

Management Accountancy

Unit 6

Cost Volume Profit Analysis Under Condition of Constraints

Lecturer: Dr. Jeetendra Dangol

Contents

- CVP analysis under condition of resource constraints
- Allocation of single resource constraint
- Allocation of multiple resource constraints using Linear Programming Model

Learning Objectives

- Apply CVP analysis under condition of resource constraints
- Apply Linear Programming Model for an allocation of multiple resources constraints

CVP Analysis Under Condition of Resources Constraints

- Limiting factor plays an important role in decision making
- The resources which are being used by a firm are scarce in nature and they put a limit on production and profitability of an enterprise
- On account of their limiting nature, they should be carefully allocated
- The correct allocation of limit resources is a difficult task
- If the resources are in excess, there is no problem of allocation
 - But such cases are very rare

CVP Analysis Under Condition of Resources Constraints

- One of the main objectives of a business enterprise is
 - to maximize the profit or minimization of cost
- Maximization of profit is possible only if all resources have been used to manufacture and sell the maximum quantities that yield the highest individual marginal contribution
- The decision is made for the best allocation of resources among the various activities

CVP Analysis Under Condition of Resources Constraints

- The resources which are available in limited quantity is known as constraints or critical factor
- The restriction imposed on utilization of machine hours, availability of raw material, availability of man power, availability of funds etc. are examples of constraints
- The constraint may be single or multiple
 - If there is single constraint, the optimum production policy can be easily established
 - However, in case of more than one constraint, the problem becomes complicated and for making correct decision linear programming technique is to be used

Single Resource Constraints

- When production mix is insignificant

Step 1	Identify the critical factor unit of each product
Step 2	Find out contribution margin per unit (CMPU) for each product
Step 3	Find out contribution margin per critical factor (CMPCF) of each product $\text{CMPCF} = \frac{\text{CMPU}}{\text{Critical Factor per unit}}$
Step 4	Decision: Highest CMPCF should be consider

Single Resource Constraints

- When production mix is significant

Step 1	Determine the product mix (PM)
Step 2	Find out the total Critical Factor mix on the basis of product mix Total Critical Factor mix (CF) = (PM1 X CF1) : (PM2 X CF2).....
Step 3	Ascertain the total production units Total Production Units = $\frac{\text{Critical Factor Available X Total Product Mix}}{\text{Total CF Mix}}$
Step 4	Determine the outputs units of each product Output units of each product = Product Mix Ratio X Total Production Units
Step 5	Find out Contribution Margin of Product Mix Total Contribution Margin = Production Units X Critical Factor per unit X Contribution Margin Per Unit
Step 6	Decision: Total highest contribution margin sales mix should be preferred

Single Resource Constraints

QUESTION 1

A firm produces two products P_1 and P_2 , processing by a single machine. The total machine hours available for the products are 48,000 hours. Data on the two products are :

	Product P_1	Product P_2
Unit Selling Price	Rs. 20	Rs. 30
Unit Variable Cost	Rs. 8	Rs. 14
Unit Machine Hours	2 hours	4 hours

Required: Allocation of machine hours between the products, when product mix is insignificant

Single Resource Constraints

SOLUTION

Calculation of Contribution margin per unit :

	Product P₁	Product P₂
Selling Price per unit	Rs. 20	Rs. 30
Variable Cost per unit	<u>Rs. 8</u>	<u>Rs. 14</u>
Contribution Margin	Rs. 12	Rs. 16

The critical factor or constraint in this problem is machine hours.

$$\text{Contribution Margin per unit of Critical Factor} = \frac{\text{CMPU}}{\text{Critical Factor per unit}}$$
$$P_1 = \frac{12}{2} = \text{Rs. 6 / Machine Hour}$$
$$P_2 = \frac{16}{4} = \text{Rs. 4 / Machine Hour}$$

Since, contribution margin per machine hour of P₁ is greater than P₂, the firm should produce maximum units of product P₁.

