Management Accountancy

Unit 6
Cost Volume Profit (CVP) Analysis under Condition of Constraints

Structure
- CVP analysis under condition of resource constraints
- Allocation of single resource constraint
- Allocation of multiple resource constraints using Linear Programming Model

After the completion of this unit, you should be able to:
- Apply CVP analysis under condition of resource constraints
- Apply Linear Programming Model for an allocation of multiple resources constraints

6.1 Introduction

Limiting factor plays an important role in decision making. The resources which are being used by a firm are scarce in nature and they put a limit on production and profitability of an enterprise. On account of their limiting nature, they should be carefully allocated. The correct allocation of limit resources is a difficult task. If the resources are in excess, there is no problem of allocation. But such cases are very rare.

One of the main objectives of a business enterprise is to maximize the profit or minimization of cost. Maximization of profit is possible only if all resources have been used to manufacture and sell the maximum quantities that yield the highest individual marginal contribution. Therefore, the decision is made for the best allocation of resources among the various activities. The resources which are available in limited quantity is known as constraints or critical factor. So, the restriction imposed on utilization of machine hours, availability of raw material, availability of man power, availability of funds etc. are examples of constraints.

The constraint may be single or multiple. If there is single constraint, the optimum production policy can be easily established. However, in case of more than one constraint, the problem becomes complicated and for making correct decision linear programming technique is to be used.

The illustrative example of single constraint is given first. Thenafter the linear programming technique is discussed.

When production mix is insignificant

Step 1	Identify the critical factor unit of each product
Step 2	Find out contribution margin per unit (CMPU) for each product
Step 3	Find out contribution margin per critical factor (CMPCF) of each product CMPCF = $\frac{\text{CMPU}}{\text{Critical Factor per unit}}$
Step 4	Decision: Highest CMPCF should be consider

When production mix is significant

Step 1	Determine the product mix (PM)
Step 2	Find out the total Critical Factor mix on the basis of prodcut mix Total Critical Factor mix (CF) = (PM1 X CF1) : (PM2 X CF2)……….
Step 3	Ascertain the total production units Total Production Units = $\frac{\text{Critical Factor Available X Total Product Mix}}{\text{Total CF Mix}}$
Step 4	Determine the outputs units of each product Output units of each product = Product Mix Ratio X Total Production Units
Step 5	Find out Contribution Margin of Product Mix Total Contribution Margin = Production Units X Critical Factor per unit X Contribution Margin Per Unit
Step 6	Decision: Total highest contribution margin sales mix should be preferred

ILLUSTRATION 1

A firm produces two products P_1 and P_2, processing by a single machine. The total machine hours available for the products are 48,000 hours. Data on the two products are :

	Product P_1	Product P_2
Unit Selling Price	Rs. 20	Rs. 30
Unit Variable Cost	Rs. 8	Rs. 14
Unit Machine Hours	2 hours	4 hours

Required: Allocation of machine hours between the products, when product mix is insignificant.

SOLUTION:

Calculation of Contribution margin per unit :

	Product P_1	Product P_2
Selling Price per unit	Rs. 20	Rs. 30
Variable Cost per unit	Rs. 8	Rs. 14
Contribution Margin	Rs. 12	Rs. 16

The critical factor or constraint in this problem is machine hours.

$$\text{Contribution Margin per unit of Critical Factor} = \frac{\text{CMPU}}{\text{Critical Factor per unit}}$$

$$P_1 = \frac{12}{2} = \text{Rs. 6 / Machine Hour}$$

$$P_2 = \frac{16}{4} = \text{Rs. 4/ Machine Hour}$$

Since, contribution margin per machine hour of P_1 is greater than P_2, the firm should produce maximum units of product P_1.

ILLUSTRATION 2

A firm produces two products A and B. Both products require a single type of raw material. The firm has an availability of 14,000 units of raw materials. Data on the two products are as follows :

Products	Raw Material Consumption	Unit Contribution Margin
A	4 units per Product	Rs. 16
B	2 units per Product	Rs. 10

Required: (i) Allocation of materials between the products, when product mix is insignificant.

(ii) Allocation where product mix of at least one is to five will be necessary.

