Course Title:

Fundamental of Thermodynamics and Heat Transfer

Lecture 6 (Week 6):

Properties of Pure Working Substances

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Learning Objective of Lecture:

To impart a great deal of knowledge to undergraduate students on the topics:

- ✓ Properties of a Two Phase Mixture
- ✓ Quality and Moisture Content
- ✓ Specific Volume, Specific Internal Energy, Specific Enthalpy, and Specific Entropy
- ✓ Other Thermodynamic Properties: Internal Energy, Enthalpy, and Specific Heats
- ✓ Development of Property Data
- ✓ Tabular and Graphical Data Presentation
- ✓ Pressure Enthalpy ($P-h$) Diagram
- ✓ Temperature Entropy ($T-s$) Diagram
- ✓ Enthalpy Entropy ($h-s$) Diagram or Mollier Diagram

3.4. Properties of a Two Phase Mixture

When the specific volume of the liquid phase of water along the saturation line 2 – 4 is measured, the same value v_2 is found at any state and is equal to v_l as the specific volume of the saturated liquid. Similarly, the specific volume of the vapor phase of water at any state along the saturation line 2 – 4 is found to be same v_4 and equal to v_g as the specific volume of the saturated vapor as shown in figure 3.14. This is because of the existence of both liquid and vapor phases at every state along the saturation line 2 – 4 at the same saturation temperature and the same corresponding saturation pressure. As the saturation temperature and the pressure are dependent within two phase mixture or saturation region, the states along the saturation line 2 – 4 cannot be fixed by using only these two dependent properties according to state postulate. Hence, a third property is necessary to complete this task and the third property will be the specific volume. Hence, from the set of properties such as pressure, temperature and specific volume, the only independent property sets that can be used to fix the state of a two phase system are the sets of properties pressure and specific volume or temperature and specific volume [1]. Thus, it would be useful to define the specific volume *v* of the total mass in two phase mixture or saturation region in terms of the specific volume of the individual phases (i.e. saturated liquid and saturated vapor) v_l and v_g. Because the specific properties are not additive in nature, to do this, a new property called quality of the two phase mixture is introduced (see section 3.4.1). To illustrate this fact let us consider three different states *A*, *B* and *C* of water defined by three different temperatures and at the constant pressure of 1 atm as

State *A*: 1 atm, 50^0C
State *B*: 1 atm, 100^0C
State *C*: 1 atm, 150^0C

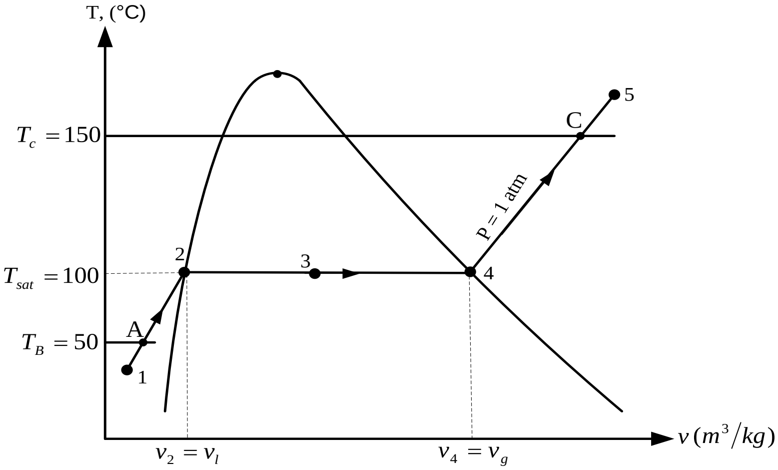

Figure 3.14. Locating states on $T - v$ diagram with reference to saturation line

To fix these above states on $T - v$ diagram, two constant temperature lines for 50°C and 150°C can be drawn for pressure 1 atm, as shown in figure 3.14. Then the state "A" can be fixed as the intersection of 50^0C temperature line and 1 atm pressure line. Similarly, state "C" can be fixed as the intersection of 150^0C temperature line and 1 atm pressure line. But a unique point for state "B" cannot be fixed because the entire line 2–4 satisfy the given condition pressure, P = 1 atm and temperature, T = 100^0C.

Thus, for a given pressure, if the temperature is less than the saturation temperature ($T < T_{sat}$), the state lies in the compressed or sub-cooled liquid region and if the temperature is higher than the saturation temperature ($T > T_{sat}$), the state lies in the superheated vapor region.

