Lecture 02: Kirchhoff's Current and Voltage Laws

Kirchoff's Current Law (KCL).

The sum of currents *entering* a node is equal to the sum of currents *leaving* that node.

$$i_1 + i_4 = i_2 + i_3 + i_5$$

Equivalent statement of KCL:

The *algebraic* sum of currents *entering* a node is equal to zero.

$$i_1 - i_2 - i_3 + i_4 - i_5 = 0$$

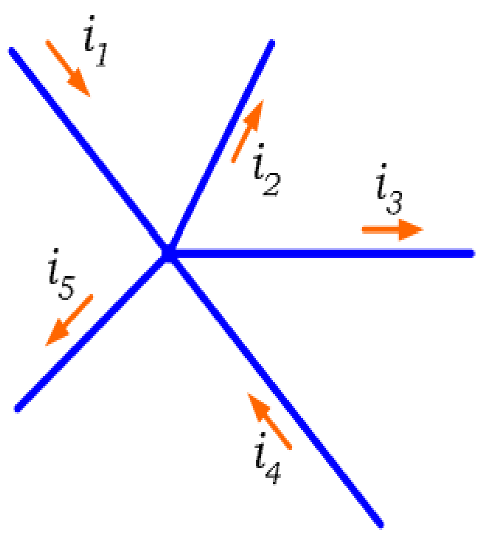

Figure 8

Example 4:

Calculate the unknown currents in the following circuits.

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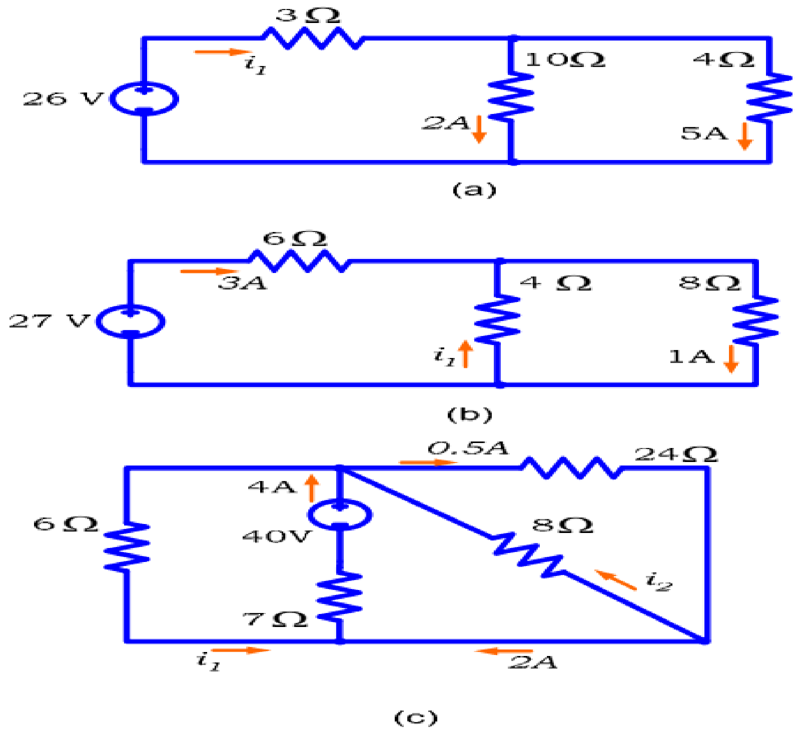


Figure 9

Solution:

a) KCL at node (a) $\Rightarrow$ $i_1 = 2 + 4 = 6A$

b) KCL at node (a) $\Rightarrow$ $3 + i_1 = 1$ $\Rightarrow$ $i_1 = -2A$

Alternatively

KCL at node (a) $\Rightarrow$ $3 + i_1 - 1 = 0$ $\Rightarrow$ $i_1 = -2A$

c) KCL at node (b) $\Rightarrow$ $i_1 - 4 + 2 = 0$ $\Rightarrow$ $i_1 = 2A$

KCL at node (c) $\Rightarrow$ $0.5 - i_2 - 2 = 0$ $\Rightarrow$ $i_2 = -1.5A$

Check KCL at node (a) $\Rightarrow$

$$-i_1 + 4 + i_2 - 0.5 = -(2) + 4 + (-1.5) - 0.5 = -4 + 4 = 0$$

KCL is also applicable to a *closed* area (super node).

