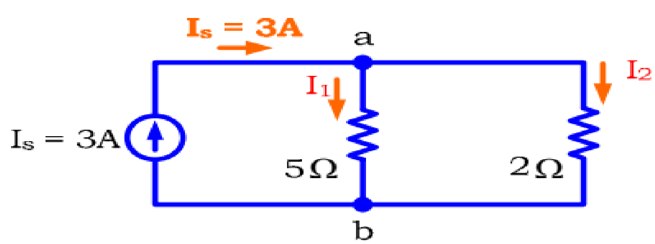


Figure 16

CDR $\Rightarrow$ *Higher* current passes through the *lower* resistance.

Example 6:

Use CDR to calculate I_1, I_2 & I_3.

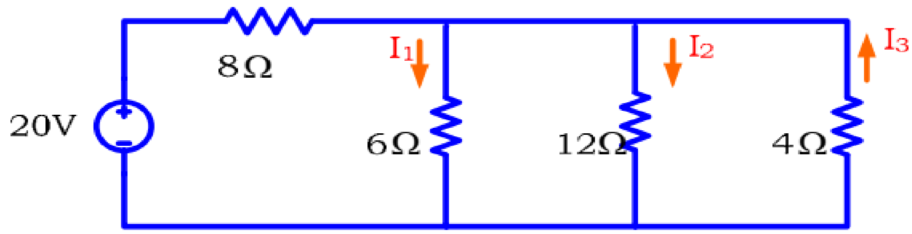


Figure 17

Solution:

$$6\Omega \;\&\; 12\Omega \text{ (in parallel)} \Rightarrow \frac{6\times 12}{6+12}=\frac{72}{18}=4\Omega$$

$$4\Omega \;\&\; 4\Omega \text{ (in parallel)} \Rightarrow \frac{4\times 4}{4+4}=\frac{16}{8}=2\Omega$$

$$\therefore R_{eq}=8+2=10\Omega$$

$$I=\frac{20}{10}=2A$$

$$\text{CDR} \Rightarrow I_4=\frac{4}{4+4}\times 2=1A$$

$$\text{CDR} \Rightarrow I_3=-\frac{4}{4+4}\times 2=-1A$$ (the minus sign is necessary in this case. Why?)

$$\text{CDR} \Rightarrow I_1=\frac{12}{6+12}\times I_4$$ (because I_4 is the total current through 6Ω & 12Ω)

$$\therefore I_1=\frac{12}{18}\times 1=\frac{2}{3}A$$

$$\text{CDR} \Rightarrow I_2=\frac{6}{6+12}\times 1=\frac{1}{3}A$$

Check KCL at super node $\Rightarrow$ $I-I_1-I_2+I_3=2-\frac{2}{3}-\frac{1}{3}+(-1)=1-1=0$ (KCL verified)

Verify that the three *parallel* resistors 6Ω, 12Ω and 4Ω have the *same* voltage.