Single Resource Constraints

QUESTION 2

A firm produces two products A and B. Both products require a single type of raw material. The firm has an availability of 14,000 units of raw materials. Data on the two products are as follows :

Products	Raw Material Consumption	Unit Contribution Margin
A	4 units per Product	Rs. 16
B	2 units per Product	Rs. 10

Required:

- Allocation of materials between the products, when product mix is insignificant.
- Allocation where product mix of at least one is to five will be necessary.

Single Resource Constraints

SOLUTION

(a) Allocation of materials between the products, when product mix is insignificant

Calculation of CM/PU of Critical Factor - Raw Material :

$$\text{Product A} = \frac{16}{4} = \text{Rs. } 4$$

$$\text{Product B} = \frac{10}{2} = \text{Rs. } 5$$

Since, product B has higher contribution margin per unit of critical factor and the company if does not have to follow product mix, it should produce maximum unit of product B to maximize its profit.

Single Resource Constraints

SOLUTION

(b) Allocation where product mix of at least one is to five will be necessary

Consumption of Raw Material per mix of Product (A:B = 1:5) :

$$A = 1 \times 4 = 4 \text{ units}$$

$$B = 5 \times 2 = 10 \text{ units}$$

$$\text{Total raw material consumption per mix} = 4 + 10 = 14 \text{ units}$$

$$\begin{aligned} \text{Total Production units} &= \text{Critical Factor available} \times \frac{\text{Total Product Mix}}{\text{Total CF Mix}} \\ &= 14,000 \times \frac{6}{14} = 6,000 \text{ units.} \end{aligned}$$

Raw Material Consumption Per Mix :

$$\text{Product A} = \frac{1}{6} \times 6,000 = 1,000 \text{ units} \times 4 \text{ units} = 4,000 \text{ units}$$

$$B = \frac{5}{6} \times 6,000 = 5,000 \text{ units} \times 2 \text{ units} = 10,000 \text{ units}$$

Contribution Margin of Product Mix :

$$A = 4,000 \text{ units} \times \text{Rs. 4 per unit} = \text{Rs. 16,000}$$

$$B = 10,000 \text{ units} \times \text{Rs. 5 per unit} = \text{Rs. 50,000}$$

Conclusion:

If product mix of at least one is to five is necessary, raw materials should allocate 4,000 units to product A and 10,000 units to product B for earning maximum contribution margin amounting Rs. 66,000 (= Rs. 16,000 + Rs. 50,000).

Multiple Resources Constraints

- Resources management under certain constraints (limitations)
- Objectives – Cost minimization or Profit maximization
- Linear Programming Problem
- Linear programming is a mathematical technique
- Linear: Constant ratio
 - The relationship are represented by straight line, i.e., $y = a + bx$
- Programming: It means taking decisions systematically
- Linear Programming: It is a systematic decision making process under given constraints on the assumption that any function is happened to be linear
- Linear Programming Problem: It is concerned with the problem of allocating limited resources among competing activities in an optimal manner.
 - By using this technique, the optimum benefit can be derived from the utilization of limited resources, maximization of contribution margin or minimization of cost subject to constraint will make possible with the help of linear programming

Multiple Resources Constraints

- Linear programming is a technique for choosing the best alternative from a set of feasible alternatives
- The objective function as well as the constraints can be expressed as linear mathematical function under linear programming technique

Multiple Resources Constraints

In order to apply linear programming there are certain requirements to be fulfilled which are as follows:

- There should be an objective which should be clearly identifiable and measurable in quantitative term. Maximization of profit, minimization of cost are some of the examples of objective function. The objective function is related with the variable which are dealt in problem.
- The activities should be distinctly identifiable and measurable in quantitative terms.
- The resources which are to be allocated must be of constraint nature and must be clearly identified and quantified.
- The linear relationship should be existed between the objective function and activities.
- There should be a series of feasible alternative course of available to the decision maker that are determined by resource constraints.