The algebraic sum of currents entering a super node is equal to zero.

$i_1 + i_2 - i_3 + i_4 - i_5 = 0$

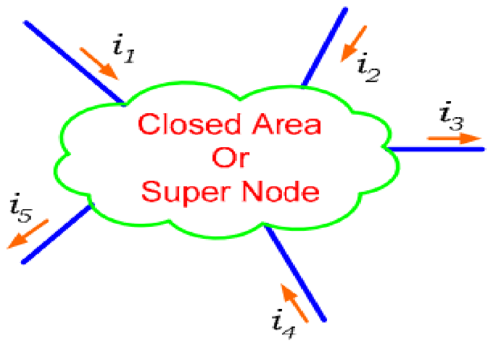


Figure 10

Example:

Calculate the currents i_1 and i_2 in the circuit shown below:

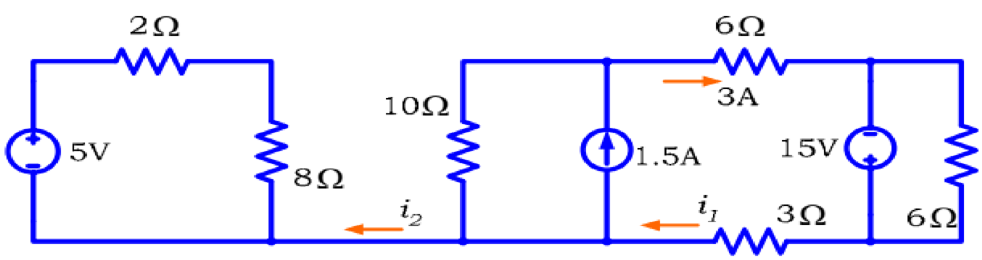


Figure 11

Solution:

KCL at super node 1 $\Rightarrow$ $3 - i_1 = 0$ $\Rightarrow$ $i_1 = 3A$

KCL at super node 2 $\Rightarrow$ $i_2 = 0$ $\Rightarrow$ $i_2 = 0A$

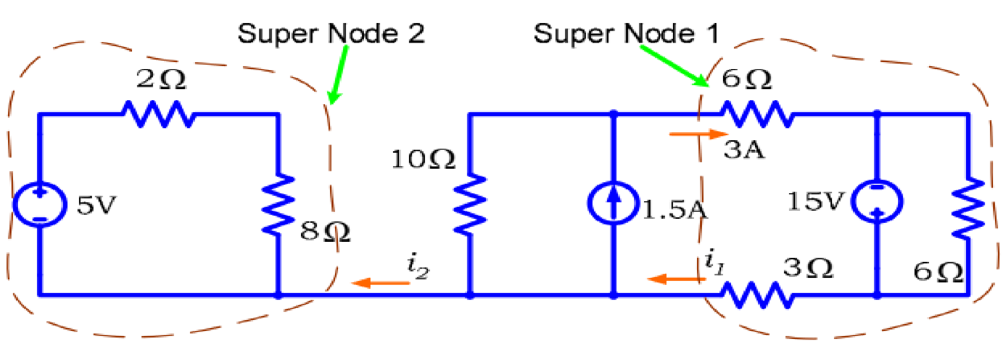


Figure 12

Lecture 02: Kirchhoff's Current and Voltage Laws

Kirchoff's Voltage Law (KVL):

The *algebraic* sum of voltages around *any closed* circuit is equal to zero.

KVL around circuit 1 (CW) $\Rightarrow$ $-v_1 - v_2 + v_3 - v_4 + v_5 = 0$ (1)

KVL around circuit 1 (CCW) $\Rightarrow$ $+v_1 + v_2 - v_3 + v_4 - v_5 = 0$ (2) [same as (1)]

CW = clockwise & CCW = counterclockwise

KVL around the outer circuit (CW) $\Rightarrow$ $-v_6 + v_8 + v_3 - v_4 + v_5 = 0$ (3)

KVL around circuit 2 (CW) $\Rightarrow$ $-v_6 + v_7 = 0$ $\Rightarrow$ $v_6 = v_7$ (parallel elements)

