

# Econometrics

## Course Calendar

Week	Main Content
Week 1	Introduction to Simple Regression
Week 2	Simple Regression
Week 3	Simple Regression: $r^2$ & Hands-on-Exercise
Week 4	Central Limit Theorem, Probability and Probability Density Function (PDF)
Week 5	Hypothesis Testing: Basics
Week 6	Simple Regression: Testing of Hypothesis

# Econometrics

## Lecture 3: $r^2$ & Simple Regression

### Hands on Exercise in Excel

Geetha Rani Prakasam, Ph.D.

Professor

# Recap

- Simple Regression- Problem of Estimation
- Least Square Criterion – OLS
- Assumptions of OLS
- Gauss Markov theorem – BLUE
- Precision of the Estimated Parameters  $\hat{\beta}_2$  &  $\hat{\beta}_1$  given by their variance and SEs

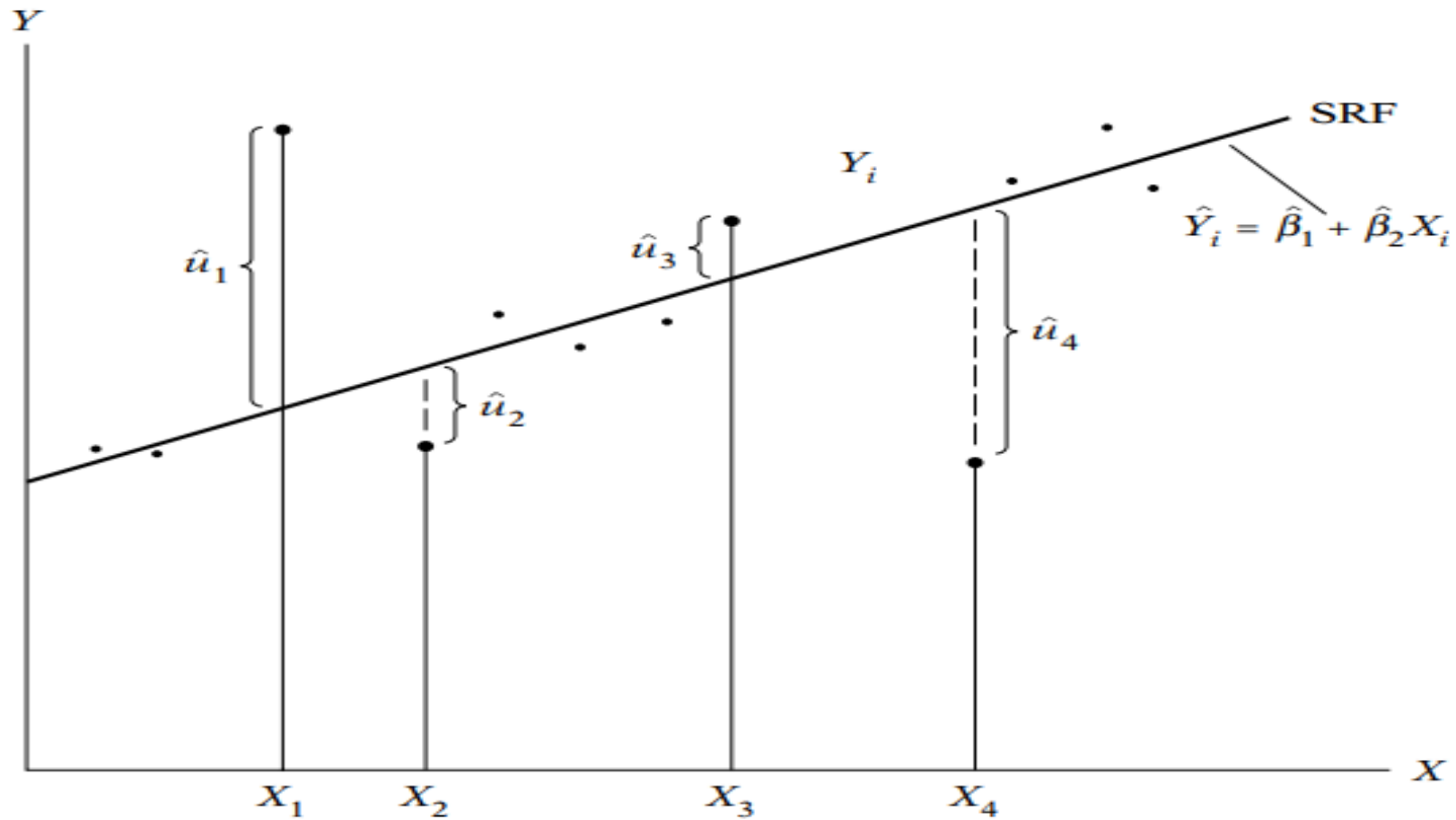
# outline

- Co-efficient of determination  $r^2$
- Recollecting the formula and estimating the simple regression parameters, intercept and slope
- Recollecting the formula and estimating the variance and standard errors of the simple regression parameters, intercept and slope
- The estimation done in excel with a numerical example
- Estimating  $r^2$  in excel
- Normality of U

### 3-5. The coefficient of determination $r^2$ : A measure of “Goodness of fit”

- How “well” the sample regression line fits the data
- Recall Fig 3.1: fitted line of SRF - there will be some positive  $u^{\wedge}_i$  and some negative  $u^{\wedge}_i$ .

