LECTURE 10: SUMS & PRODUCTS IN MATLAB

In this section we look at sums and products in both numerical and symbolic forms using the commands sum, prod, symsum and symprod.

3.4.1 Simple "numerical" sums

Suppose we want to evaluate the three sums

$$S_1 = \sum_{r=1}^{10} r, \qquad S_2 = \sum_{i=1}^{100} i^2, \qquad S_3 = \sum_{k=10}^{100} \frac{1}{(k+1)^2}.$$

One approach is to construct an appropriate vector and then add up the elements:

```
>> r = 1:10;
>> S1 = sum(r)
```

For S_2:

```
>> i = 1:100;
>> isqr = i.^2;
>> sum(isqr)
>> % or directly:
>> S2 = sum(i.^2)
```

Note that we must use the "dot" component-wise vector operations:

```
>> k = 10:100;
>> S3_num = sum(1 ./ (k+1).^2)
```

Exercise 3.5 Use MATLAB to evaluate the following

$$\text{(i)} \sum_{r=1}^{10}(r^2+3r-2), \qquad \text{(ii)} \sum_{i=1}^{50} 2^i, \qquad \text{(iii)} \sum_{k=1}^{10} e^{-\sqrt{k}}, \qquad \text{(iv)} \sum_{k=1}^{100} \frac{1}{\sqrt{k}}.$$

□

3.4.2 Products

Products can be approached in a similar fashion using the prod command. If you have not seen it before, $\prod$ denotes a product in the same way as $\sum$ denotes a sum. For example,

$$\prod_{r=1}^{5}(r+2) = 3 \times 4 \times 5 \times 6 \times 7 = 2520.$$

□

Exercise 3.6 Use MATLAB to evaluate the following:

$$\text{(i)} \prod_{r=1}^{5}(r+2), \qquad \text{(ii)} \prod_{r=1}^{10} r^2.$$

□

3.4.3 Symbolic manipulation of sums and products

The previous approach uses standard MATLAB (without the Symbolic Math Toolbox) and works for adding up numbers. But what about sums with variables in the limits? Or infinite sums? For this we use the `symsum` command.

Let us now consider a more general case, where the upper limit is replaced by a free variable, n. For example:

$$S_n = \sum_{r=1}^{n} r.$$

Our previous list idea won't work here, instead:

```
>> syms r n
>> Sn = symsum(r, r, 1, n)
```

(the first `r` here is the summand, the second is the variable to sum over, see `help symsum`). Now that we have S_n we can manipulate it in the usual ways:

```
>> subs(Sn, n, 10)
>> % which is the same as:
>> sum(1:10)
```

Infinite sums can be evaluated in this way too.

```
>> clear all
>> syms a k
>> S = symsum(a^k, k, 1, inf)
```

Why does this output look confusing? Recall that the geometric series converges only for certain values of a. Let's use the assumptions features of the symbolic toolbox to clear things up a bit:

```
>> syms a b
>> assume(b, 'real')
>> assumeAlso(b >= 0)
>> assumeAlso(b < 1)
>> assumptions(b)  % summarize assumptions on b
>> subs(S, a, b)   % replace a with b
```

Actually, that was "heavy handed", how about:

```
>> % this will clear the assumptions on b
>> b = sym('b', 'clear')
>> % want the magnitude of b to be less than 1
>> assume(abs(b) < 1)
>> assumptions(b)  % summarize assumptions on b
>> subs(S, a, b)   % replace a with b
```

Instead of using `subs` after the fact, you could also make the assumptions first:

```
>> clear all
>> syms a k
>> assume(abs(a) < 1)
>> S = symsum(a^k, k, 1, inf)
```

Exercise 3.7 Use MATLAB and the Symbolic Math Toolbox to evaluate the following:

$$\text{(i)} \ \sum_{k=0}^{n-1} a^k, \qquad \text{(ii)} \ \sum_{k=0}^{\infty} \frac{1}{k!}.$$

□

Exercise 3.8 Use MATLAB and the Symbolic Math Toolbox to help verify the following:

$$\text{(i)} \ \sum_{n=1}^{\infty} \frac{2}{(n+1)(n+2)} = 1, \qquad \text{(ii)} \ \sum_{k=0}^{\infty} \frac{k^2+k-1}{(k+2)!} = 0, \qquad \text{(iii)} \ \sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$$

□

Exercise 3.9 By embedding one `symsum` command within another, evaluate the double sums

$$\text{(i)} \ \sum_{m=0}^{10}\sum_{n=0}^{10}(2n+1)e^m, \qquad \text{(ii)} \ \sum_{n=0}^{10}\sum_{m=0}^{10}(2n+1)e^m, \qquad \text{(iii)} \ \sum_{m=0}^{10}\sum_{n=0}^{m}(2n+1)e^m,$$

where (i), (ii) should be approximately `4.2162e+06` and (iii) should be `3.814e+06`.) □

Exercise 3.10 Use `symprod` to evaluate the following:

$$\text{(i)} \ \prod_{r=1}^{10} r^2, \qquad \text{(ii)} \ \prod_{r=1}^{7}(1-q^r), \qquad \text{(iii)} \ \prod_{r=1}^{\infty} q^r, \text{ where } q \in \mathbb{R}.$$

Check that substituting $q = 2$ gives `-78129765` in (ii). Does the Symbolic Math Toolbox give reasonable answers in (iii)? □