

3A.1 Derivation of Least-Squares Estimates

Differentiating Eq. (3.1.2) partially with respect to $\hat{\beta}_1$ and $\hat{\beta}_2$, we obtain

$$\frac{\partial(\sum \hat{u}_i^2)}{\partial \hat{\beta}_1} = -2 \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) = -2 \sum \hat{u}_i \quad (1)$$

$$\frac{\partial(\sum \hat{u}_i^2)}{\partial \hat{\beta}_2} = -2 \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) X_i = -2 \sum \hat{u}_i X_i \quad (2)$$

Setting these equations to zero, after algebraic simplification and manipulation, gives the estimators given in Eqs. (3.1.6) and (3.1.7).