Fig 3.1 Least-squares criterion.



Source: Basic Econometrics by Damodar Gujarati, Page 59.

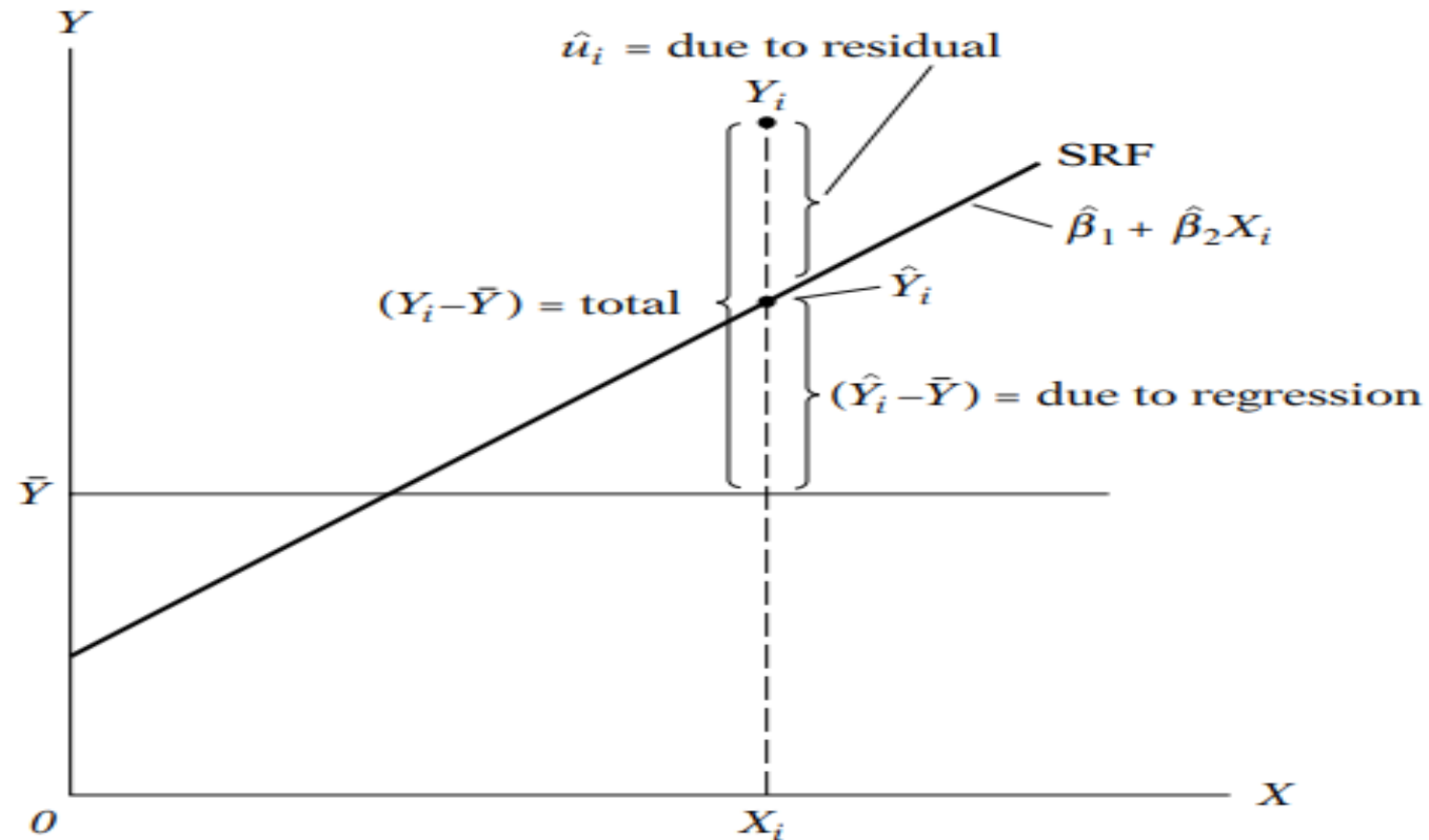
# The coefficient of determination $r^2$

- What we hope for is that these residuals around the regression line are as small as possible.
- The **coefficient of determination**  $r^2$  (**two-variable case**) or  $R^2$  (multiple regression) is a summary measure that tells how well the sample regression line fits the data.
- To compute this  $r^2$ , we proceed as follows: Recall that
- $Y_i = \hat{Y}_i - \hat{u}_i$  -----(2.6.3)
- or in the deviation form:  $y_i = \hat{y}_i - \hat{u}_i$  -----(3.5.1)