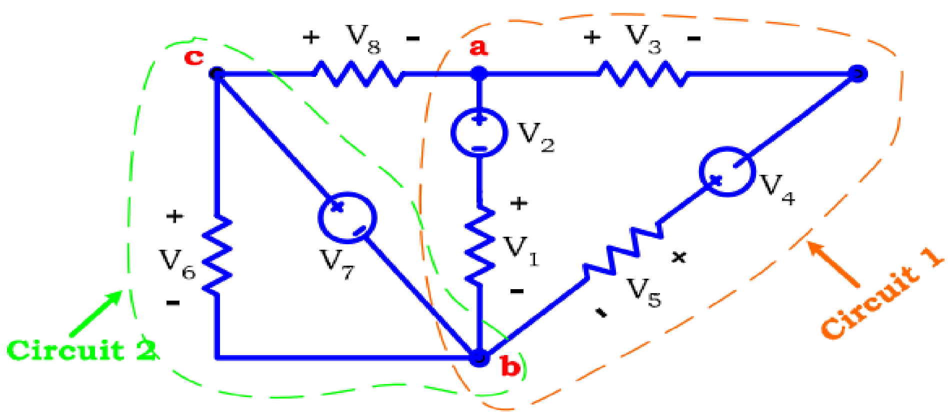


Figure 1

Alternative KVL Statement:

The *algebraic* sum of voltages between two nodes is *independent* of the path taken from the first node to the second node.

KVL *Node a* $\xrightarrow{path1\&2}$ *Node b* $\Rightarrow$ $+v_2 + v_1 = +v_3 - v_4 + v_5$ (4) [same as (1)]

KVL *Node a* $\xrightarrow{path2\&3}$ *Node b* $\Rightarrow$ $+v_3 - v_4 + v_5 = -v_8 + v_6$ (5) [same as (3)]

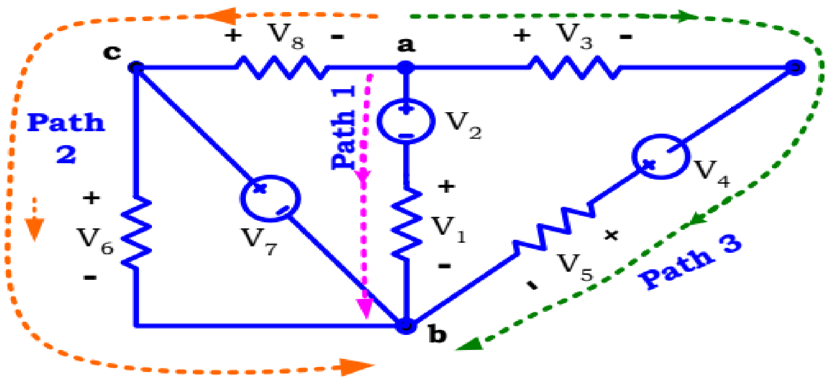


Figure 2

Example:

Calculate the unknown voltages in the given circuit.

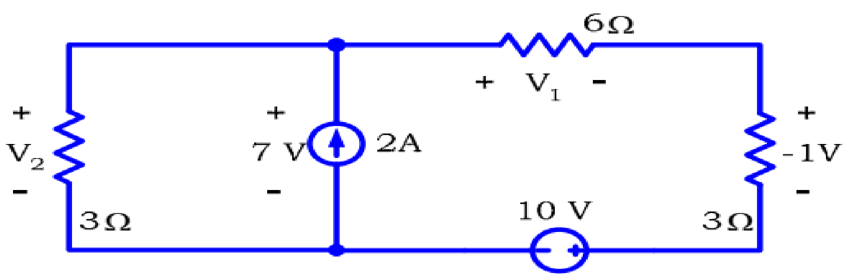


Figure 3

Solution:

Applying KVL:

Right-hand circuit (CW) $\Rightarrow$ $-(7)+v_1+(-1)+10=0$ $\Rightarrow$ $v_1=-2V$

Right-hand circuit (CCW) $\Rightarrow$ $+(7)-(10)-(-1)-v_1=0$ $\Rightarrow$ $v_1=-2V$