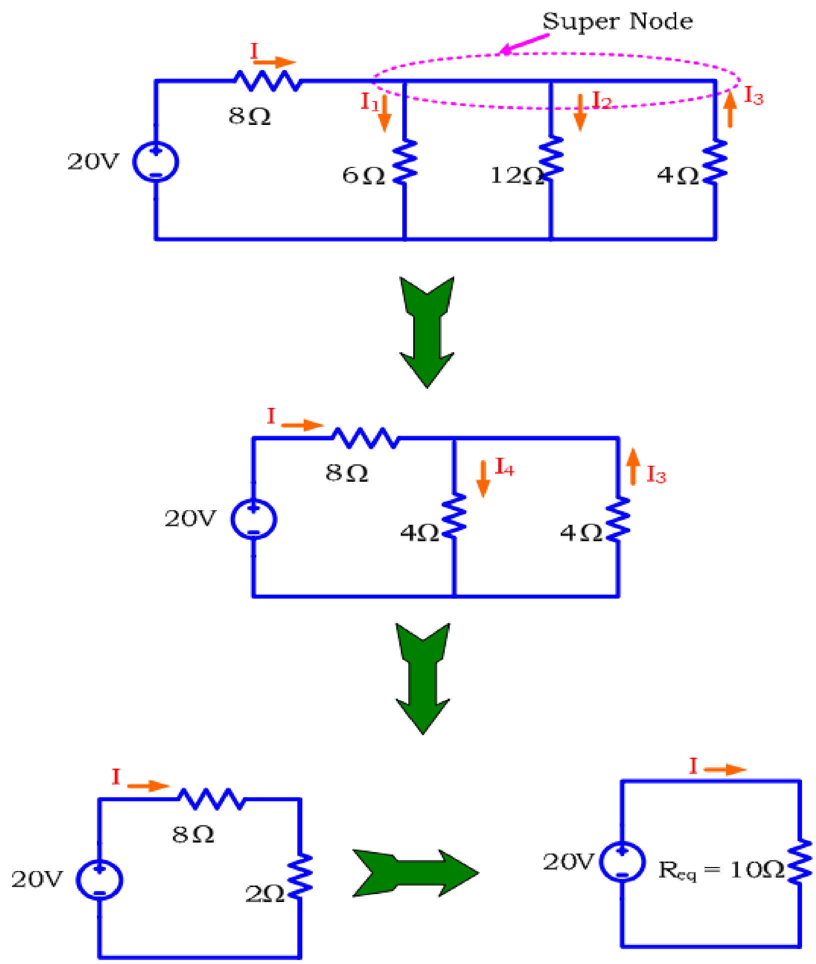


Figure 18

Solutions for Circuits Containing More than One Source

For Circuit with more than one source, we systematically apply KVL, KCL and Ohm's Law. It is obvious by now that:

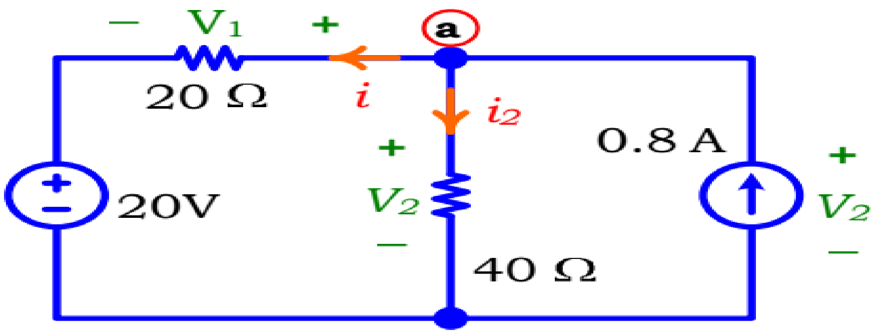
- **Currents through elements in series are equal. (Referred to as simple node)**
- **Voltages across elements in parallel are equal. (Referred to as simple loop)**

Thus

- **Apply KCL only for nonsimple nodes, ie for nodes connecting more than two elements. (Elements not in series).**
- **Apply KVL only for nonsimple loops, ie for loops whose elements are not connected at both pairs of nodes. (Elements not in parallel)**

Example 8

Determine the current i in the circuit of



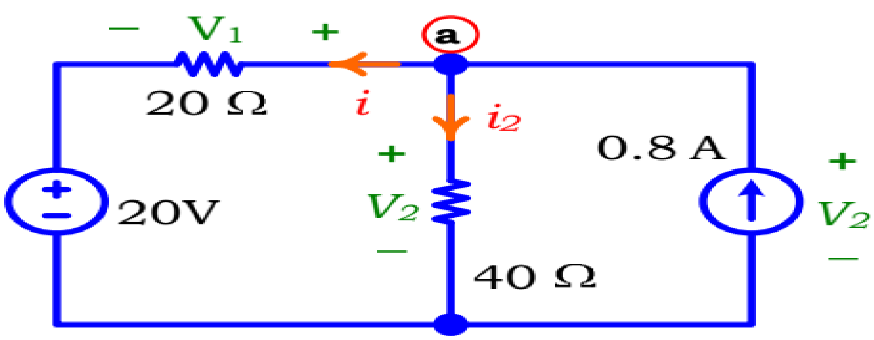
Solution:

$$-i_2 - i + \tfrac{4}{5} = 0$$
$$i_2 = \tfrac{3}{5}\,A$$

$$-v_2 + v_1 + 20 = 0$$
$$-(40 \times \tfrac{3}{5}) + 20i + 20 = 0$$
$$i = \tfrac{1}{5}\,A$$