Multiple Resources Constraints

- **Methods of Linear Programming Problem**

There are two methods of solving linear programming problem

(i) Graphical method used for only two variables

(ii) Simplex method used for two or more than two variables

Linear Programming Problem

The following steps are required for solving linear programming problems

Step 1: Formulation of LPP

Identify the Problem

Write the Objective Function

Write the Constraints Functions

Step 2: Draw a Graph which Represents all the Constraints

Step 3: Feasible Solutions

Step 4: Optimum Solution

Linear Programming Problem

Step 1: Formulation of LPP

Identify the Problem

- At first, we have to formulate the LPP
- Formulation of LPP is become define the decision variables representing product mix
- Very beginning of formulation of the LPP, first requirement is to identify the goal in terms of objective function, for example, maximization of contribution, maximization of profit, minimization of cost etc.

Linear Programming Problem

Step 1: Formulation of LPP

Write the Objective Function

- It is also known as the criterion function
- It helps us to get actual product mix which makes maximum profit or incurs minimum cost
- An algebraic expression of the firm's goal is known as the objective function, e.g., suppose the goal is to maximize contribution
- Maximization of contribution would be obtained by producing two products x and y respectively
- The contribution margin per unit of x and y are Rs. 4 and Rs. 2 respectively.
- In that case, if z is the total contribution margin, algebraic expression of goal will be :

$$z = 4x + 2y$$

Linear Programming Problem

Step 1: Formulation of LPP

Write the Constraint Function

- Constraints are limitations or restrictions imposed upon the decision variables/available resources.
- Algebraic expression of the limitations on resources faced by the firm has to be determined
- Normally, constraints function will be always inequalities form, e.g., \leq or \geq or $=$

Linear Programming Problem

Step 1: Formulation of LPP

Write the Constraint Function

- The lesser than or equal to (\leq) will involve when the resource is related to maximum available, for example:
- X : Variable representing use of raw materials
- Limitation : Maximum raw materials available 10 units per day
- Possible solutions
- $X = 0, X = 1, X = 2, X = 3, \dots, X = 9, X = 10$ (Possible solutions = 11)
- $X \leq 10$

Linear Programming Problem

Step 1: Formulation of LPP

Write the Constraint Function

- The greater than or equal to (\geq) will involve when the resource is related to minimum requirement, for example:
- Y : Variable representing use of Direct Labour Hours per day
- Limitation : Minimum 100 DLHs requirement per day
- $Y \geq 100$

Linear Programming Problem

Step 1: Formulation of LPP

Write the Constraint Function

- In the same way, equal to ($=$) will involve when resources need exactly equal to, for example:
- Z : Production units
- Limitation : Production units exactly equal to 200 units
- $Z = 200$

Linear Programming Problem

Step 1: Formulation of LPP

Write the Constraint Function

- For instance, suppose the first product x requires 2 kg. of raw material and second product y requires 5 kg. of raw material per unit output. The total raw material available is 300 kg.
- x = Production units of x
- Y = Production units of y
- The constraint can be expressed as: $2x + 5y \leq 300$

Linear Programming Problem

Step 2: Draw a Graph which Represents all the Constraints

- The graph is drawn after the expression of all constraints.
- The graph drawn should represent all the constraints.
- The feasible region is also identified in this step.
- The area where all points lie within space simultaneously satisfying all the constraints is known as feasible region.
 - For greater than or equal to constraints the feasible region will be the area above the constraint lines.
 - On the contrary, it will be below constraint lines for less than or equal to constraints.

Linear Programming Problem

Step 3: Feasible Solutions

- All the possible solutions of LPP satisfying the given constraints are called feasible solutions
- So, obtain the point on feasible region that optimizes the objective function

Linear Programming Problem

Step 4: Optimum Solution

- The best solution out of the feasible solutions which makes maximum profit or incurs minimum cost is called optimum solution

Multiple Resources Constraints

QUESTION 3 (Maximization case)

A firm produces two products, A and B. The following information regarding the materials and labours required for the products are given :

Particulars	Product A	Product B	Total Available
Raw Material	2 kg	3 kg	60 kg
Labour Hours	4 DLH	3 DLH	96 DLH
Contribution Margin	Rs. 40	Rs. 35	—

Required:

Graphical optimal solution for maximize contribution

Utilization of each constraint at the optimal activity level

Multiple Resources Constraints

SOLUTION

Step 1: Formulation of LPP

Let x and y be the number of units of outputs for the products A and B respectively.

