Automatic Control Systems

Lecture-4

Modeling Electrical Systems

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Modeling Electrical Systems

Session Objectives

By the end of this session, learners will be able to:

- ➢Define the system transfer function
- ➢Explain Laws used to determine system transfer function
- ➢Describe parameters considered in modeling electrical systems
- ➢Determine transfer function of various electrical networks.

Transfer function definition

The transfer function of a linear time invariant differential equation system is defined as the ratio of the **Laplace transform of the output** (**response function**) to the **Laplace transform of the input** (**driving function**)under **the assumption that all initial conditions are zero**.

Ahmed Nassef, Automatic Control Systems, Salman Bin Abdulaziz University, slide 32

Comments on Transfer Functions

1. A transfer function of a system is a mathematical model in that it is an operational method of expressing the differential equation that relates the output variable to the input variable.

2. The transfer function is a property of a system itself, independent of the magnitude and nature of the input or driving function.

3. All initial conditions of the system are set to zero.

Comments on Transfer Functions(cont’d)

4. The transfer function does not provide any information about physical structure of a system.

5. If the transfer function of a system is unknown, it may be established experimentally by introducing known inputs and studying the output of the system.

K. Ogata (1997), Modern Control Engineering, 3rd Edition, Prentice Hall, page 48.

Laws governing Electrical Circuits

Basic laws governing electrical circuits are:

1. *Kirchoff's Current Law(KCL)*
2. *Kirchoff's Voltage Law(KVL)*

Kirchoff's Current Law(KCL)

Kirchhoff's current law (node law) states that the algebraic sum of all currents entering and leaving a node is zero.

This law can also be stated as follows: The sum of currents entering a node is equal to the sum of currents leaving the same node.

Kirchoff's Voltage Law(KVL)

Kirchhoff's voltage law (loop law) states that at any given instant the algebraic sum of the voltages around any loop in an electrical circuit is zero.

This law can also be stated as follows: The sum of the voltage drops is equal to the sum of the voltage rises around a loop.

Network Parameters

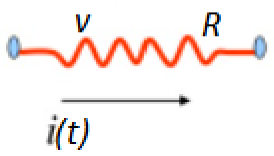
Parameters to consider while modeling electrical systems are the following:

- *Resistor*
- *Inductor*
- *Capacitor*

Smarajit Gosh (2007), Control systems, Pearson Education, page 110.