SOLUTION:

(i) Calculation of CMPU of Critical Factor - Raw Material :

Product A = $\frac{16}{4}$ = Rs. 4

Product B = $\frac{10}{2}$ = Rs. 5

Since, product B has higher contribution margin per unit of critical factor and the company if does not have to follow product mix, it should produce maximum of product B to maximise its profit.

(ii) Consumption of Raw Material per mix of Product (A:B = 1:5) :

A = 1 × 4 = 4 units

B = 5 × 2 = 10 units

Total raw material consumption per mix = 4 + 10 = 14 units

Total Production units = Critical Factor available × $\frac{\text{Total Product Mix}}{\text{Total CF Mix}}$

= 14,000 × $\frac{6}{14}$ = 6,000 units.

Raw Material Consumption Per Mix :

Product A = $\frac{1}{6}$ × 6,000 = 1,000 units × 4 units = 4,000 units

B = $\frac{5}{6}$ × 6,000 = 5,000 units × 2 units = 10,000 units

Contribution Margin of Product Mix :

A = 4,000 units × Rs. 4 per unit = Rs. 16,000

B = 10,000 units × Rs. 5 per unit = Rs. 50,000

Conclusion: If product mix of at least one is to five is necessary, raw materials should allocate 4,000 units to product A and 10,000 units to product B for earning maximum contribution margin amounting Rs. 66,000 (= Rs. 16,000 + Rs. 50,000).

6.2 Linear Programming Model

Linear programming is a mathematical technique. It is a systematic decision making process under given constraints on the assumption that any function is happened to be linear. It is concerned with the problem of allocating limited resources among competing activities in an optimal manner. By using this technique, the optimum benefit can be derived from the utilization of limited resources, maximization of contribution margin or minimization of cost subject to constraint will make possible with the help of linear programming.

Linear programming is a technique for choosing the best alternative from a set of feasible alternatives. The objective function as well as the constraints can be expressed as linear mathematical function under linear programming technique.

In order to apply linear programming there are certain requirements to be fulfilled which are as follows:

(i) There should be an objective which should be clearly identifiable and measurable in quantitative term. Maximization of profit, minimization of cost are some of the examples of objective function. The objective function is related with the variable which are dealt in problem.

(ii) The activities should be distinctly identifiable and measurable in quantitative terms.

(iii) The resources which are to be allocated must be of constraint nature and must be clearly identified and quantified.

(iv) The linear relationship should be existed between the objective function and activities.

(v) There should be a series of feasible alternative course of available to the decision maker that are determined by resource constraints.

Methods of Linear Programming Problem

There are two methods of solving linear programming problem :

(i) Graphical method used for only two variables.

(ii) Simplest method used for two or more than two variables.

To use the graphic method for solving linear problems, the following steps are required :

Step 1: Identify the Problem

At first, we have to formulate the linear progamming problem (LPP). Formulation of LPP is become define the decision variables representing product mix. Very beginning of formulation of the LPP, first requirement is to identify the goal in terms of objective function, for example, maximization of contribution, maximization of profit, minimization of cost etc.

Step 2: Write the Objective Function

It is also known as the criterian function. It helps us to get actual product mix which makes maximum profit or incurrs minimum cost. An algebraic expression of the firm's goal is known as the objective function, e.g., suppose the goal is to maximize contribution. Maximization of contribution would be obtained by producing two products x and y respectively. The contribution margin per unit of x and y are Rs. 4 and Rs. 2 respectively. In that case, if z is the total contribution margin, algebraic expression of goal will be :

$$z = 4x + 2y$$

Step 3: Write the Constraints

Constraints are limitations or restrictions imposed upon the decision varaibles/available resources. Algebraic expression of the limitations on resources faced by the firm has to be determined in this step. Normally, constraints function will be always inequalities form, e.g., $\leq$ or $\geq$ or =. The lesser than or equal to ($\leq$) will involve when the resource is related to maximum available. The greater than or equal to ($\geq$) will involve when the resource is related to minimum requirement. In the same way equal to (=) will involve when resources need exactly equal to. For instance, suppose the first product x requires 2 kg. of raw material and second product y requires 5 kg. of raw material per unit output. The total raw material available is 300 kg. The constraint can be expressed as:

$$2x + 5y \leq 300$$

Step 4: Draw a Graph which Represents all the Constraints

The graph is drawn after the expression of all constraints. The graph drawn should represent all the constraints. The feasible region is also identified in this step. The area where all points lie within space simultaneously satisfying all the constraints is known as feasible region. For greater than or equal to constraints the feasible region will be the area above the constraint lines. On the contrary, it will be below constraint lines for less than or equal to constraints.