### 3-5. The coefficient of determination $r^2$ : A measure of “Goodness of fit”

- Squaring Equation 3.5.1 on both sides and summing over the sample, we obtain
- $\sum y_i^2 = \sum \hat{y}_i^2 + \sum u_i^2 + 2\sum \hat{y}_i u_i$
- $= \sum \hat{y}_i^2 + \sum u_i^2$
- $= \beta^2 \sum x_i^2 + \sum u_i^2$  -----(3.5.2)
- $TSS = ESS + RSS$  -----(3.5.3)
- shows that the total variation in the observed Y values about their mean value can be partitioned into two parts, one attributable to the regression line and the other to random forces because not all actual Y observations lie on the fitted line. Geometrically, we have Figure 3.9.

Fig 3.9: Breakdown of the variation of  $Y_i$  into two components



Source: Basic Econometrics by Damodar Gujarati, Page 83.

3-5. The coefficient of determination  $r^2$ : A measure of “Goodness of fit”

- Fig 3.9 - Breakdown of the variation of  $Y_i$  into two components
- The  $r^2$  can be interpreted as the fraction of sample variation in  $y$  that is explained by  $x$

Note 1: in the simple regression analysis,  $r^2$  is the same as the square of the correlation

coefficient,  $r$ .

$$= \frac{\sum (Y_i - \bar{Y})^2}{\sum (Y_i - \bar{Y})^2} + \frac{\sum \hat{u}_i^2}{\sum (Y_i - \bar{Y})^2}$$

## 3-5. The coefficient of determination $r^2$ : A measure of “Goodness of fit”

- Note 2:
- $TSS = \sum y_i^2 =$  Total Sum of Squares
- $ESS = \sum \hat{Y}_i^2 = \beta^2 \sum x_i^2 =$  Explained Sum of Squares
- $RSS = \sum u_i^2 =$  Residual Sum of Squares
- Now dividing Equation 3.5.3 by TSS on both sides, we obtain

$$1 = \frac{ESS}{TSS} + \frac{RSS}{TSS} ; \text{ or}$$

$$1 = r^2 + \frac{RSS}{TSS} ; \quad \text{or} \quad r^2 = 1 - \frac{RSS}{TSS}$$

----- (3.5.4)  
11

## 3-5. The coefficient of determination $r^2$ : **A measure of “Goodness of fit”**

- We now define  $r^2 = ESS/TSS$
- $r^2$  is coefficient of determination, it measures the proportion or percentage of the total variation in Y explained by the regression Model
- Two properties of  $r^2$  may be noted:
- It is a nonnegative quantity
- $0 \leq r^2 \leq 1$ ;
- A quantity closely related to but conceptually very much different from  $r^2$  is the **coefficient of correlation**;
- $r = \pm\sqrt{r^2}$  is sample correlation coefficient

# Summary and Conclusions

- The basic framework of regression analysis is the **CLRM**.
- The CLRM is based on a set of assumptions.
- Based on these assumptions, the least-squares estimators take on certain properties summarized in the Gauss–Markov theorem, that in the class of linear unbiased estimators, the least-squares estimators have minimum variance.
- In short, they are **BLUE**.

# Summary and Conclusions

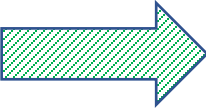
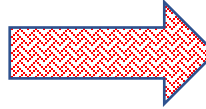
- The precision of OLS estimators is measured by their **standard errors**: we shall see how the standard errors enable one to draw inferences on the population parameters, the  $\beta$  coefficients – Test of Significance.
- The overall goodness of fit of the regression model is measured by the **coefficient of determination,  $r^2$** .
- **It tells what proportion of the variation in the dependent variable, or regressand, is explained by the explanatory variable, or regressor.** This  $r^2$  lies between 0 and 1; the closer it is to 1, the better is the fit.

# Estimating the Regression Parameters

- $\beta^{\wedge}_2 = \sum x_i y_i / \sum x_i^2$  -----(3.1.6)

- Small / lower case letter denote the deviation from mean values

- $\beta^{\wedge}_1 = \bar{y} - \beta^{\wedge}_2 \bar{x}$  -----(3.1.7)

- Estimating the regression parameters  $\beta^{\wedge}_2$  &  $\beta^{\wedge}_1$  Using MSExcel
- Steps to be completed before estimating  $\beta^{\wedge}_2$
- $\sum x_i y_i$  and  $\bar{y}$   the product of the deviations from the mean values of  $\bar{x}$
- $\sum x_i^2$   the product of the deviations from the mean values of  $\bar{x}$

Data on the Following Table on Y and X Variables, N=15

PRICE OF GOLD (Y)	X= Consumer Price Index(CPI) 1[982-84 = 100]
147.98	60.6
193.44	65.2
307.62	72.6
612.51	82.4
459.61	90.9
376.01	96.5
423.83	99.6
360.29	103.9
317.3	107.6
367.87	109.6
446.5	113.6
436.93	118.3
381.28	124
384.08	130.7
362.04	136.2