Objective function : maximize contribution:

$$Z = 40x + 35y$$

Subject to constraints:

Raw material constraint : $2x + 3y \leq 60$

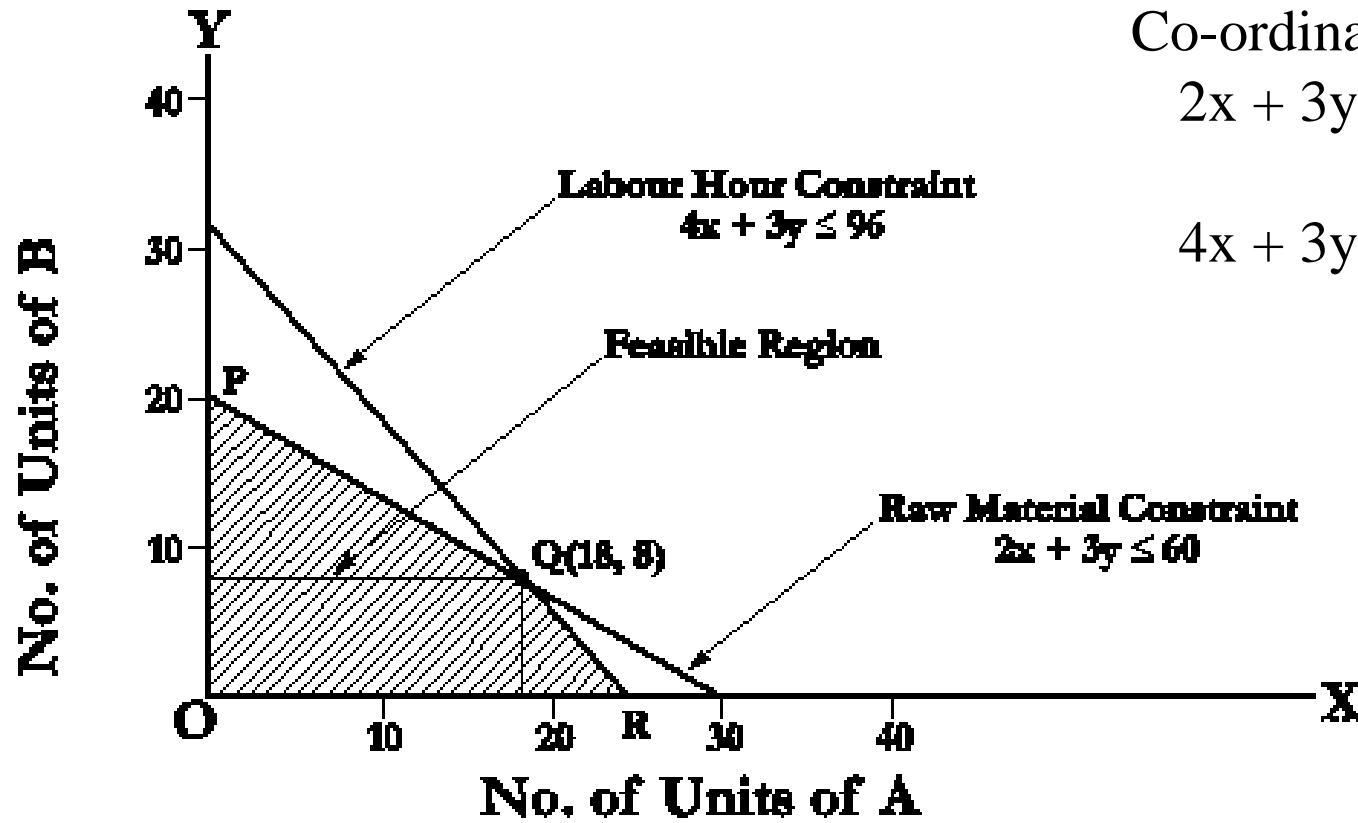
Labour hour constraint : $4x + 3y \leq 96$