Similarly, let us consider three different states D, E and F of water defined by three different pressures and the constant temperature of 100°C as

State D: 1.5 atm, 100^0C
State E: 1 atm, 100^0C
State F: 0.5 atm, 100^0C

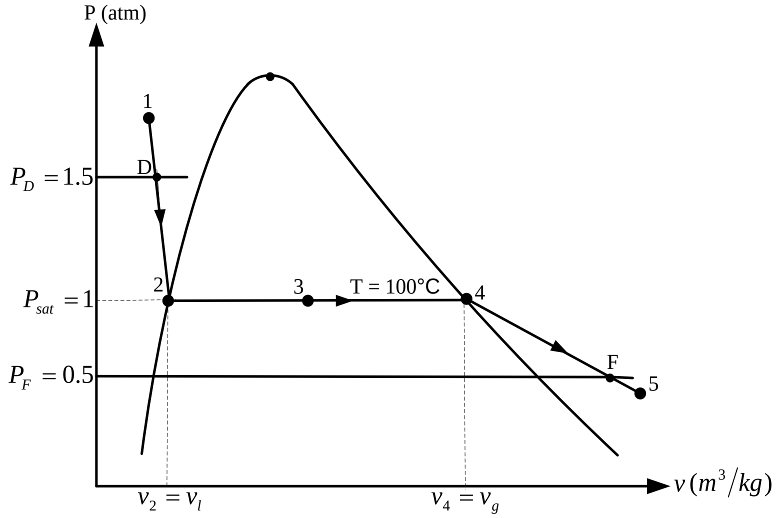

Figure 3.15. Locating states on $P - v$ diagram with reference to saturation line

To fix these states on $P - v$ diagram, two constant pressure lines for 0.5 atm and 1.5 atm can be drawn for temperature 100°C as shown in figure 3.15. Then the state "D" can be fixed as intersection of 1.5 atm pressure line and 100^0C temperature line and state "F" as intersection of 0.5 atm pressure line and 100^0C temperature line. Here again, a unique point for state "E" cannot be fixed, because the entire line $2 - 4$ satisfy the given condition pressure, P = 1 atm and temperature, T = 100^0C.

Thus, for a given temperature, if the pressure of a state is higher than the saturation pressure $(P > P_{sat})$, the state lies in the compressed or subcooled liquid region and if the pressure is less than the saturation pressure $(P < P_{sat})$, the state lies in the superheated vapor region.

3.4.1. Quality

It can be concluded from the above examples that the states A, C, D and F can be fixed with the help of the given values of pressure and temperature at these states. But the states B and E cannot be located uniquely due to constant values of given pressure and temperature of all the states along the saturation line 2 – 4. This difference can be explained with reference to state postulate. In single phase region for compressed liquid or superheated vapor state, pressure and temperature are independent; the states can therefore be fixed with the help of the given values of pressure and temperature. But in two-phase mixture (saturation) region, the pressure and the temperature are no more independent, and the states cannot be fixed with the help of the values of dependent pressure and temperature. Hence, to fix a state within the two phase mixture or saturation region, a third property is necessary which is called quality. Quality is defined as the ratio of mass of saturated vapor and the total mass of the two phase mixture. It is also called dryness fraction and is denoted by x.

$$x = \frac{m_g}{m} = \frac{m_g}{m_l + m_g} \qquad \text{……… (3.19)}$$

where m_g is the mass of saturated vapor, m_l is the mass of saturated liquid, and m is the total mass of the two phase mixture.

For saturated liquid state, $m = m_l$ or $m_g = 0$

$$\therefore \quad x = 0 \qquad \text{……… (3.20)}$$

For saturated vapor state, $m = m_g$ or $m_l = 0$

$$\therefore \quad x = 1 \qquad \text{……… (3.21)}$$

Hence, quality varies from 0 to 1 $(0 \leq x \leq 1)$ or from 0% to 100% within the saturation region as shown in figure 3.16.

If the quality value is given for a state, then without doing any verification it can be said that the state lies in the two phase mixture region. If the quality of a state is 0, then the state is a saturated liquid state which lies on saturated liquid line and if the quality of a state is 1 or 100%, then the state is a saturated vapor state which lies on saturated vapor line.