	PRICE OF GOLD (Y)	X= Consumer Price Index(CPI) 1[982-84 = 100]	Deviation from $\bar{x}$ = x	Deviation from $\bar{y}$ = Y	xy	$x^2$
	147.98	60.6	-40.18	-223.84	8993.86	1614.43
	193.44	65.2	-35.58	-178.38	6346.74	1265.94
	307.62	72.6	-28.18	-64.20	1809.14	794.11
	612.51	82.4	-18.38	240.69	-4423.89	337.82
	459.61	90.9	-9.88	87.79	-867.37	97.61
	376.01	96.5	-4.28	4.19	-17.94	18.32
	423.83	99.6	-1.18	52.01	-61.37	1.39
	360.29	103.9	3.12	-11.53	-35.97	9.73
	317.3	107.6	6.82	-54.52	-371.82	46.51
	367.87	109.6	8.82	-3.95	-34.83	77.79
	446.5	113.6	12.82	74.68	957.41	164.35
	436.93	118.3	17.52	65.11	1140.74	306.95
	381.28	124	23.22	9.46	219.68	539.17
	384.08	130.7	29.92	12.26	366.84	895.21
	362.04	136.2	35.42	-9.78	-346.38	1254.58
<b>TOTAL (<math>\Sigma</math>)</b>	<b>5577.29</b>	<b>1511.7</b>	<b>0.00</b>	<b>0.00</b>	<b>13674.81</b>	<b>7423.92</b>
AVERAGE	371.82	100.78				

	X= Consumer					
	PRICE OF	Price Index(CPI)	Deviation from	Deviation		
	GOLD (Y)	1[982-84 = 100]	$\bar{x}$ ; $\bar{x} - X_i = x_i$	from $\bar{y}$ ;		
				$\bar{y} - Y_i = y_i$	$x_i y_i$	$x_i^2$
	(1)	(2)	(3)	(4)	(5)	(6)

- Deviation from  $\bar{x}$  means the difference between  $\bar{x}$  from each of the  $X_i$  denoted by lower case  $x_i$  in column 3 and is calculated by =C3-C19;
- =C3-\$C\$19 - “\$” is used to fix the cell – which contains the  $\bar{x}$
- Deviation from  $\bar{y}$  means the difference between  $\bar{y}$  from each of the  $Y_i$  and noted as lower case  $y_i$  in column 4 & is calculated by =B3-B19;
- =B3-\$B\$19 – “\$” is used to fix the cell – that contains the  $\bar{y}$
- $x_i y_i$  = multiplication of values in column 3 and 4
- $x_i^2$  = square the values in column 3
- Apply these obtained values in the formula for  $\beta^{\wedge}_2 = \frac{\sum x_i y_i}{\sum x_i^2}$

Slope co-efficient:  $\beta^{\wedge}_2 = \sum x_i y_i / \sum x_i^2$

- $\beta^{\wedge}_2 = \sum x_i y_i / \sum x_i^2$
- $\sum x_i y_i = 13674.81$
- $\sum x_i^2 = 7423.92$
- **$13674.81 / 7423.92 = 1.841993$**

Estimating Intercept co-efficient:  $\beta^{\wedge}_1 = \bar{y} - \beta^{\wedge}_2 \bar{x}$

- $\beta^{\wedge}_1 = \bar{y} - \beta^{\wedge}_2 \bar{x}$
- $\bar{y} = 371.82$
- $\bar{x} = 100.78$
- $\beta^{\wedge}_2 = 1.841993$
- $\beta^{\wedge}_1 = 371.82 - 1.841993 * 100.78$
- $\beta^{\wedge}_1 = 186.183$

## Variance and SE of least-squares estimates

- $\text{var}(\hat{\beta}_2) = \sigma^2 / \sum x_i^2$  (3.3.1)

- $\text{se}(\hat{\beta}_2) = \sqrt{\text{Var}(\hat{\beta}_2)}$  (3.3.2)

- $\text{var}(\hat{\beta}_1) = \sigma^2 \sum x_i^2 / n \sum x_i^2$  (3.3.3)

- $\text{se}(\hat{\beta}_1) = \sqrt{\text{Var}(\hat{\beta}_1)}$  (3.3.4)

- Where

- $\hat{\sigma}^2 = \sum u_i^2 / (n - 2)$

## Reference

Chapter 3: Two-Variable Regression Model: The Problem of Estimation **in** Basic Econometrics by Domodar Gujarati

Chapter 4: THE NORMALITY ASSUMPTION:

**Classical Normal Linear Regression Model**

**(CNLRM), in** Basic Econometrics by Domodar Gujarati

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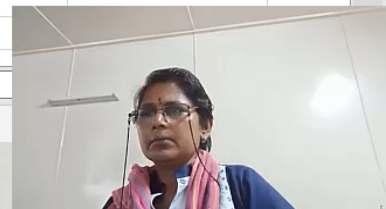
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	A	B	C	D	E	F	G	H	I	J	K
1			<b>X= Consumer Price Index(CPI) 1[982-84 = 100]</b>								
2		<b>PRICE OF GOLD</b>		<b>xi</b>	<b>yi</b>	<b>y<sup>2</sup></b>	<b>x<sup>2</sup></b>	<b>xiyi</b>	<b>Y<sup>^</sup> = 186.18 + 1.841 X</b>	<b>u<sup>^</sup></b>	<b>u<sup>2</sup></b>
3		147.98	60.6								
4		193.44	65.2								
5		307.62	72.6								
6		612.51	82.4								
7		459.61	90.9								
8		376.01	96.5								
9		423.83	99.6								
10		360.29	103.9								
11		317.3	107.6								
12		367.87	109.6								
13		446.5	113.6								
14		436.93	118.3								
15		381.28	124								
16		384.08	130.7								
17		362.04	136.2								
18											
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	A	B	C	D	E	F	G	H	I	J	K	L	M	N
1			<b>X= Consumer Price Index(CPI) 1[982-84 = 100]</b>											
2		<b>PRICE OF GOLD</b>		<b>x</b>	<b>y</b>	<b>x<sup>2</sup></b>	<b>y<sup>2</sup></b>	<b>xy</b>	<b>Y<sup>^</sup> = 186.18 + 1.841 X</b>	<b>u<sup>^</sup></b>	<b>u<sup>2</sup></b>	<b>X<sup>2</sup></b>		
3		147.98	60.6	-40.18	-223.84	1614.43	50104.05	8993.86	297.74	149.76	22429.44	3672.36		
4		193.44	65.2	-35.58	-178.38	1265.94	31819.19	6346.74	306.21	112.77	12717.79	4251.04		
5		307.62	72.6	-28.18	-64.20	794.11	4121.55	1809.14	319.84	12.22	149.25	5270.76		
6		612.51	82.4	-18.38	240.69	337.82	57932.00	-4423.89	337.88	-274.63	75422.52	6789.76		
7		459.61	90.9	-9.88	87.79	97.61	7707.20	-867.37	353.53	-106.08	11253.62	8262.81		
8		376.01	96.5	-4.28	4.19	18.32	17.56	-17.94	363.84	-12.17	148.19	9312.25		
9		423.83	99.6	-1.18	52.01	1.39	2705.11	-61.37	369.54	-54.29	2947.01	9920.16		
10		360.29	103.9	3.12	-11.53	9.73	132.93	-35.97	377.46	17.17	294.81	10795.21		
11		317.3	107.6	6.82	-54.52	46.51	2972.36	-371.82	384.27	66.97	4485.20	11577.76		
12		367.87	109.6	8.82	-3.95	77.79	15.60	-34.83	387.95	20.08	403.35	12012.16		
13		446.5	113.6	12.82	74.68	164.35	5577.20	957.41	395.32	-51.18	2619.64	12904.96		
14		436.93	118.3	17.52	65.11	306.95	4239.40	1140.74	403.97	-32.96	1086.34	13994.89		
15		381.28	124	23.22	9.46	539.17	89.50	219.68	414.46	33.18	1101.18	15376		
16		384.08	130.7	29.92	12.26	895.21	150.32	366.84	426.80	42.72	1824.89	17082.49		
17		362.04	136.2	35.42	-9.78	1254.58	95.64	-346.38	436.92	74.88	5607.64	18550.44		
18	TOTAL	5577.29	1511.7	0.00	0.00	7423.92	167679.60	13674.81	5575.74		142490.863	159773.05		
19	AVERAGE	371.82	100.78											

Sheet1 3.1 3.2

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16		384.08	130.7	29.92	12.26	895.21	150.32	366.84	426.80	42.72	1824.89	17082.49		
17		362.04	136.2	35.42	-9.78	1254.58	95.64	-346.38	436.92	74.88	5607.64	18550.44		
18	TOTAL	5577.29	1511.7	0.00	0.00	7423.92	167679.60	13674.81	5575.74		142490.863	159773.05		
19	AVERAGE	371.82	100.78											
20														
21	n=15	$\sigma^2 =$	10960.83559	$\sigma =$	104.694009	$b_2 =$	1.841993		$RSS = 1 - \frac{\sum u^2}{\sum y^2}$ $r^2 = 0.150219462$					
22					$b_1 =$	186.1833								
23		var( $b_2$ )=	1.47642077											
24		s.e( $b_2$ )=	1.215080561											
25														
26														
27		var( $b_1$ )=	15726.14997											
28		s.e ( $b_1$ )=	125.4039472											
29														
30														
31														
32				X	Y	Y^								
33				60.6	147.98	297.7446								
34				65.2	193.44	306.2132								
35				72.6	307.62	319.8366								
36				82.4	612.51	337.8784								



# Interpretation of Regression Coefficients

- $\hat{Y} = 186.18 + 1.841 X$   
(125.404) (1.215)
- The intercept coefficient in the first row is 186.18 indicate when X takes value zero, what is the price of Gold Price.
- The slope coefficient (parameter estimate) is 1.84.
- So, for every unit increase in CPI (X), a 1.84 unit increase in Gold Price (Y) is predicted.
- The second row represent the standard errors (se) associated with the intercept and slope coefficients.
- The standard error is used for testing whether the parameter is significantly different from 0 by dividing the parameter estimate by the standard error to obtain a **t** value ( will learn later).