Step 5: Feasible Solutions

All the possible solutions of LPP satisfying the given constraints are called feasible solutions. So, obtain the point on feasible region that optimizes the objective function.

Step 6: Optimum Solution

The best solution out of the feasible solutions which makes maximum profit or incurs minimum cost is called optimum solution.

ILLUSTRATION 3 (MAXIMIZATION CASE)

A firm produces two products, A and B. The following information regarding the materials and labours required for the products are given :

Particulars	Product A	Product B	Total Available
Raw Material	2 kg	3 kg	60 kg
Labour Hours	4 DLH	3 DLH	96 DLH
Contribution Margin	Rs. 40	Rs. 35	–

Required: (i) Graphical optimal solution for maximize contribution.
(ii) Utilization of each constraint at the optimal activity level.

SOLUTION:

Let x and y be the number of units of outputs for the products A and B respectively.

(i) Objective function : maximize contribution:

$$Z = 40x + 35y$$

(ii) Subject to constraints:

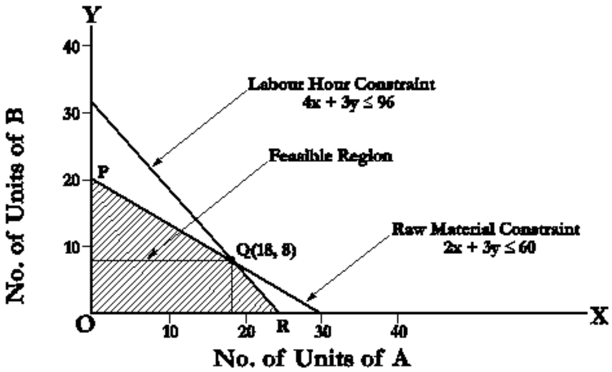
Raw material constraint : $2x + 3y \le 60$

Labour hour constraint : $4x + 3y \le 96$

Non-negative constraint : $x, y \ge 0$

(iii) Co-ordinate of equation for plotting graph:

2x + 3y = 60;	If x = 0, y = 20	[0,20]
	If y = 0, x = 30	[30,0]
4x + 3y = 96;	If x = 0, y = 32	[0,32]
	If y = 0, x = 24	[24,0]



(iv) Point O, P, Q and R of feasible region would give the optimal solution for contribution margin maximization : $Z = 40x + 35y$

Feasible Points	Coordinate x and y	Z = 40x + 35y
Point O	(0,0)	$(40 \times 0) + (35 \times 0) = 0$
Point P	(0,20)	$(40 \times 0) + (35 \times 20) = 700$
Point Q	(18,8)	$(40 \times 18) + (35 \times 8) = 1,000$ (Maximum)
Point R	(24,0)	$(40 \times 24) + (35 \times 0) = 960$

(vi) Optimum Solution:

Optimal solution is 18 units of A and 8 units of B with maximum contribution margin amounting Rs. 1,000.

Verification regarding utilization of each constraint at the optimal activity level :

Raw Material Constraint :

$2x + 3y = 60$

or, $(2 \times 18) + (3 \times 8) = 60$ (Raw material fully used)

Labour Hour Constraint :

$4x + 3y = 96$

or, $(4 \times 18) + (3 \times 8) = 96$ (Labour hour fully utilized)

Notes: Verification is not compulsory.

Woring note for determination of coordinate point Q:

$2x + 3y = 60$ ...... (i)

$4x + 3y = 96$ ..... (ii)

By substracting (i) from (ii), we get

$2x = 36$

$\therefore \quad x = 18$

Substituting the value of x in (i), we have,

$2.18 + 3y = 60$

or, $3y = 60 - 36$

∴ $y = 8$

Hence, $x = 18$, $y = 8$.

ILLUSTRATION 4 (MINIMIZATION CASE)

Mixture x and y are the two sources for obtaining chemicals A and B. Nepal Agricultural Research Institute suggested to a firm to spread out at least 2,100 kg. of A and not less than 3,000 kg. of B to raise its agricultural products. Mixture x and y are available in bags weighing 100 kg. each and they cost Rs. 100 and Rs. 60 respectively. Mixture x contains A and B equivalent to 10kg and 40kg respectively, while mixture y contains the ingredients to equivalent of 25 kg. each.