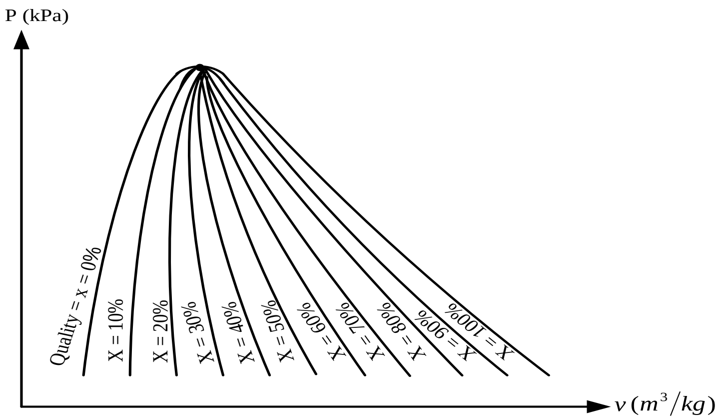

Figure 3.16. Distribution of quality lines on P–v diagram

The region of a state can also be identified by comparing the specific volume v of the state with the corresponding specific volumes of the saturated liquid v_l and the saturated vapor v_g at the given pressure and temperature of that state. If the specific volume of a state is less than the specific volume of the saturated liquid ($v < v_l$), then the state lies on the subcooled or compressed liquid region. If the specific volume of a state is greater than the specific volume of the saturated vapor ($v > v_g$), then the state lies on the superheated vapor region. If the specific volume of a state is greater than the specific volume of the saturated liquid and less than the specific volume of the saturated vapor ($v_l < v < v_g$), then the state lies within the two phase mixture or saturation region.

3.4.2. Moisture Content

Moisture content is defined as the ratio of mass of saturated liquid and the total mass of the two phase mixture. This property can also be used to fix a state within the two phase mixture (saturation) region. It is denoted by y.

$$y = \frac{m_l}{m} = \frac{m - m_g}{m} = 1 - x \qquad \text{………. (3.22)}$$

3.4.3. Specific Properties of a Two-Phase Mixture

Specific properties of the two-phase mixture like specific volume, specific internal energy, specific enthalpy and specific entropy can be expressed in term of quality. For example, specific volume of a two phase mixture is given by the ratio of total volume of two-phase mixture and the total mass of the two phase mixture, i.e.

$$v = \frac{V}{m} = \frac{V_l}{m} + \frac{V_g}{m}$$

where V_l is the volume of saturated liquid, and V_g is the volume of saturated vapor.

or, $$v = \frac{V_l}{m_l}\frac{m_l}{m} + \frac{V_g}{m_g}\frac{m_g}{m} = v_l(1-x)+v_g x$$

where v_l is the specific volume of saturated liquid, and v_g is the specific volume of saturated vapor.

or, $$v = v_l - v_l x + v_g x = v_l + x(v_g - v_l)$$

$$\therefore \quad v = v_l + x v_{lg} \qquad \text{………(3.23)}$$

where $v_{lg} = v_g - v_l$, is the change in specific volume during vaporization.

Similarly, other specific properties of the two-phase mixture can be derived and expressed in terms of quality in the same way as in the specific volume as below:

Specific internal energy, $u = u_l + x u_{lg}$ ……….. (3.24)

Specific enthalpy, $h = h_l + x h_{lg}$ ………. (3.25)

Specific entropy, $s = s_l + x s_{lg}$ ………. (3.26)

Thus, it is common practice to use quality and one other specific property or intensive property (e.g. temperature and pressure) to fix the state along the two phase line 2–4. This is because the specific volume and other specific properties of two phase system are determined only when quality is specified at a given pressure and temperature (by using equations from 3.23 to 3.26).

3.5. Other Thermodynamic Properties

3.5.1. Internal Energy and Enthalpy

In the study of thermodynamics internal energy is one of the important thermodynamic property with other properties like temperature, pressure, and specific volume. Internal energy which is denoted by *U* incorporates the microscopic forms of energy which result from the molecular motions and positions (e.g., internal kinetic and potential energy). The specific internal energy (*u = U/m*) can be found for any pure substance as a function of any two independent intensive properties as per the state postulate.

The expression *U + PV* occurs so frequently in thermodynamic calculations that it has been defined a new property called enthalpy and it is denoted by *H*:

$$H = U + PV \qquad \text{…….(3.27)}$$

Hence, enthalpy is defined as the summation of internal energy and product of pressure and volume. The specific enthalpy is expressed as

$$h = H/m = u + Pv \quad \text{………(3.28)}$$

To evaluate the specific enthalpy at a given state, values of specific internal energy u and specific volume v must be found from some data source. Because all the properties on the right hand side of above equation (3.28) can be found as a function of two independent intensive properties, it follows that the specific enthalpy can also be tabulated as a function of the same two independent intensive properties.

