## NYQUIST STABILITY CRITERION:

In control theory and stability theory, the **Nyquist stability criterion**, discovered by Swedish-American electrical engineer Harry Nyquist at Bell Telephone Laboratoriesin 1932,[1] is a graphical technique for determining the stability of a dynamical system. Because it only looks at the Nyquist plot of the open loop systems, it can be applied without explicitly computing the poles and zeros of either the closed-loop or open-loop system (although the number of each type of right-half-plane singularities must be known). As a result, it can be applied to systems defined by non-rational functions, such as systems with delays. In contrast to Bode plots, it can handletransfer functions with right half-plane singularities. In addition, there is a natural generalization to more complex systems with multiple inputs and multiple outputs, such as control systems for airplanes.

The Nyquist criterion is widely used in electronics and control system engineering, as well as other fields, for designing and analyzing systems with feedback. While Nyquist is one of the most general stability tests, it is still restricted to linear, time-invariant (LTI) systems. Non-linear systems must use more complex stability criteria, such as Lyapunov or the circle criterion. While Nyquist is a graphical technique, it only provides a limited amount of intuition for why a system is stable or unstable, or how to modify an unstable system to be stable. Techniques like Bode plots, while less general, are sometimes a more useful design tool. In control theory and stability theory, the **Nyquist stability criterion**, discovered by Swedish-American electrical engineer Harry Nyquist at Bell Telephone Laboratoriesin 1932,[1] is a graphical technique for determining the stability of a dynamical system. Because it only looks at the Nyquist plot of the open loop systems, it can be applied without explicitly computing the poles and zeros of either the closed-loop or open-loop system (although the number of each type of right-half-plane singularities must be known). As a result, it can be applied to systems defined by non-rational functions, such as systems with delays. In contrast to Bode plots, it can handletransfer functions with right half-plane singularities. In addition, there is a natural generalization to more complex systems with multiple inputs and multiple outputs, such as control systems for airplanes.

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## The Nyquist criterion

We first construct **the Nyquist contour**, a contour that encompasses the right-half of the complex plane:

- a path traveling up the $j\omega$ axis, from $0 - j\infty$ to $0 + j\infty$.
- a semicircular arc, with radius $r \to \infty$, that starts at $0 + j\infty$ and travels clock-wise to $0 - j\infty$.

The Nyquist contour mapped through the function $1 + G(s)$ yields a plot of $1 + G(s)$ in the complex plane. By the Argument Principle, the number of clock-wise encirclements of the origin must be the number of zeros of $1 + G(s)$ in the right-half complex plane minus the poles of $1 + G(s)$ in the right-half complex plane. If instead, the contour is mapped through the open-loop transfer function $G(s)$, the result is the Nyquist Plot of $G(s)$. By counting the resulting contour's encirclements of -1, we find the difference between the number of poles and zeros in the right-half complex plane of $1 + G(s)$. Recalling that the zeros of $1 + G(s)$ are the poles of the closed-loop system, and noting that the poles of $1 + G(s)$ are same as the poles of $G(s)$, we now state

**The Nyquist Criterion**:

Given a Nyquist contour $\Gamma_s$, let $P$ be the number of poles of $G(s)$ encircled by $\Gamma_s$, and $Z$ be the number of zeros of $1 + G(s)$ encircled by $\Gamma_s$. Alternatively, and more importantly, $Z$ is the number of poles of the closed loop system in the right half plane. The resultant contour in the $G(s)$-plane, $\Gamma_{G(s)}$ shall encircle (clock-wise) the point $(-1 + j0)$ $N$ times such that $N = Z - P$.

- If the system is originally open-loop unstable, feedback is necessary to stabilize the system. Right-half-plane (RHP) poles represent that instability. For closed-loop stability of a system, the number of closed-loop roots in the right half of the s-plane must be zero. Hence, the number of counter-clockwise encirclements about $-1 + j0$ must be equal to the number of open-loop poles in the RHP. Any clockwise encirclements of the critical point by the open-loop frequency response (when judged from low frequency to high frequency) would indicate that the feedback control system would be destabilizing if the loop were closed. (Using RHP zeros to "cancel out" RHP poles does not remove the instability, but rather ensures that the system will remain unstable even in the presence of feedback, since the closed-loop roots travel between open-loop poles and zeros in the presence of feedback. In fact, the RHP zero can make the unstable pole unobservable and therefore not stabilizable through feedback. If the open-loop transfer function $G(s)$ has a zero pole of multiplicity $l$, then the Nyquist plot has a discontinuity at $\omega = 0$. During further analysis it should be assumed that the phasor travels $l$ times clock-wise along a semicircle of infinite radius. After applying this rule, the zero poles should be neglected, i.e. if there are no other unstable poles, then the open-loop transfer function $G(s)$ should be considered stable.