Required: (i) Formulate the problem in a linear programming format.

(ii) Graphical optimal solution for buying bags to minimize the cost.

SOLUTION:

Given :

Mixture	A	B	Cost
x	10 kg	40 kg	Rs. 100
y	25 kg	25 kg	Rs. 60
Requirement	2,100 kg	3,000 kg	

(i) Linear Programming Model :

Objective function : Minimise cost, $C = 100x + 60y$

Subject to constraints :

Requirement of A, $10x + 25y \geq 2{,}100$

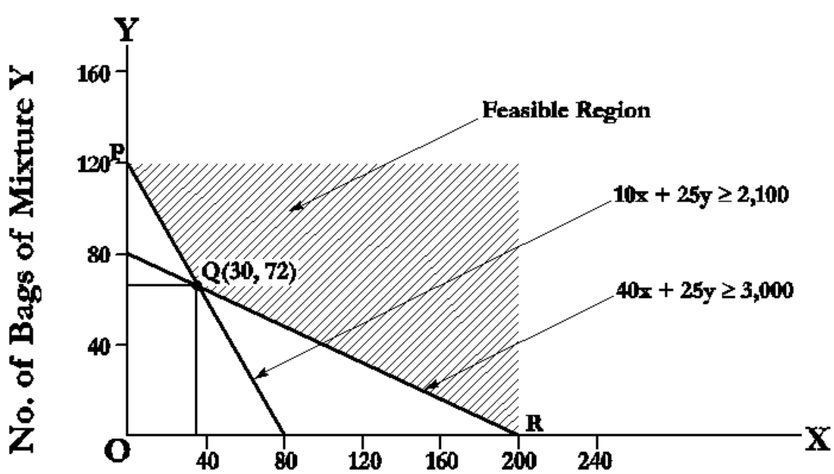
Requirement of B, $40x + 25y \geq 3{,}000$

Non-negative Constraint, $x, y \geq 0$

(ii) Graphical Method :

Co-ordinate of equation for plotting graph :

$10x + 25y = 2100$;	If $x = 0$, $y = 84$	[0,84]
	If $y = 0$, $x = 210$	[210,0]
$40x + 25y = 3{,}000$;	If $x = 0$, $y = 120$	[0,120]
	If $y = 0$, $x = 75$	[75,0]



(iii) Point P, Q and R of feasible region would give the optimal solution for minimize costs : $C = 100x + 60y$

Feasible Points	Coordinate x and y	C = 100x + 60y
Point P	(0,120)	$(100 \times 0) + (60 \times 120) = 7{,}200$ (Minimum)

Point Q	(30,72)	$(100 \times 30) + (60 \times 72) = 7,320$
Point R	(200,0)	$(100 \times 200) + (60 \times 0) = 20,000$

(Iv) Optmum Solution:

The minimum cost is at point 'P' with amounting Rs. 7,200. Hence the optimal point is 0,120, i.e., none of mixture x should be purchased and y should be purchased 120 bags.

ILLUSTRATION 5

A multiple product company's projected income statement for the coming period is as under :

Particulars		Product x	Product y	Total
	Production Units	10,000	10,000	20,000
	Sales Revenue	Rs. 3,00,000	Rs. 4,00,000	Rs. 7,00,000
Less :	Variable Cost:			
	Raw Material @ Rs. 4/unit	80,000	1,60,000	2,40,000
	Labour @ Rs. 4/DLH	1,20,000	80,000	2,00,000
	Total Variable Cost	2,00,000	2,40,000	4,40,000
	Contribution Margin	1,00,000	1,60,000	2,60,000
Less :	Fixed Cost:			
	Joint Cost			50,000
	Departmental	40,000	40,000	80,000
	Total Fixed Cost			1,30,000
	Net Income			1,30,000

Incessant rain in the country has damaged all crops and company expects some short supply of a raw material. Similarly, high demand of skilled man power in Gulf countries will create constraint on labour for the year. The company projects the following position for the year :

Raw Material = 50,000 units.

Labour = 45,000 DLH.

Required : (i) Overall company break-even in units.

(ii) Use of Linear Programming model to maximize profit under constraint.

