Center of pressure:

It is the point where the aerodynamic moment is zero and is defined as:

$$M'_{LE} = -N' x_{cp}$$

$$x_{cp} = \frac{-M'_{LE}}{-N'}$$

Dimensional Analysis:

Dimensional Analysis is a powerful tool that allows us to study the variation of one variable in terms of other variables.

In aerodynamics, we are usually interested in forces. Intuitively, we expect aerodynamic forces to be a function of the following variables:

1. Free stream speed V_∞. Why? $L = \frac{1}{2}\rho_\infty V_\infty^2 C_l S'$.
2. Free stream density ρ_∞.
3. Viscosity of fluid (μ) i.e. $\tau = \mu \frac{\partial u}{\partial y}$.
4. The size of the body. i.e. chord length C.
5. In compressible flow ($M \rangle 0.3$), the speed of sound (a_u) is usually used to study the influence of compressibility on aerodynamic forces and moments.

So, in summary, the force F is

$$F = f(\rho_\infty, V_\infty, \mu_\infty, c, a_\infty)$$

1. How many physical variables, N?
   6 physical variables: F, $\rho_\infty, V_\infty, \mu_\infty, c$ and a_∞ ==> So, N=6
2. How many basic dimensions, K?
   Mass, Length and time, K=3
3. How many dimensionless parameters?
   # of dimensionless = N-K=3, we will call them π_1, π_2, π_3

4. Dimensionless parameters:

So, $f_2(\pi_1, \pi_2, \pi_3) = 0$

let us choose ρ_∞, V_∞ and c as an arbitrary selected set of K physical variables. Then

$$\pi_1 = f_3(\rho_\infty, V_\infty, C, F)$$
$$\pi_2 = f_4(\rho_\infty, V_\infty, C, \mu_\infty)$$
$$\pi_3 = f_5(\rho_\infty, V_\infty, C, a_\infty)$$

5. finding the relation of each parameter to physical variables ρ_∞, V_∞ and c

Consider π_1 let us assume that $\pi_1 = \rho_\infty^a V_\infty^b C^e F$

$$\pi_1 = \rho_\infty^a V_\infty^b C^e F$$
$$(ML^{-3})^a (LT^{-1})^b L^e MLT^{-2}$$

for M: a+1=0
for L: -3a+b+e+1=0
for T: -b-2=0

$\Rightarrow$ a=-1
b=-2
e=-2

$$\Rightarrow \pi_1 = \rho_\infty^{-1} V_\infty^{-2} C^{-2} F = \frac{F}{\rho_\infty V_\infty^2 C^2} = \frac{F}{\rho_\infty V_\infty S} = C_F$$

where S is the area.
Similarly, assume

$$\pi_2 = \frac{\rho_\infty V_\infty C}{\mu_\infty}$$ This is called the Reynolds number.

$$\pi_3 = \frac{V_\infty}{a}$$ This is called Mach number.

6. Write the dimensionless function:

$$f_2\left(\frac{F}{1/2\rho_\infty V_\infty^2 S}, \frac{\rho_\infty V_\infty C}{\mu_\infty}, \frac{V_\infty}{a_\infty}\right) = 0$$

or $f_2\left(C_F, \mathrm{Re}, M_\infty\right)$

the force coefficient is a function of the Re and Mach numbers.

$$\boxed{C_F = f\left(\mathrm{Re}, M_\infty\right)}$$

This is an important result in aerodynamics particularly when dealing with wings. The force and moment coefficients are a function of Re, M_∞ and angle of attack α

$$C_L = f_1(\mathrm{Re}, M_\infty, \alpha)$$

$$C_D = f_2(\mathrm{Re}, M_\infty, \alpha)$$

$$C_M = f_3(\mathrm{Re}, M_\infty, \alpha)$$

The question : what is the speed of the water inside the water tunnel needed for the tow flows to be similar?

For the two flows to be similar:

$$\mathrm{Re}_a = \mathrm{Re}_w$$

$$\frac{\rho_a V_a da}{\mu_a} = \frac{\rho_w V_w dw}{\mu_w}$$

$$\Rightarrow \frac{V_w}{V_a} = \frac{\rho_a}{\rho_w} \cdot \frac{\mu_w}{\mu_w} = \frac{1.225}{1000} \cdot \frac{1x10^{-3}}{1.81x10^{-5}} = \frac{1.225}{1.81x10}$$

$$\Rightarrow V_w = \frac{1.225}{18.1} V_a$$

setting the water tunnel to this speed will assure us that force and moments coefficients in both wind and water tunnel are almost the same.

Similarly, the pressure and skin friction coefficients are almost the same. However, the forces and moments are not the same.

ρ_1 ρ_2

$$\rho_1 = \rho_2 \quad \forall_1 \neq \forall_2 \text{ and } M_1 \neq M_2$$

but $$\rho_1 = \frac{M_1}{V_1} = \frac{M_2}{V_2} = \rho_2$$