3.5.2. Specific Heats

Specific heat of a pure substance is defined as heat required for raising the temperature of a unit mass of the substance by 1^0C. This definition of the specific heat is applicable for solid and liquid substances. But in case of a gas, heat required to raise temperature of unit mass of the gas by 1^0C is different for the constant volume and the constant pressure process. For a gas therefore two specific heats are defined: specific heat at constant volume (c_V) and specific heat at constant pressure (c_P) [2].

3.5.2.1. Specific Heat at Constant Volume

Heat supplied to an ideal gas during a constant volume process in which no displacement work occurs is equal to the change in internal energy as per the first law of thermodynamics ($\delta Q = dU$). Hence, specific heat of an ideal gas at constant volume can be defined in terms of change in internal energy [2].

For this let us refer to the state postulate or two property rule, i.e., specific internal energy of a pure substance at a state can be determined if two other independent intensive properties of that state are known e.g. its temperature and specific volume which are two very useful measurable properties. Mathematically,

$$u = u(T, v) \quad \text{………. (3.29)}$$

Then the change in specific internal energy for a pure substance undergoing a thermodynamic process can be expressed as function of partial derivatives of two independent variables as

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv \quad \text{………. (3.30)}$$

For a constant volume process, putting $dv = 0$, we get

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT \qquad \text{………. (3.31)}$$

which can be expressed in simpler form as

$$du = c_v dT \qquad \text{………. (3.32)}$$

where,

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v \qquad \text{………. (3.33)}$$

is called specific heat at constant volume.

Hence, specific heat at constant volume for a pure substance e.g. an ideal gas is defined as the change in specific internal energy per degree change in temperature during a constant volume process.

3.5.2.2. Specific Heat at Constant Pressure

Heat supplied to an ideal gas during a constant pressure process in which the displacement work ($P.dV$) occurs is equal to the change in enthalpy ($\delta Q = dU + \delta W = dU + PdV = d(U + PV) = dH$) as per the first of thermodynamics Hence, specific heat of a pure substance at a constant pressure can be defined in terms of change in enthalpy [2].

For this let us again refer to the state postulate for specific enthalpy of any pure substance which tells that the specific enthalpy can be expressed as the function of any two independent intensive properties e.g. temperature and pressure, i.e.

$$h = h(T, P) \qquad \text{………. (3.34)}$$

Then the change in specific enthalpy is given as

$$dh = \left(\frac{\partial h}{\partial T}\right)_P dT + \left(\frac{\partial h}{\partial P}\right)_T dP \qquad \text{………. (3.35)}$$

For a constant pressure process, putting $dP = 0$, we get

$$dh = \left(\frac{\partial h}{\partial T}\right)_P dT \qquad \text{………. (3.36)}$$

which can be expressed in simpler form as

$$dh = c_P dT \qquad \text{………. (3.37)}$$

where,

$$c_P = \left(\frac{\partial h}{\partial T}\right)_P \qquad \text{………. (3.38)}$$

is called specific heat at constant pressure.

Hence, specific heat at constant pressure for a pure substance, e.g. an ideal gas is defined as the change in specific enthalpy per degree change in temperature during a constant pressure process.

3.5.3. Relationship between Specific Heats for an Ideal Gas

For an ideal gas specific internal energy can be shown to be function of temperature only, i.e. from equation 3.32,

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v = \frac{du}{dT} \qquad \text{……….. (3.39)}$$

Similarly, it follows that the specific enthalpy is also a function of temperature only for an ideal gas. Then from equation (3.38),

$$c_p = \left(\frac{\partial h}{\partial T}\right)_P = \frac{dh}{dT} \qquad \text{………….(3.40)}$$

From the definition of the specific enthalpy and the use of the ideal gas relation,

$$h = u + Pv = u + RT \qquad \text{…………..(3.41)}$$

By taking derivative of above equation (3.41) with respect to temperature, it becomes

$$\frac{dh}{dT} = \frac{du}{dT} + R$$

By using equations (3.39) and (3.40),

$$c_p = c_v + R$$

$$\Rightarrow\ c_p - c_v = R \qquad \text{...…….(3.42)}$$

Diving both sides of equation (3.42) by c_v and substituting the ratio of two specific heats (c_p/c_v) by γ , we get

$$c_v = \frac{R}{\gamma - 1} \qquad \text{...........(3.43)}$$

Similarly, dividing both sides of equation (3.42) by c_p and substituting the ratio of two specific heats (c_p/c_v) by γ , we get

$$c_p = \frac{\gamma}{\gamma - 1} R \qquad \text{...........(3.44)}$$

Thus, for an ideal gas the specific heat at constant pressure c_p must always be greater than the specific heat at constant volume c_v and the difference between two specific heats is equal to the characteristic or particular gas constant (R) of the ideal gas.