Non-negative constraint : $x, y \geq 0$

Multiple Resources Constraints

SOLUTION

Step 2: Draw a Graph which Represents all the Constraints



Co-ordinate of equation for plotting graph:

$$2x + 3y = 60; \text{ If } x = 0, y = 20 [0,20]$$

$$\text{If } y = 0, x = 30 [30,0]$$

$$4x + 3y = 96; \text{ If } x = 0, y = 32 [0,32]$$

$$\text{If } y = 0, x = 24 [24,0]$$

Multiple Resources Constraints

SOLUTION

Step 3: Feasible Solutions

Point O (0,0), P (0,20), Q (18,8)
and R (24,0) of feasible region

Working note for determination of coordinate point Q:

$$2x + 3y = 60 \dots\dots (i)$$

$$4x + 3y = 96 \dots\dots (ii)$$

By subtracting (i) from (ii), we get

$$2x = 36$$

$$\therefore x = 18$$

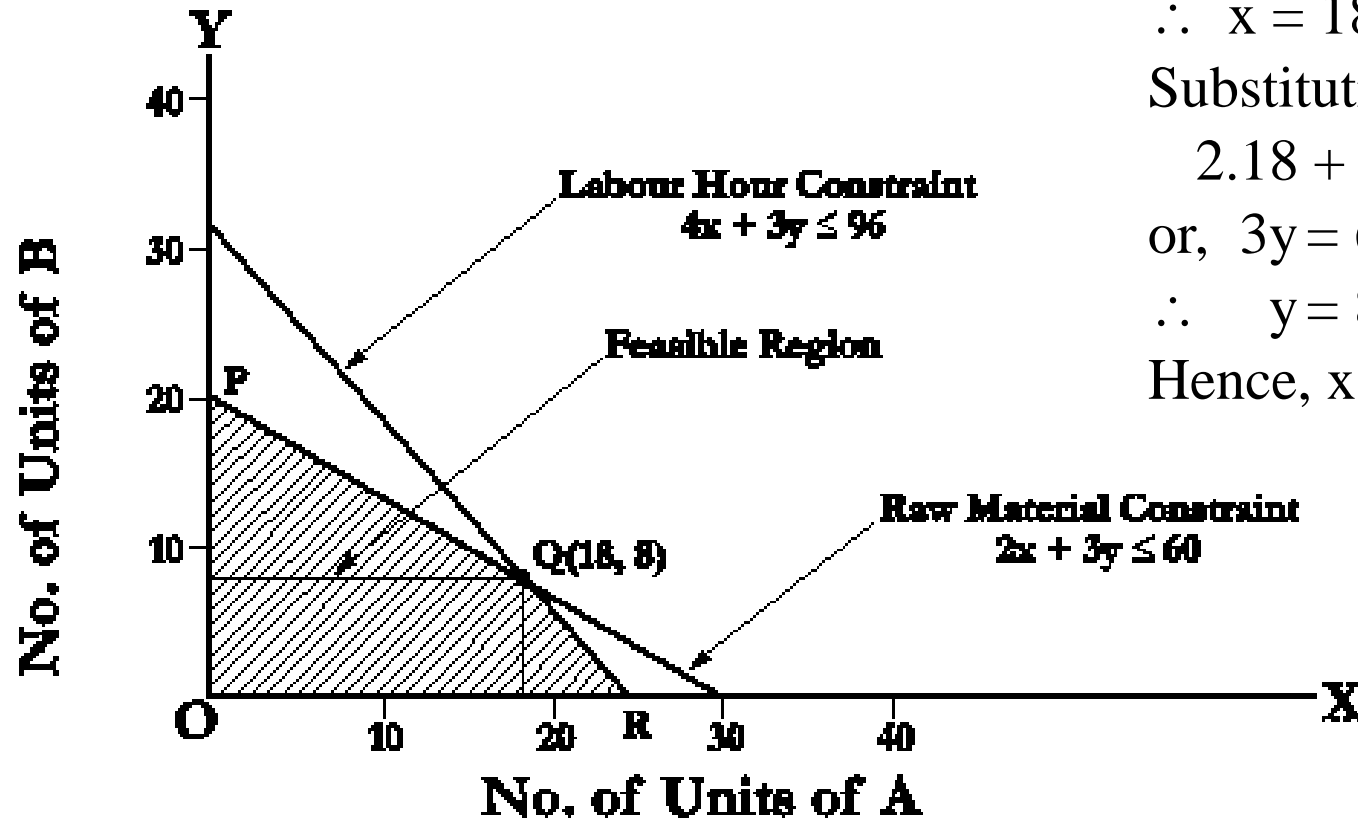
Substituting the value of x in (i), we have,