Element Laws
1. Resistor

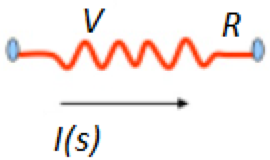
Time Domain



$v=R.i(t)$

$i=(1/R).v$

S-Domain

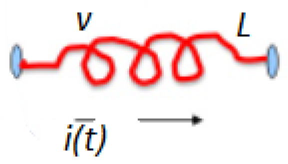


$V=R.I(s)$

$I=(1/R).V$

Element Laws (cont'd)
2. Inductor

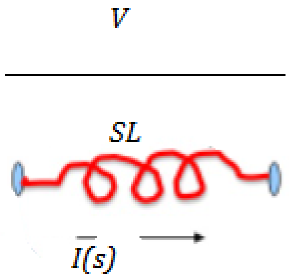
Time Domain



$v=L(di/dt)$

$$i(t) = \frac{1}{L}\int_{to}^{t} vdt$$

S-Domain

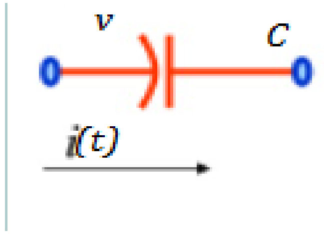


$V=SLI(s)$

$$I(s) = \frac{V(s)}{SL}$$

Element Laws (cont'd)
3. Capacitor

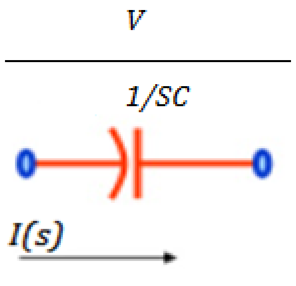
Time Domain



$i = C(dv/dt)$

$$v(t) = \frac{1}{C}\int_{to}^{t} i dt$$

S-Domain

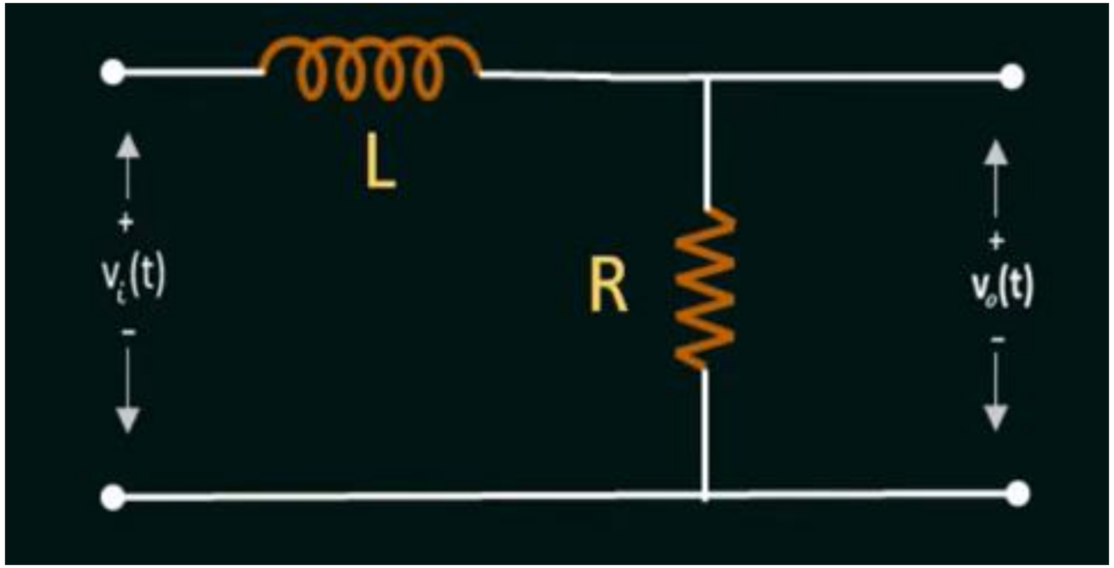


$I = SCV(s)$

$$V(s) = \frac{I(s)}{CS}$$

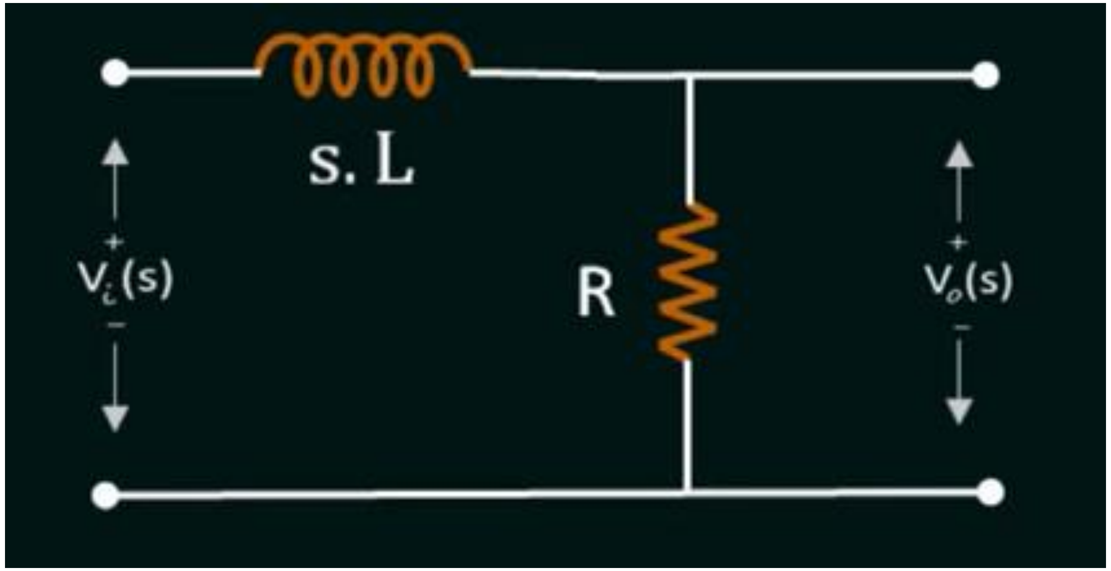
Modeling RL series Circuit

Find the transfer function of the following RL circuit



Time Domain

Solution:



S-Domain

Using KVL,

$$V_i(s) = SLI(s) + RI(s) \ldots\ldots\ldots\ldots\ldots. (1)$$

$$V_o(s) = RI(s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

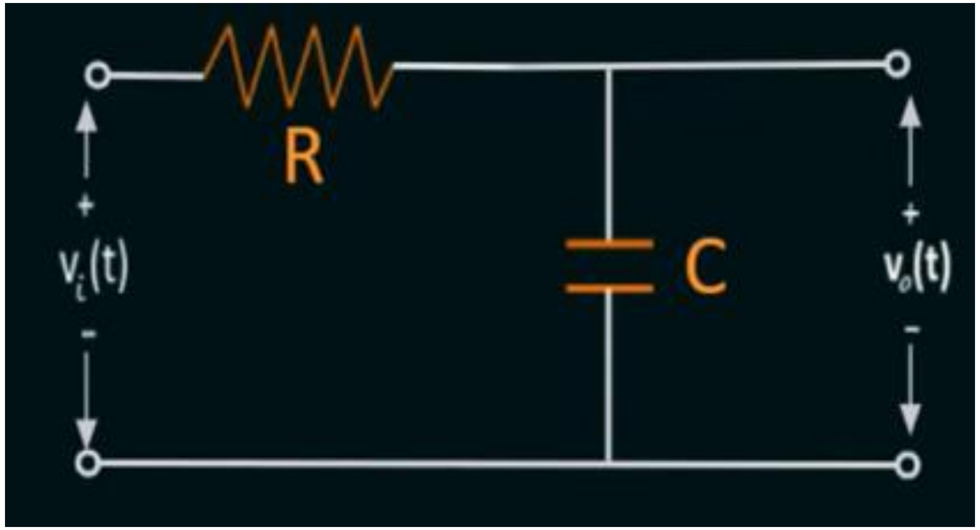
$$\frac{V_o(s)}{V_i(s)} = \frac{RI(s)}{SLI(s) + RI(s)}$$

The system transfer function is

$$\frac{V_o(s)}{V_i(s)} = \frac{R}{SL+R}$$

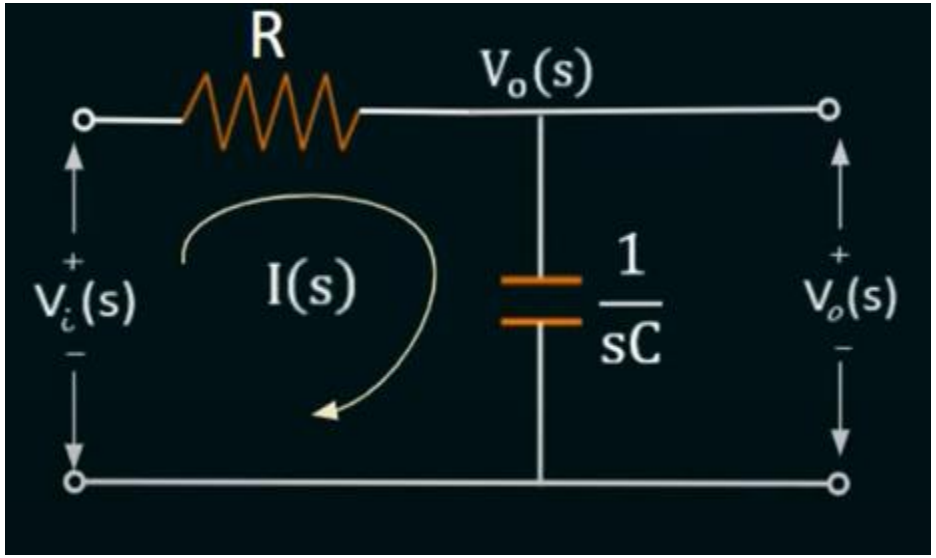
Modeling RC series Circuit

Find the transfer function of RC series circuit



Time Domain

Solution:



S-Domain

Apply KVL,

$$V_i(s) = RI(s) + \frac{1}{SC} I(s) \dots\dots\dots\dots\dots (1)$$

$$V_o(s) = \frac{1}{SC} I(s) \dots\dots\dots\dots\dots\dots\dots\dots. (2)$$

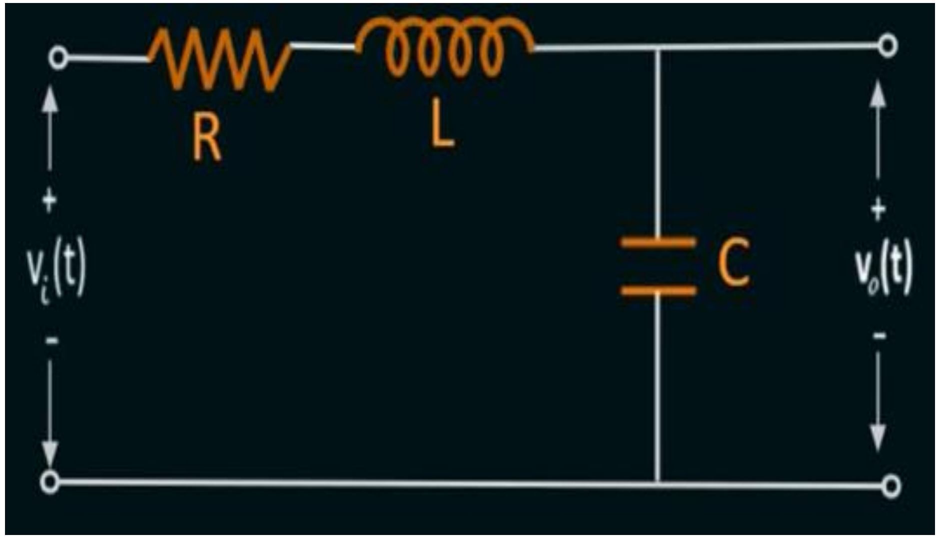
$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{SC} I(s)}{RI(s) + \frac{1}{SC} I(s)}$$

The system transfer function is

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{SC}}{R + \frac{1}{SC}} = \frac{1}{RCS + 1}$$

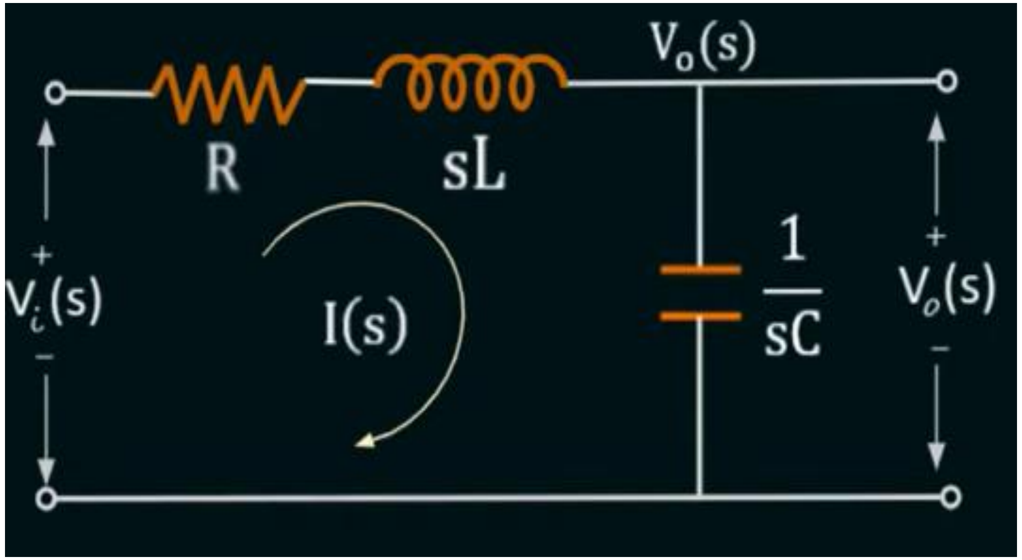
Modeling RLC series Circuit

Find the transfer function of the following RLC series circuit



Time Domain

Solution:



S-Domain

Apply KVL,

$$V_i(s) = RI(s) + SLI(s) + \frac{1}{SC}I(s) \ldots\ldots..(1)$$

$$V_o(s) = \frac{1}{SC}I(s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{SC}I(s)}{RI(s) + SLI(s) + \frac{1}{SC}I(s)}$$

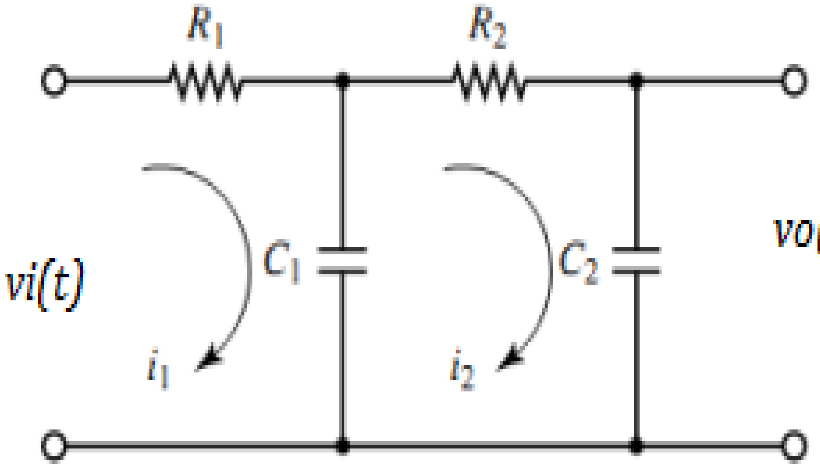
The system transfer function

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{SC}}{\frac{RCS + CLS^2 + 1}{SC}}$$