Example 8

Determine the current i in the circuit

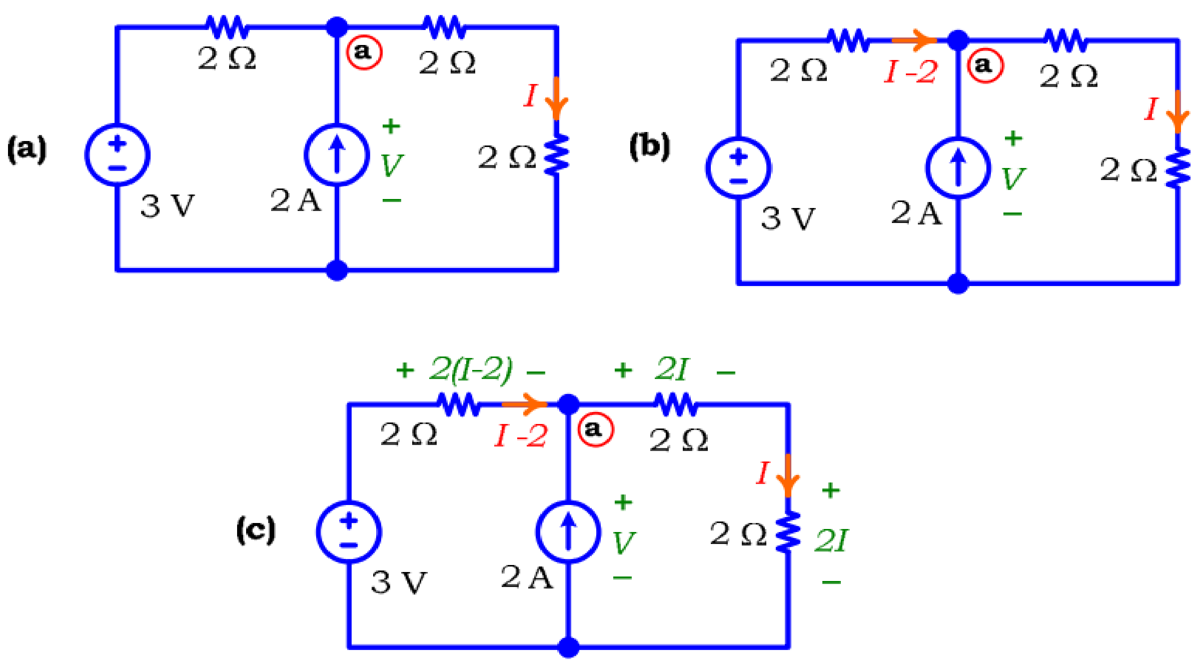


Solution:

$$i = \tfrac{1}{5}\,A$$
$$i_2 = \tfrac{3}{5}\,A$$

Example 9

Determine the voltage V and current I in the circuit



Solution:

We don't know the voltage or current associated with each resistor, so we cannot apply Ohm's law or KVL. Hence we apply KCL at the single nonsimple node a to obtain the current through the 3-V source in terms of I as I-2 as shown in Fig. Next apply Ohm's law to all resistors (in terms of I) then apply KVL around the outside loop for which we know the voltages across all elements. This yields:

$$-3V = 2(I-2) + 2I + 2I$$

Solving yields:

$$I = \tfrac{7}{6} A$$

Apply KVL we obtain V:

$$\mathbf{V} = 2I + 2I$$

$$\mathbf{V} = 3V - 2(I-2)$$

$$\mathbf{V} = \tfrac{14}{3} V$$