Some Fundamental Aerodynamic variables

There are some important variables which we encounter when we talk about aerodynamic problems. These variables are pressure, density, temperature and velocity.

All of above variables are point properties meaning that these variables change from one point to another. However, p, T, ρ are scalar quantities (no directions) where V is vector quantity.

The pressure at a point is defined as

$$p = \lim_{dA \to 0}\left(\frac{dF}{dA}\right) \Rightarrow F = \int \rho dA$$

The density at a point is defined as

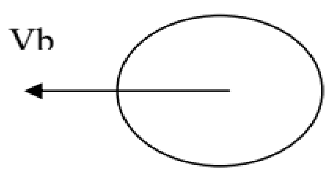
$$\rho = \lim_{dv \to 0} \frac{dm}{dv}$$

Where

dv: elemental volume
dm: mass of fluid inside dv

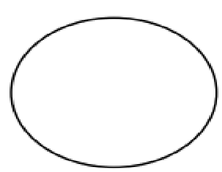
Temperature becomes important when you study high speed aerodynamics.
For the velocity, consider as object moving in the air at speed V_b and the air around this object is still.

$V_\infty = 0$



We can make our reference point on the object. This means I am riding the ball and will see the air coming toward me at V_b. That is

$$V'_\infty = \vec{V}_b - \dot{V}_\infty$$
$$V_\infty = V_b$$



Aerodynamic Forces and Moments:

The aerodynamic forces and moments on a body are due to tow basic sources:

1. Pressure (p) distribution over the body surface.
2. Shear Stress (τ) distribution over the body surface.

Let us take an airfoil as an example. We want to know the lift, drag and moments acting on the airfoil. First, let's sketch the pressure and shear Stress acting on the airfoil:

The pressure is always normal to the surface while the shear is always tangential to the surface.
Let's consider the case where $\alpha = 0$

Sign convention of θ is $+ve$ clockwise
$-ve$ Counter-clockwise

<u>Contribution of the upper surface to the normal force (N) and axial force (A)</u>

$$N'_u = \int_{LE}^{TE} -p_u \cos(-\theta) ds_u + \int \tau_u \sin(-\theta) ds_u$$

$$= -\int_{LE}^{TE} p_u \cos(\theta) ds_u - \int \tau_u \sin(\theta) ds_u$$

$$\cos(-\theta) = \cos\theta$$
$$\sin(-\theta) = -\sin\theta$$

$$A'_u = \int_{LE}^{TE} p_u \sin(-\theta) ds_u + \int \tau_u \cos(-\theta) ds_u$$

$$= -\int_{LE}^{TE} p_u \sin(\theta) ds_u + \int \tau_u \cos(\theta) ds_u$$

Similarly, for the lower surface, θ is positive

$$N_e' = \int_{LE}^{TE} p_l \cos(\theta) ds_l - \int \tau_l \sin(\theta) ds_l$$

$$A_l' = \int_{LE}^{TE} p_l \sin(\theta) ds_l + \int \tau_l \cos(\theta) ds_l$$

Therefore,

$$N' = N_u' + N_l'$$

$$A' = A'_u + A'_l$$

If $\alpha = 0$

$$L = N' \cos\alpha - A' \sin\alpha$$

$$D = N' \sin\alpha + A' \cos\alpha$$

Moment:

Sing convention

Moments which tend to increase α are positive

Moments which tend to decrease α are negative.

Now, let's go back to the normal and axial forces, we want to calculate the moment due to these forces.

For the upper surface:

$$dM'_u = dN'_u \cdot x + dA'_u \cdot y$$

For the lower surface:

$$dM'_l = -dN'_l \cdot x - dA'_l \cdot (-y)$$

Then, the moment around the leading edge is given by:

$$M'_{LE} = \int_{LE}^{TE} [N'_u\, x + A'_u\, y] ds_u + \int_{LE}^{TE} [-N'_l\, x + A'_l\, y] ds_l$$

In the aerodynamic literature coefficients (dimensionless) are more usually used than forces.

Once we compute or measure forces, the coefficients can be computed by dividing by the dynamic pressure and area. For the moment coefficient we divide by the chord of the wing in addition to dynamic pressure and area.

The *dynamic pressure* is defined by:

$$q_\infty = \frac{1}{2}\rho_\infty V_\infty^2$$

$\Rightarrow$ Lift coefficient: $C_l = \dfrac{L}{\frac{1}{2}\rho_\infty V_\infty^2 S}$

Drag coefficient: $C_D = \dfrac{D}{\frac{1}{2}\rho_\infty V_\infty^2 S}$

Moment coefficient: $C_M = \dfrac{M}{\frac{1}{2}\rho_\infty V_\infty^2 S l}$ where l is the characteristic length

Pressure coefficient: $C_p = \dfrac{p - p_\infty}{\frac{1}{2}\rho_\infty V_\infty^2}$

Skin friction: $C_f = \dfrac{\tau}{\frac{1}{2}\rho_\infty V_\infty^2}$

Recall:

$$L = N\cos\alpha - A\sin\alpha$$
$$D = N\sin\alpha + A\cos\alpha$$

divide by $\frac{1}{2}\rho_\infty V_\infty^2 S$

$$C_l = C_N\cos\alpha - C_A\sin\alpha$$
$$C_D = C_N\sin\alpha + C_A\cos\alpha$$