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3.6. Development of Property Data

In engineering problems, for any working substance, the six basic thermodynamic properties required are: pressure, temperature, volume, internal energy, enthalpy and entropy. The values of these properties must be known at difference pressure for analyzing the thermodynamic applications or devices and are determined theoretically or experimentally. However, these properties of a pure substance within saturation region or near saturation region cannot be determined theoretically, i.e., using the mathematical equations (e.g. ideal gas equations). Hence, the values of all properties under such conditions are determined experimentally and presented in the form of tables so called *steam table* and graphs or charts. For doing this, careful and detailed experimental measurements are necessary for most substances to obtain the values of the minimum set of dependent properties as a function of a chosen set of independent properties such as pressure and temperature. Many methods have been tried for the presentation of the experimental data of all the properties and the most useful are tabular data and graphical data presentation. Both methods are equally in use for thermodynamic calculations. However, the tabular data presentation is more useful for calculating thermodynamic properties.

3.6.1. Tabular Data Presentation

In thermodynamic calculation the property tables for pure compressible substances are common in use. Properties for a given state can be determined from a property table. When detailed property data are to be presented for a wide range of states, so that a single equation of state is not sufficient. It is necessary to present the data in tabular form. Because the commonly measured properties are temperature and pressure, usually data are presented with temperature and pressure used as the independent properties and the other properties like specific volume, specific internal energy, specific enthalpy and specific entropy are treated as dependent.

Property tables for the water are tabulated in the most of the textbook of the thermodynamics. The properties values of the specific volume, specific internal energy, specific enthalpy and specific entropy are obtained experimentally and presented in the tabular form. In the textbook four separate tables are provided for these properties of the water (or steam).

(i) The properties of water at *saturation condition*, i.e., properties of the saturated liquid and saturated vapor are tabulated with different pressures and corresponding saturation temperatures.

(ii) The properties of water at *saturation condition*, i.e., properties of the saturated liquid and saturated vapor are tabulated with different temperatures with corresponding saturation pressures.

In thermodynamics usually property table for saturated liquid and that for saturated vapor are combined together and given the name saturation table. In these tables, the values of the properties of saturated liquid are denoted by subscript *l* and that of saturated vapor are denoted by subscript *g*. Further, difference of these properties of saturated vapor and saturated liquid are also tabulated and denoted by subscript *lg*. For example, for specific volume, $v_{\mathrm{lg}} = v_g - v_l$.

(iii) Similarly, *compressed liquid table* provides the properties of water for compressed or sub-cooled liquid state for different pressures and temperatures. Since liquid is incompressible substance, the change in properties with pressure is very little for compressed or sub-cooled liquid. Hence, the values of the properties were tabulated only for high pressures, but for low pressures the values of the properties like specific volume, specific internal energy, specific enthalpy, and specific entropy are taken from the saturation table at the corresponding value of given temperature of saturated liquid with slight error. It should be noted that while taking properties values from saturated liquid table, the values are taken from temperature table rather than pressure table because at low pressure the properties are independent upon pressure.

(iv) *Superheated steam table* gives the properties of water for superheated vapor state for each value of different pressures and temperatures independently. The properties are very sensitive to pressure for superheated vapor.

Linear Interpolation for Tabulated Data

For the substance whether in saturated liquid or saturated vapor or superheated vapor condition, when the values of the properties corresponding to any pressure and temperature are not provided in the tabulated data, then linear interpolation method is used to find the properties values corresponding to that given pressure and temperature. The interval between tables entries in complete tabulation are usually chosen so that accurate values for states that fall between entries are found by using this method. In general the following formula is used to find the properties values:

$$y = y_1 + \frac{x - x_1}{x_2 - x_1}(y_2 - y_1) \qquad \text{..........(3.45)}$$

where y is the value of required property (to be found) for given property x.