$$2 \cdot 18 + 3y = 60$$

$$\text{or, } 3y = 60 - 36$$

$$\therefore y = 8$$

Hence, $x = 18, y = 8$.



Multiple Resources Constraints

SOLUTION

Step 4: Optimum Solution

Point O, P, Q and R of feasible region would give the optimal solution for contribution margin maximization

Feasible Points	Coordinate x and y	$Z = 40x + 35y$
Point O	(0,0)	$(40 \times 0) + (35 \times 0) = 0$
Point P	(0,20)	$(40 \times 0) + (35 \times 20) = 700$
Point Q	(18,8)	$(40 \times 18) + (35 \times 8) = 1,000$ (Maximum)
Point R	(24,0)	$(40 \times 24) + (35 \times 0) = 960$

Optimal solution is 18 units of A and 8 units of B with maximum contribution margin amounting Rs. 1,000.

Multiple Resources Constraints

Verification regarding utilization of each constraint at the optimal activity level :

Raw Material Constraint :

$$2x + 3y = 60$$

or, $(2 \times 18) + (3 \times 8) = 60$ (Raw material fully used)

Labour Hour Constraint :

$$4x + 3y = 96$$

or, $(4 \times 18) + (3 \times 8) = 96$ (Labour hour fully utilized)

Notes: Verification is not compulsory.

Multiple Resources Constraints

QUESTION 4 (Minimization case)

Mixture x and y are the two sources for obtaining chemicals A and B. Nepal Agricultural Research Institute suggested to a firm to spread out at least 2,100 kg. of A and not less than 3,000 kg. of B to raise its agricultural products. Mixture x and y are available in bags weighing 100 kg. each and they cost Rs. 100 and Rs. 60 respectively. Mixture x contains A and B equivalent to 10 kg and 40 kg respectively, while mixture y contains the ingredients to equivalent of 25 kg each.

Required:

- (i) Formulate the problem in a linear programming format
- (ii) Graphical optimal solution for buying bags to minimize the cost.

Multiple Resources Constraints

SOLUTION

Step 1: Formulation of LPP

Given :

Mixture	A	B	Cost
x	10 kg	40 kg	Rs. 100
y	25 kg	25 kg	Rs. 60
Requirement	2,100 kg	3,000 kg	

Objective function : Minimize cost, $C = 100x + 60y$

Subject to constraints :