SOLUTION:

Selling Price per unit :

$$x = \frac{3,00,000}{10,000} = \text{Rs. } 30$$

$$y = \frac{4,00,000}{10,000} = \text{Rs. } 40$$

Variable Cost per unit :

$$x = \frac{2,00,000}{10,000} = \text{Rs. } 20$$

$$y = \frac{2,40,000}{10,000} = \text{Rs. } 24$$

Contribution Margin per unit :

x = 30 - 20 = Rs. 10

y = 40 - 24 = Rs. 16

Sales mix ratio = A:B = 10,000 : 10,000 = 1:1

Total Fixed Cost = Rs. 1,30,000

(i) Overall Break-even Points in units $= \dfrac{\text{Fixed Cost}}{\text{Weighted Average Contribution Margin}} = \dfrac{1,30,000}{\left(10 \times \frac{1}{2}\right) + \left(16 \times \frac{1}{2}\right)}$

$$= \frac{1,30,000}{13} = 10,000 \text{ units.}$$

(ii) Calculation of Material and Labour hour per unit :

	Product x	Product y
Material:		
Total Units	$\frac{80,000}{4} = 20,000$	$\frac{1,60,000}{4} = 40,000$
Material required per Unit	$\frac{20,000}{10,000} = 2$ units	$\frac{40,000}{10,000} = 4$ units
Labour :		
Total DLH	$\frac{1,20,000}{4} = 30,000$	$\frac{80,000}{4} = 20,000$
DLH per Unit	$\frac{30,000}{10,000} = 3$ DLH	$\frac{20,000}{10,000} = 2$ DLH

Thus, we have

Product	Raw Materials	DLH	CMPU
x	2 units	3 Hrs.	Rs. 10
y	4 units	2 Hrs.	Rs. 16
Availability	50,000 units	45,000 DLH	

Linear Programming Format :

Objective Function :

Maximize Contribution Margin, $Z = 10x + 16y$

Subject to Constraints :

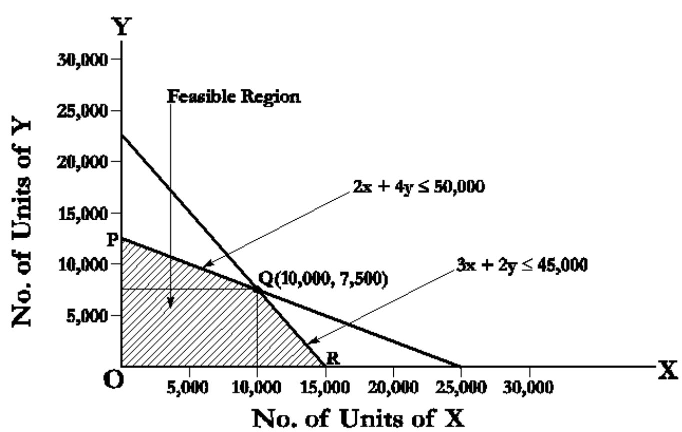
Raw material, $2x + 4y \leq 50,000$

Labour Hours, $3x + 2y \leq 45,000$

Non-negative Constraint, $x, y \geq 0$

Co-ordinate of Equation for Plotting Graph :

$2x + 4y = 50,000$;	If x = 0, y = 12,500	[0; 12,500]
	If y = 0, x = 25,000	[25,000; 0]
$3x + 2y = 45,000$;	If x = 0, y = 22,500	[0,22,500]
	If y = 0, x = 15,000	[15,000; 0]



Point O, P, Q and R of feasible region would give the optimal solution for maximisation of profit.

Feasible Points	Coordinate x and y	Z = 10x + 16y	
Point O	(0,0)	$(10 \times 0) + (16 \times 0)$	= 0
Point P	(0,12,500)	$(10 \times 0) + (16 \times 12,500)$	= 2,00,000

Point Q	(10,000 , 7,500)	$(10 \times 10{,}000) + (16 \times 7{,}500)$	= 2,20,000 (Maximum)
Point R	(15,000, 0)	$(10 \times 15{,}000) + (16 \times 0)$	= 1,50,000

Point Q shows highest contribution margin. Hence, product x and y should be produced 10,000 and 75,000 units respectively for maximization of profit Rs. 90,000 (= Rs. 2,20,000 – Rs. 130,000).

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