## THE NORMALITY ASSUMPTION: Classical Normal Linear Regression Model (CNLRM)

- Classical theory of statistical inference consists of two branches, namely, **estimation and hypothesis testing**.
- So far, we covered estimation of the parameters of the (two-variable) linear regression model. Using the method of OLS we were able to estimate the parameters  $\beta_1$ ,  $\beta_2$ , and  $\sigma^2$ .
- Under the assumptions of the CLRM, we show that the estimators of these parameters,  $\hat{\beta}_1$ ,  $\hat{\beta}_2$  and  $\hat{\sigma}^2$ , satisfy several desirable statistical properties, such as unbiasedness, minimum variance, etc (**the BLUE property**)
- Since these are estimators, their values will change from sample to sample. Therefore, these estimators are random variables.

## Classical Normal Linear Regression Model

- But estimation is half the battle. Hypothesis testing is the other half.
- Our objective is not only to estimate the SRF, but also to use it to draw inferences about the PRF.
- Thus, we would like to find out how close  $\hat{\beta}_1$  is to the true  $\beta_1$  or how close  $\hat{\sigma}^2$  is to the true  $\sigma^2$ .
- Therefore, since  $\hat{\beta}_1$ ,  $\hat{\beta}_2$ , and  $\hat{\sigma}^2$  are random variables, we need to find out their probability distributions, for without that knowledge we will not be able to relate them to their true values.

## 4.1 The Probability Distribution of Disturbances $u_i$

- Since the method of OLS does not make any assumption about the probabilistic nature of  $u_i$ , it is of little help for the purpose of drawing inferences about the PRF from the SRF, the Gauss–Markov theorem notwithstanding.
- This void can be filled if we are willing to assume that the  $u$ 's follow some probability distribution. In the regression context it is usually assumed that the  $u$ 's follow the normal distribution.
- Adding the normality assumption for  $u_i$  to the assumptions of the classical linear regression model (CLRM) discussed earlier, we obtain what is known as the **classical normal linear regression model (CNLRM)**.

## 4-2. The normality assumption

- CNLR assumes that each  $u_i$  is distributed normally  $u_i \sim N(0, \sigma^2)$  with:
  - Mean  $= E(u_i) = 0$  (Assn. 3)--- (4.2.1)
  - Variance  $= E(u_i^2) = \sigma^2$  (Assn. 4)----- (4.2.2)
  - Cov( $u_i, u_j$ ) =  $E(u_i, u_j) = 0$  ( $i \neq j$ ) (Assn. 5) (4.2.3)
- The assumptions given above can be more compactly stated as  $u_i \sim N(0, \sigma^2)$  ----- (4.2.4)
- Note: For two normally distributed variables, the zero covariance or correlation means independence of them, so  $u_i$  and  $u_j$  are not only uncorrelated but also independently distributed. Therefore
- $u_i \sim \mathbf{NID}(0, \sigma^2)$  ----- (4.2.5)
- *is Normal and Independently Distributed*

## 4-2. Why the normality assumption?

1. With a few exceptions, the distribution of sum of a large number of independent and identically distributed random variables tends to a normal distribution as the number of such variables increases indefinitely. It is the **Central Limit Theorem (CLT)** that provides a theoretical justification for the assumption of normality of  $u_i$ .
2. A variant of the **CLT** states that, if the number of variables is not very large or they are not strictly independent, their sum may still be normally distributed
3. Under the normality assumption for  $u_i$ , the OLS estimators  $\beta^{\wedge}_1$  and  $\beta^{\wedge}_2$  are also normally distributed.

## 4-2. Why the normality assumption?

- (4) The normal distribution is a comparatively simple distribution involving only two parameters (mean and variance)
- (5) If we are dealing with a small, or finite, sample size, (less than 100 observations), the normality assumption assumes a critical role. It not only helps us to derive the exact probability distributions of OLS estimators but also enables us to use the  $t$ ,  $F$ , and  $\chi^2$  statistical tests for regression models.
- (6) Finally, in *large samples*,  $t$  and  $F$  statistics have *approximately the  $t$  and  $F$  probability distributions* so that the  $t$  and  $F$  tests that are based on the assumption that the error term is normally distributed can still be applied validly.

## 4-3. Properties of OLS estimators under the normality assumption

- **With the normality assumption the OLS estimators  $\beta^{\wedge}_1$ ,  $\beta^{\wedge}_2$  and  $\sigma^{\wedge 2}$  have the following properties:**
  1. They are unbiased
  2. They have minimum variance. Combined 1 and 2, they are efficient estimators.
  3. Consistency, that is, as the sample size increases indefinitely, the estimators converge to their true population values.