- If the open-loop transfer function $G(s)$ is stable, then the closed-loop system is unstable for *any* encirclement of the point -1.

- If the open-loop transfer function $G(s)$ is *unstable*, then there must be one *counter* clock-wise encirclement of -1 for each pole of $G(s)$ in the right-half of the complex plane.
- The number of surplus encirclements (greater than N+P) is exactly the number of unstable poles of the closed-loop system.
- However, if the graph happens to pass through the point $-1 + j0$, then deciding upon even the marginal stability of the system becomes difficult and the only conclusion that can be drawn from the graph is that there exist zeros on the $j\omega$ axis.

POLAR PLOTS:

the polar coordinate system is a two-dimensional coordinate system in which each point on a plane is determined by a distance from a reference point and an anglefrom a reference direction. Angles in polar notation are generally expressed in either degrees or radians (2π rad being equal to 360°). Degrees are traditionally used in navigation, surveying, and many applied disciplines, while radians are more common in mathematics and mathematical physics.[9]

In many contexts, a positive angular coordinate means that the angle $\phi$ is measuredcounterclockwise from the axis.

## Polar equation of a curve

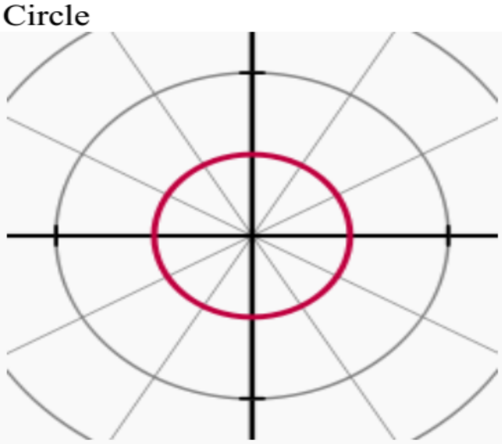
The equation defining an algebraic curve expressed in polar coordinates is known as a *polar equation*. In many cases, such an equation can simply be specified by defining *r* as afunction of $\phi$. The resulting curve then consists of points of the form $(r(\phi), \phi)$ and can be regarded as the graph of the polar function $r$.

Different forms of symmetry can be deduced from the equation of a polar function $r$. If $r(-\phi) = r(\phi)$ the curve will be symmetrical about the horizontal (0°/180°) ray, if $r(\pi - \phi) = r(\phi)$ it will be symmetric about the vertical (90°/270°) ray, and if $r(\phi - \alpha) = r(\phi)$ it will be rotationally symmetric by α clockwise and counterclockwise about the pole.

Because of the circular nature of the polar coordinate system, many curves can be described by a rather simple polar equation, whereas their Cartesian form is much more intricate. Among the best known of these curves are the polar rose, Archimedean spiral, lemniscate,limaçon, and cardioid.

For the circle, line, and polar rose below, it is understood that there are no restrictions on the domain and range of the curve.

Circle



A circle with equation $r(\phi) = 1$

The general equation for a circle with a center at $(r_0, \gamma)$ and radius $a$ is

$$r^2 - 2rr_0\cos(\varphi - \gamma) + r_0^2 = a^2.$$

This can be simplified in various ways, to conform to more specific cases, such as the equation

$$r(\varphi) = a$$

for a circle with a center at the pole and radius $a$.[14]

When $r_0 = a$, or when the origin lies on the circle, the equation becomes

$$r = 2a\cos(\varphi - \gamma).$$

In the general case, the equation can be solved for $r$, giving

$$r = r_0\cos(\varphi - \gamma) + \sqrt{a^2 - r_0^2\sin^2(\varphi - \gamma)},$$

the solution with a minus sign in front of the square root gives the same curve