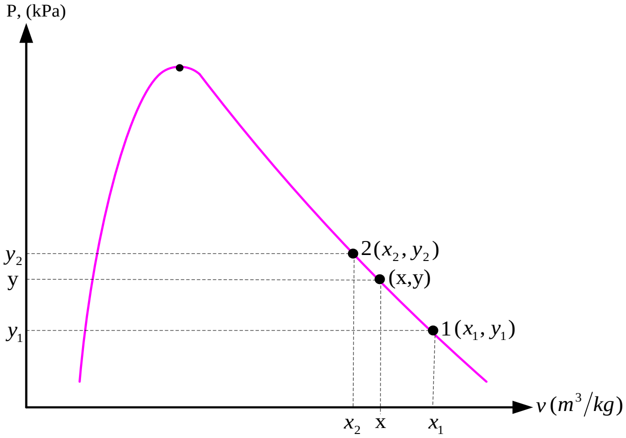

Figure 3.18. Linear interpolation for pressure

Equation (3.45) represents a straight line passing through two given points 1 (x_1, y_1) and 2 (x_2, y_2) with respect to x and y axes as given in figure 3.18. In this diagram a small portion of the saturation curve from state 1 to state 2 is assumed as a straight line where these two points or states for instant lie on saturated vapor line. Let us take an example to illustrate the interpolation method; if the values of specific volumes of the saturated vapor at 125 kPa and 150 kPa are given in the table, then that value of specific volume at 140 kPa can be found by linear interpolation as follows. In this problem, the given property x (= 140 kPa) is the pressure and the required property y (= v_g =?) is the specific volume of the saturated vapor. Then from the table,

x_1 = 125 kPa, $y_1 = v_g(125\ kPa) = 1.3752 m^3/kg$, and
x_2 = 150 kPa, $y_2 = v_g(150\ kPa) = 1.1595\ m^3/kg.$

Now, the value of $y\ (= v_g)$ can be calculated by using above equation (3.45) as

$$v_g = 1.3752 + \frac{140 - 125}{150 - 125}(1.1595 - 1.3752)$$

v_g= 1.24578 $m^3/kg.$

Conversely, for given property $x = v_g = 1.24578\ m^3/kg$ the required property, say pressure (y) can be calculated by the same formula and its value will be $y = P = 140$ kPa just by interchanging x and y values as

$$x_1 = v_g(125\ kPa) = 1.3752 m^3/kg, \quad y_1 = 125 \text{ kPa, and}$$
$$x_2 = v_g(150\ kPa) = 1.1595\ m^3/kg, \quad y_2 = 150 \text{ kPa.}$$

3.6.2. Graphical Data Presentation

Thermodynamic properties for given states can be determined from a graph or chart and thermodynamic processes can also be plotted graphically. Therefore, it is very useful to be able to visualize thermodynamic processes on co-ordinates of various pairs of thermodynamic properties. Graphical representation of processes on the phase or property diagrams plotted in terms of two pairs of properties enables visualization and analysis of many practical systems. Some useful property diagrams are plotted on co-ordinates of P–v, T–v, P–T, P–h, h–s and T–s diagrams etc. By the use of these diagrams one can easily locate and find the thermodynamic properties. The property diagrams such as P–v, T–v, and P–T are already discussed in detail and presented in the previous sections. Now, the P–h, h–s and T–s diagrams for water are given below.

Pressure–Enthalpy (*P*–*h*) diagram

The pressure-enthalpy diagram is commonly used in refrigeration cycles and throttling processes. The P–h diagram for pure substance such as water is shown in figure 3.19 [3]. The constant temperature and constant entropy lines are also shown in the diagram. The constant quality lines are also drawn in the same diagram.