$$= \frac{1}{CLS^2 + RCS + 1}$$

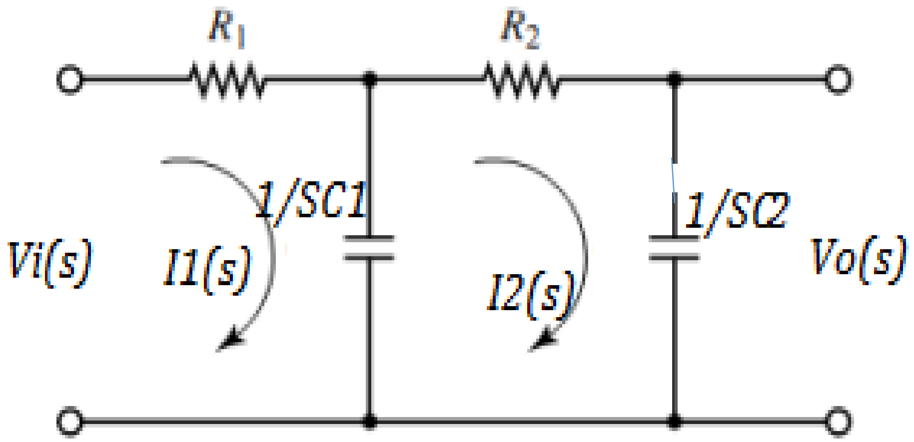
Modeling RLC series Circuit

Example 2: Find the mathematical model (transfer function) of the following circuit



→ Time Domain

Solution:



S-Domain

Apply KVL,

For loop 1:

$$V_i(s) = R_1 I_1(S) + \frac{1}{SC_1}\left(I_1(s) - I_2(s)\right)$$

$$= \left(R_1 + \frac{1}{SC_1}\right) I_1(s) - \frac{1}{SC_1} I_2(s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

For loop 2:

$$R_2 I_2(S) + \frac{1}{SC_2} I_2(s) + \frac{1}{SC_1}\left(I_2(s) - I_1(s)\right) = 0$$

$$R_2 I_2(S) + \frac{1}{SC_2} I_2(s) + \frac{1}{SC_1} I_2(s) - \frac{1}{SC_1} I_1(s) = 0$$

$$(R_1 + \frac{1}{SC_1} + \frac{1}{SC_2}) I_2 - \frac{1}{SC_1} I_1(s) = 0 \ldots\ldots\ldots\ldots\ldots\ldots\ldots. (2)$$

For Loop 3:

$$\frac{1}{SC_2} I_2(s) = V_o(s) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

From equation (2),

$$I_1(s) = \left(R_2 C_1 S + \frac{C_1}{C_2} + 1\right) I_2(s) \dots\dots\dots\dots(4)$$

Putting equation (4) into equation (1), we get

$$V_i(s) = \left(R_1 + \frac{1}{SC_1}\right)\left(R_2 C_1 S + \frac{C_1}{C_2} + 1\right) I_2(s) - \frac{1}{SC_1} I_2(s)$$

$$V_i(s) = \left(\frac{R_1 R_2 C_1 C_2 S^2 + R_1 C_1 S + R_1 C_2 S + R_2 C_2 S + 1}{SC_2}\right) I_2(s) \dots\dots(5)$$

The system transfer function

$$\frac{V_O(s)}{V_I(s)} = \frac{\frac{I_2(s)}{SC_2}}{\left(\frac{R_1R_2C_1C_2S^2 + R_1C_1S + R_1C_2S + R_2C_2S + 1}{SC_2}\right) I_2(s)}$$

$$= \frac{1}{R_1R_2C_1C_2S^2 + (R_1C_1 + R_1C_2 + R_2C_2)S + 1}$$

The term $R_1 C_2 S$ in the denominator of the transfer function represents the interaction of the two simple RC circuits.

References

1. Ahmed Nassef, Automatic Control Systems, Salman Bin Abdulaziz University.
2. K. Ogata (1997), Modern Control Engineering, 3rd Edition, Prentice Hall.
3. Smarajit Gosh (2007), Control systems, Pearson Education.

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