Requirement of A, $10x + 25y \geq 2,100$

Requirement of B, $40x + 25y \geq 3,000$

Non-negative Constraint, $x, y \geq 0$

Multiple Resources Constraints

SOLUTION

Step 2: Draw a Graph which Represents all the Constraints

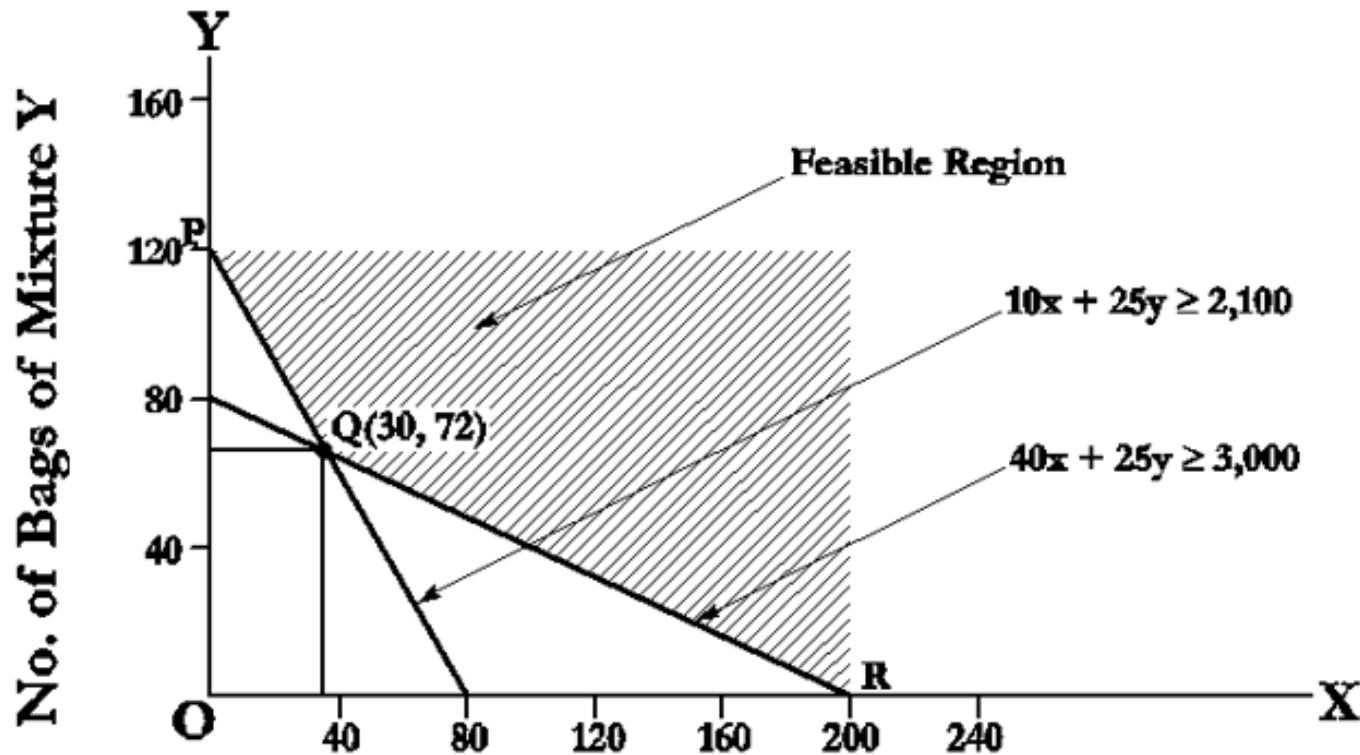
Co-ordinate of equation for plotting graph :

$$10x + 25y = 2100; \text{ If } x = 0, y = 84 [0,84]$$

$$\text{If } y = 0, x = 21 [210,0]$$

$$40x + 25y = 3,000; \text{ If } x = 0, y = 120 [0,120]$$

$$\text{If } y = 0, x = 75 [75,0]$$



Multiple Resources Constraints

SOLUTION

Step 3: Feasible Solutions

Point P, Q and R of feasible solutions

Working note for determination of coordinate point Q:

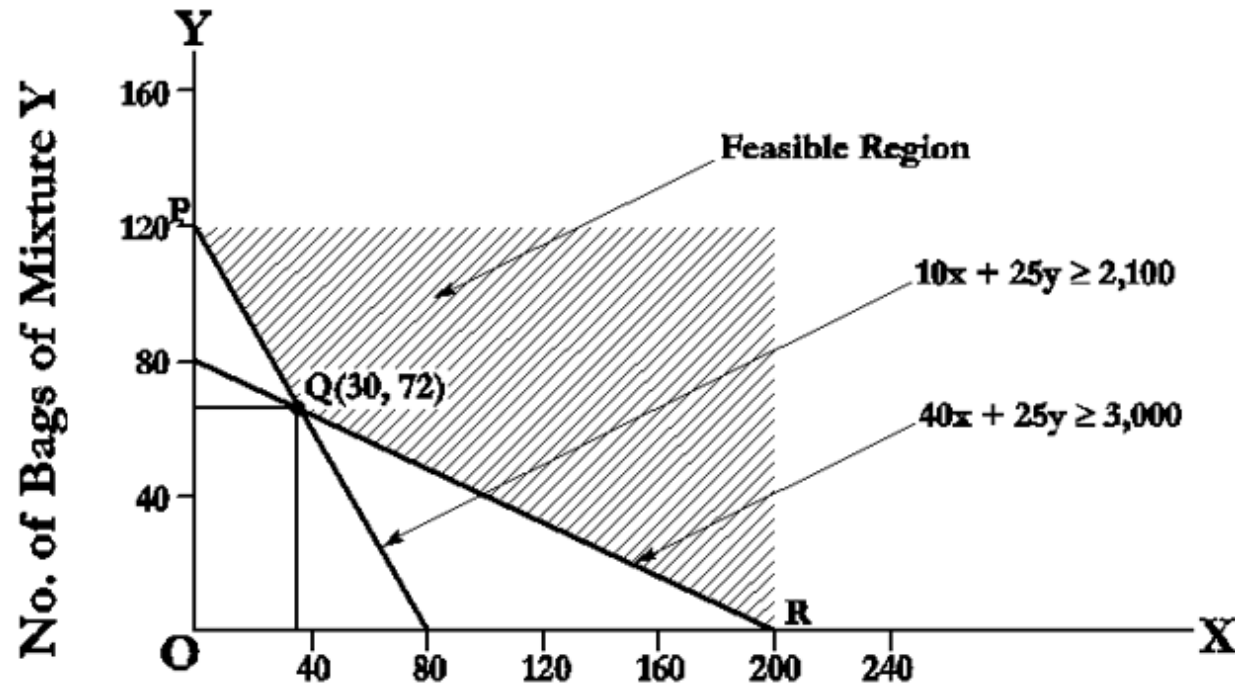
$$10x + 25y = 2,100 \dots\dots (i)$$

$$40x + 25y = 3,000 \dots\dots (ii)$$

$$\therefore x = 30$$

$$\therefore y = 72$$

Hence, $x = 30$, $y = 72$, Q (30,72)



Multiple Resources Constraints

SOLUTION

Step 4: Optimum Solution

Point P, Q and R of feasible region would give the optimal solution for minimize costs
: $C = 100x + 60y$

Feasible Points	Coordinate x and y	$C = 100x + 60y$
Point P	(0,120)	$(100 \times 0) + (60 \times 120) = 7,200$ (Minimum)
Point Q	(30,72)	$(100 \times 30) + (60 \times 72) = 7,320$
Point R	(200,0)	$(100 \times 200) + (60 \times 0) = 20,000$

Optimal solution

The minimum cost is at point 'P' with amounting Rs. 7,200. Hence the optimal point is 0,120, i.e., none of mixture x should be purchased and y should be purchased 120 bags.

References

- Atkinson, A. A., Kaplan, R. S., Matsumura, E. M., Young, S. M., & Kumar, G. A. (2104). *Management accounting: Information for decision-making and strategy execution*. Pearson Education
- Colin, D. (1995). *Management and cost accounting*. ELBS, London
- Dangol, R. M. & Dangol, J. (2019). *Management accountancy*. Taleju Prakashan, Kathmandu
- Hilton. R. W. & Platt, D. E. (2014). *Managerial accounting: Creating value in a global business environment*. McGraw-Hill Education
- Horngren, C. T., Datar, G. F., Rajan, M. V., & Lttner, C. (2011). *Cost accounting: A managerial emphasis*. Pearson Education

Thank You