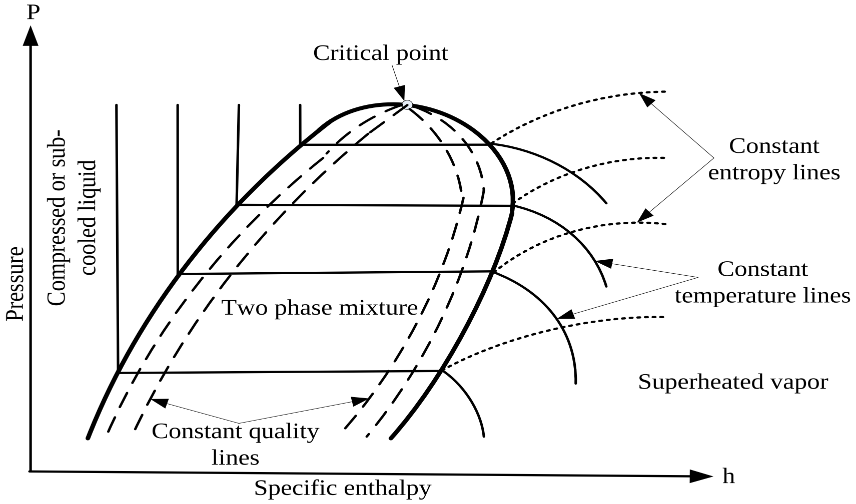

Figure 3.19. $P - h$ diagram of water

Entropy ($T-s$) Diagram

The temperature-entropy ($T-s$) diagram for a pure substance such as water is shown in the figure 3.20 [4]. The $T-s$ diagram is used to calculate indirectly the magnitude of specific heat transfer to the working substance and is equal to $q = \int Tds$, which indicates the area under process curve in $T-s$ diagram. Thus, this diagram permits observation of change in the temperature of the working substance and makes it possible to find the quantity of heat involved in the process. Therefore, it is extensively used in the investigation of thermodynamic processes and cycles [4].

The constant pressure and constant volume lines are drawn in the diagram. The constant quality lines are also shown in the same diagram.

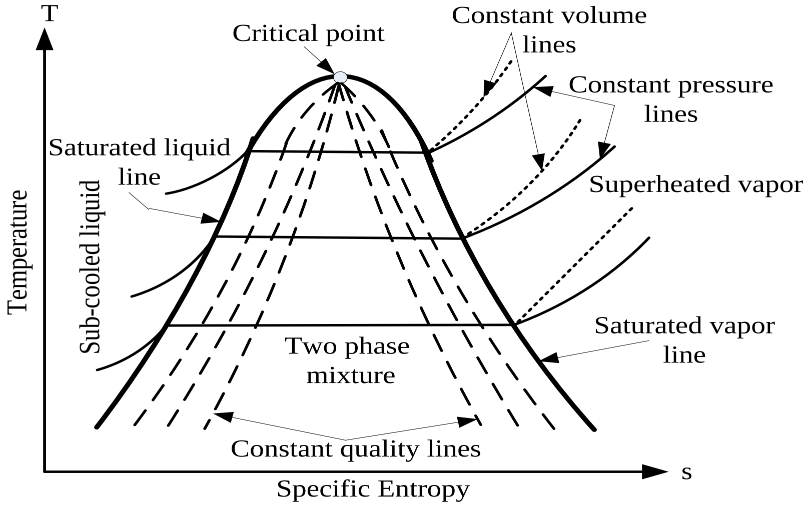

Figure 3.20. $T-s$ diagram of water

Enthalpy-Entropy (h–s) Diagram or Mollier Diagram

Since in engineering mostly the change of specific enthalpy and the amount of specific heat supplied or rejected are needed to be calculated, the enthalpy-entropy ($h-s$) diagram is very important and useful. The change in specific enthalpy and the amount of specific heat cannot directly be calculated from $T-s$ diagram. This problem was solved by developing $h-s$ diagram by Dr. Mollier in 1904. Therefore, the $h-s$ diagram is also known as *Mollier diagram* [4]. From Mollier diagram heat transfer and then work transfer can directly be calculated as a length since steam generation is done at constant pressure and for constant pressure heating it reveals that $\delta q = dh$ (i.e. $q = h_2 - h_1$), whereas on a $T-s$ diagram it is represented by an area. Hence, it facilitates the study of different thermodynamic steady work or flow applications or devices such as turbine, compressor, pump, fan, nozzle, diffuser, heat exchanger etc.