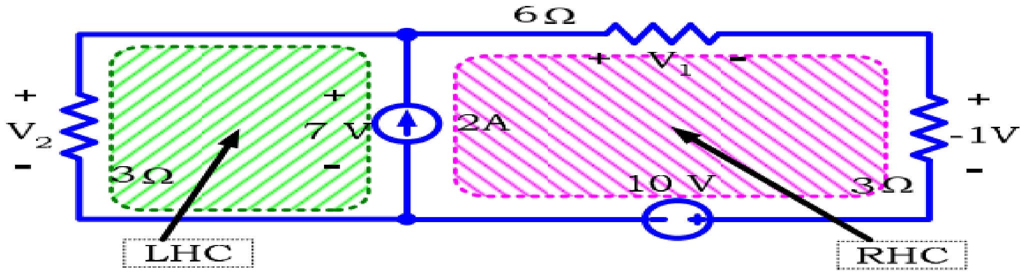
$Node\ a \xrightarrow{path1\&2} Node\ b$ $\Rightarrow$ $+v_1=+(7)-(10)-(-1)$ $\Rightarrow$ $v_1=-2V$

Same answer in all cases.

Left-hand circuit (CW) $\Rightarrow$ $+(7)-(v_2)=0$ $\Rightarrow$ $v_2=7V$

$Node\ a \xrightarrow{path3\&4} Node\ c$ $\Rightarrow$ $+v_2=+7$ $\Rightarrow$ $v_2=7V$

Same answer in both cases.



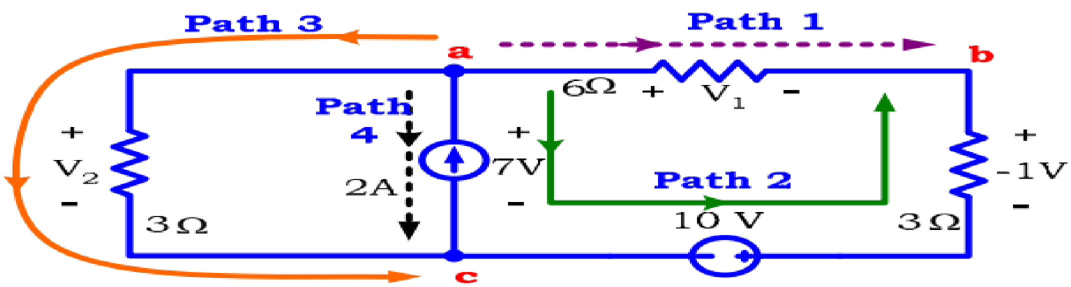
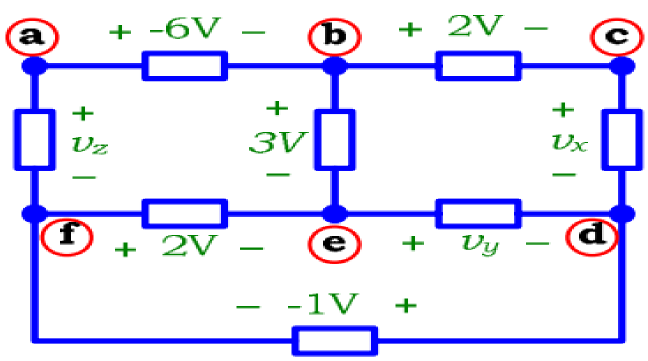


Figure 4

Example: (KVL)

Determine voltages v_x, v_y, v_z in the circuit of fig….by applying KVL.



Solution:

KVL around the loop abcfa

$$-v_z + (-6) + 3 - 2 = 0$$

$$\Rightarrow v_z = -6 - 2 + 3 = -5V \qquad (1)$$

KVL around the loop fedef

$$2 + v_y + (-1) = 0$$

$$\Rightarrow v_y = -2 + 1 = -1V \qquad (2)$$

KVL around the loop bcdeb

$$-3+2+v_x-v_y=0 \qquad (3)$$

To get vx we can substitute v_y from (2) into (3) to get:

$$v_x=+3-2+v_y=1+(-1)=0$$

$$\Rightarrow v_x=0V$$

Note: We can also apply KVL around the loop febcdf to get v_x directly:

$$2-3+2+v_x+(-1)=0$$

$$\Rightarrow v_x=-2+3-2+1=0V$$