### 4-3. Properties of OLS estimators under the normality assumption

4.  $\beta_1$  (being a linear function of  $u_i$ ) is normally distributed with

$$\text{Mean : } E(\hat{\beta}_1) = \beta_1 \text{ ..... (4.3.1)}$$

$$\text{var}(\hat{\beta}_1) = \sigma^2 \sum x_i^2 / n \sum x_i^2 \text{ ..... (3.3.3) 4.3.2)}$$

Or more compactly, it can be written as

$$\hat{\beta}_1 \sim N(\beta_1, \sigma_{\hat{\beta}_1}^2); \text{ ..... (4.3.3)}$$

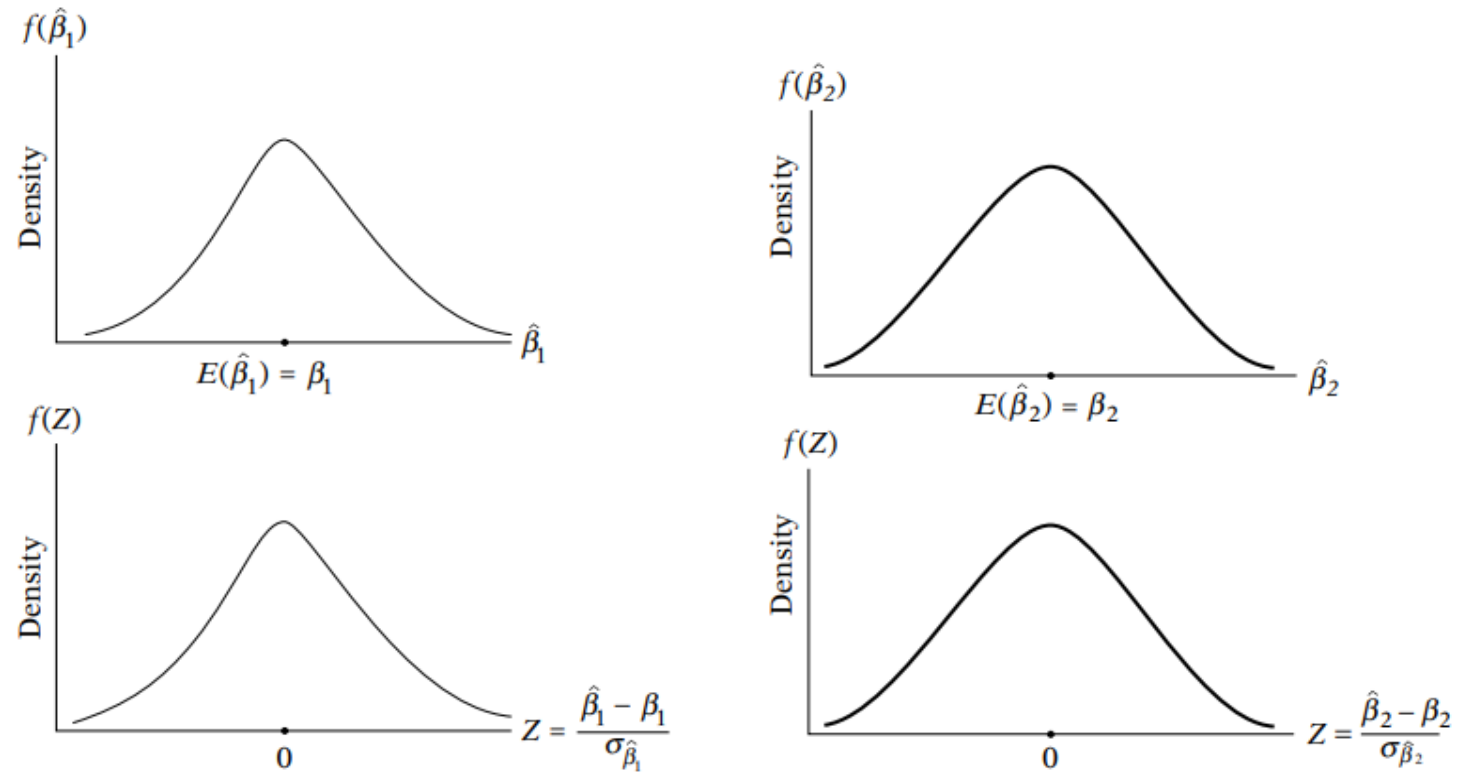
Then by the properties of the normal distribution, the variable  $Z$ , which is defined as

$$E(\hat{\beta}_2) = \beta_2 \text{ ..... (4.3.4)}$$

$$\text{var}(\hat{\beta}_2) = \sigma^2 / \sum x_i^2 \text{ ..... (3.3.1) (4.3.5)}$$

$$Z = (\hat{\beta}_1 - \beta_1) / \sigma_{\hat{\beta}_1} \text{ is } \sim N(0,1) \text{ ..... (4.3.6)}$$

# Fig 4.1 Probability distribution of Betas



**FIGURE 4.1** Probability distributions of  $\hat{\beta}_1$  and  $\hat{\beta}_2$ .

Source: Basic Econometrics by Damodar Gujarati, Page 111.