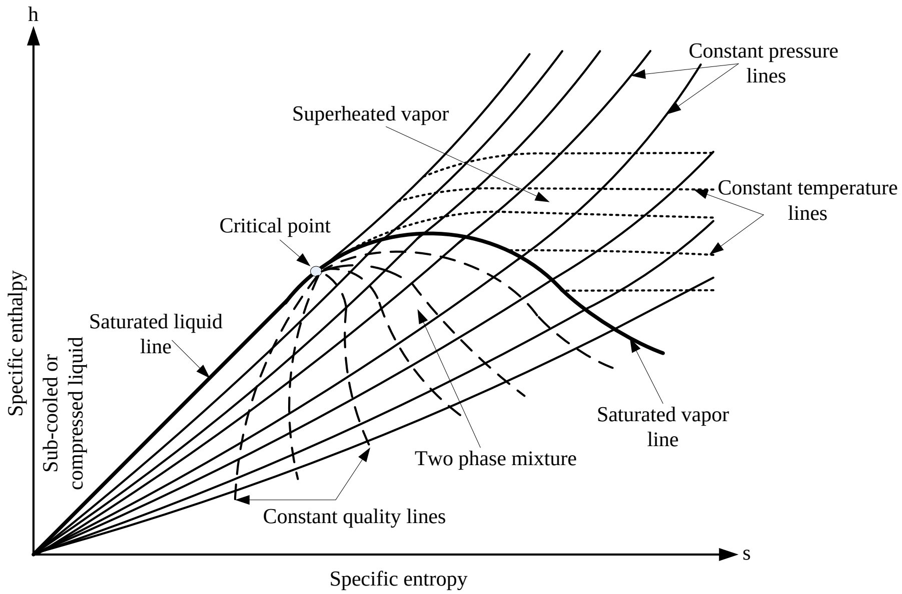

Figure 3.21. h–s diagram of water

The h–s diagram for a pure substance such as water is shown in the figure 3.21 [4]. The constant pressure and constant temperature lines are given in the diagram. The constant quality lines are also shown in the same diagram.

Lecture Highlights

- *Quality or dryness fraction:* It is defined as the ratio of mass of saturated vapor and the total mass of the two phase mixture, i.e. $x = m_g/m$. It has a value ranges from 0 to 1.
- *Moisture content or liquid fraction:* It is defined as the ratio of mass of saturated liquid and the total mass of two phase mixture, i.e. $y = m_l/m = 1 - x$.
- *Necessity of quality:*
  Both pressure and temperature are independent for sub-cooled or compressed liquid states and superheated vapor states. Hence, states can be fixed with given pressure and temperature. But for two phase mixture or saturation region, pressure and temperature are no more independent and states cannot be fixed with dependent temperature and pressure as per state postulate. Therefore, quality is necessary to fix the states within the saturation region.
- *Specific properties of a two phase mixture:*
  - Specific volume, $v = v_l + x v_{\mathrm{lg}}$ where $v_{\mathrm{lg}} = v_g - v_l$
  - Specific internal energy, $u = u_l + x u_{\mathrm{lg}}$ where $u_{\mathrm{lg}} = u_g - u_l$
  - Specific enthalpy, $h = h_l + x h_{\mathrm{lg}}$ where $h_{\mathrm{lg}} = h_g - h_l$

  - Specific entropy, $s = s_l + xs_{\mathrm{lg}}$ where $s_{\mathrm{lg}} = s_g - s_l$

- *Enthalpy:* It is the summation of internal energy and the product of pressure and volume and mathematically, $H = U + PV$ or, specific enthalpy $h = u + Pv$.
- *Identification of states:*
  - For a given specific volume, if specific volume is lower than specific volume of saturated liquid, its state will be sub-cooled or compressed liquid and if specific volume is higher than specific volume of saturated vapor, the state will be superheated vapor and if it lies between these values, the state will be two phase mixture, i.e.,
    $v < v_l$ implies the state is sub-cooled or compressed liquid.
    $v > v_g$ implies the state is superheated vapor.
    $v_l < v < v_g$ implies the state is two phase mixture.
    The same rule is applied for specific internal energy *(u)*, enthalpy *(h)* and entropy *(s)*.
  - For a given pressure, if temperature of a state is less than the saturation temperature, its state will be a sub-cooled or compressed liquid and if temperature is higher than the saturation temperature, the state will be a superheated vapor, i.e.
    $T < T_{sat}$ implies the state is sub-cooled or compressed liquid.
    $T > T_{sat}$ implies the state is superheated vapor.
  - For a given temperature, if pressure of a state is higher than the saturation pressure, its state will be a sub-cooled or compressed liquid and if pressure is less than the saturation pressure, the state will be superheated vapor, i.e.,
    $P > P_{sat}$ implies the state is sub-cooled or compressed liquid.
    $P < P_{sat}$ implies the state is superheated vapor.
- In the absence of sub-cooled or compressed liquid data, a general approximation is to treat a sub- cooled or compressed liquid as a saturated liquid at the given *temperature*, hence

  $$z \cong z_l$$

  where z stands for v, u, s, or h.
- *Specific heats of an ideal gas:*
  - *Specific heat at constant volume:* It is defined as the change in specific internal energy per degree change in temperature during a constant volume process. Mathematically,

    $$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

  - *Specific heat at constant pressure:* It is defined as the change in specific internal enthalpy per degree change in temperature during a constant pressure process. Mathematically,

    $$c_P = \left(\frac{\partial h}{\partial T}\right)_P$$

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