### 4-3. Properties of OLS estimators under the normality assumption

5.  $\hat{\beta}_2$  is normally distributed  $\sim N(\beta_2, \sigma^2_{\hat{\beta}_2})$

And  $Z = (\hat{\beta}_2 - \beta_2) / \sigma_{\hat{\beta}_2}$  is  $\sim N(0,1)$

6.  $(n-2) \sigma^2 / \sigma^2$  is distributed as the

$$\chi^2_{(n-2)}$$

7.  $\hat{\beta}_1$  and  $\hat{\beta}_2$  are distributed independently of  $\sigma^2$ . They have minimum variance in the entire class of unbiased estimators, whether linear or not. They are best unbiased estimators (BUE)

8. Let  $u_i$  is  $\sim N(0, \sigma^2)$  then  $Y_i$  is  $\sim$

$$N[E(Y_i); \text{Var}(Y_i)] = N[\beta_1 + \beta_2 X_i; \sigma^2]$$

To sum up:

- The important point to note is that the normality assumption enables us to derive the probability, or sampling, distributions of  $\hat{\beta}_1$  and  $\hat{\beta}_2$  (both normal) and  $\hat{\sigma}^2$  (related to the chi square).
- This simplifies the task of establishing confidence intervals and testing (statistical) hypotheses.
- Note that, with the assumption that  $u_i \sim N(0, \sigma^2)$ ,  $Y_i$ , being a linear function of  $u_i$ , is itself normally distributed with the mean and variance given by
  - $E(Y_i) = \beta_1 + \beta_2 X_i$  -----(4.3.7)
  - $\text{var}(Y_i) = \sigma^2$  ----- (4.3.8)
- More neatly, we can write
- $Y_i \sim N(\beta_1 + \beta_2 X_i, \sigma^2)$  -----(4.3.9)

## 4-4. The method of Maximum likelihood (ML)

- ML is point estimation method with some stronger theoretical properties than OLS (Appendix 4.A in the Basic Econometrics book)
- The estimators of coefficients  $\beta$ 's by OLS and ML are identical. They are true estimators of the  $\beta$ 's
- (ML estimator of  $\sigma^2$ ) =  $\sum u_i^2/n$  (is biased estimator)
- (OLS estimator of  $\sigma^2$ ) =  $\sum u_i^2/n-2$  (is unbiased estimator)
- When sample size (n) gets larger the two estimators tend to be equal
- **Probability distributions related to the Normal Distribution: The t,  $\chi^2$ , and F distributions** (details in Appendix A in the Basic Econometrics book)

## Summary and Conclusions

1. Classical *normal linear regression model (CNLRM)*.
2. This model differs from CLRM in that it specifically assumes that the disturbance term *ui entering the regression model is normally distributed*. The CLRM does not require any assumption about the probability distribution of *ui* ; *it only requires that the mean value of ui is zero and its variance is a finite constant*.
3. The theoretical justification for the normality assumption is the **central limit theorem**.
4. Without the normality assumption, under the other assumptions discussed in Chapter 2, the Gauss–Markov theorem showed that the OLS estimators are BLUE.

# Summary and Conclusions

- 5. With the additional assn. of normality, the OLS estimators are not only best unbiased estimators (BUE) but also follow well-known probability distributions. The OLS estimators of the intercept and slope are themselves normally distributed and the OLS estimator of the variance of  $ui$  ( $= \hat{\sigma}^2$ ) is related to the chi-square distribution.
- 6. This knowledge is important and useful in drawing inferences about the values of the population parameters.
- 7. An alternative to the least-squares method is the method of **ML**. To use this method, however, one must make an assumption about the probability distribution of the disturbance term  $ui$ . *In the regression context, the assumption most popularly made is that  $ui$  follows the normal distribution.*

# Summary and Conclusions

- 8. Under the normality assn., the ML and OLS estimators of the intercept and slope parameters are identical. However, the OLS and ML estimators of the variance of  $u_i$  are different. In large samples these two estimators converge.
- 9. Thus the ML method is generally called a large-sample method. The ML method is of broader application, that regression models that are nonlinear in the parameters.
- 10. Largely rely on the OLS method for reasons: (a) Compared to ML, the OLS is easy to apply; (b) the ML and OLS estimators of  $\beta_1$  and  $\beta_2$  are identical; and (c) even in moderately large samples the OLS and ML estimators of  $\sigma^2$  do not differ vastly.

# Reference

- Chapter 4: Classical Normal Linear Regression Model (CNLRM), **in Basic Econometrics Book by Domodar Gujarati**

# What next?

- Normality Assumption
- Testing of hypothesis
- Prior to that recollecting the basic statistics for understanding Testing of hypothesis
- Central Limit Theorem