Automatic Control Systems

Lecture-6

System Block Diagrams and Perform Reduction Techniques of Block Diagrams

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Session Objectives

By the end of this session, learners will be able to:

- Definition of basic elements of block diagram
- Explain rules for block diagram reduction
- Explain procedures followed for block diagram reduction
- Perform block diagram reductions

Introduction

Block diagrams show the interaction between system elements for easy determination of overall system transfer function.

Block diagram is a pictorial representation of each system.

Each element of a complicated system is represented by a block.

The main aim of this lecture is to introduce block diagram representation and the rules to solve any complicated systems.

Definition of basic elements of block diagram

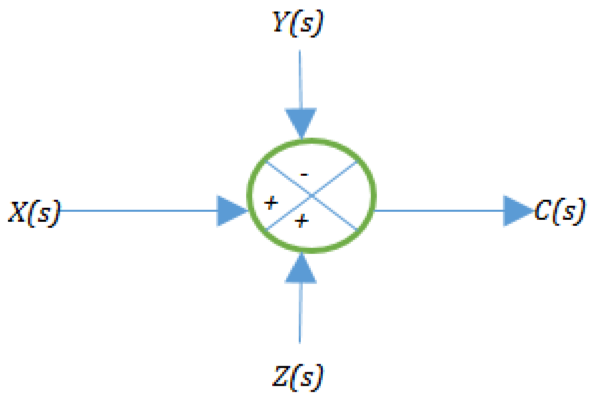
- **Block diagram**: The pictorial representation of the cause and effect relationship between input and output of a physical system is known as block diagram.

Block diagram with gain (s-domain)

- **Output**: The value of input multiplied by the block gain is known as output.

$$C(s) = G(s).R(s)$$

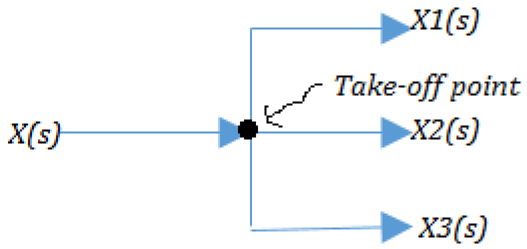
- **Summing point**: At summing point, two or more signals can be added or subtracted.



$$X(s), Z(s) and\ Y(s) are\ inputs\ while\ C(s) is\ an\ output.$$

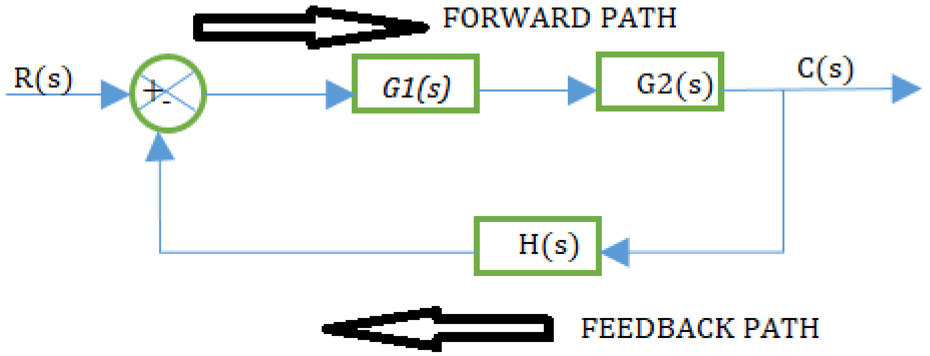
It can be written as $C(s) = X(s) + Z(s) - Y(s)$

- **Take off point**: The point at which the output signal of any block can be applied to two or more points is known as take off point.



- **Forward path**: The direction of flow of signal from input to output is known as forward path.

- **Feed back path**: The direction of flow of signal from output to input is known as feedback path.

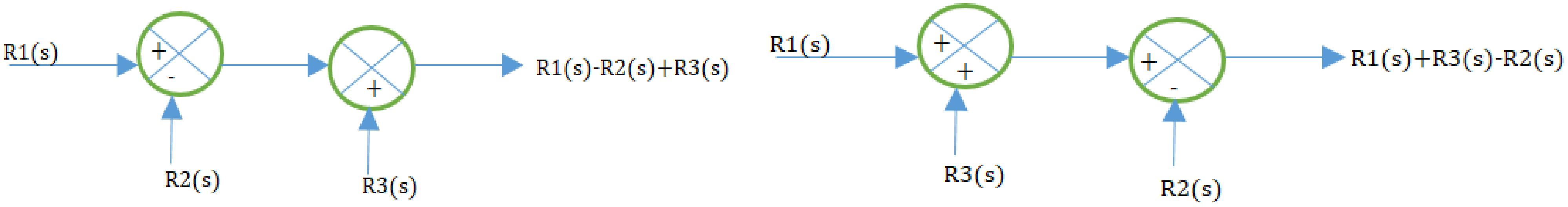


Block diagram reduction

In many practical situations, the block diagram of a Single Input-Single Output (SISO), feedback control system may involve several feedback loops and summing points. In principle, the block diagram of (SISO) closed loop system, no matter how complicated it is, it can be reduced to the standard single loop form

Rules for block diagram reduction

Rule 1: Associative law

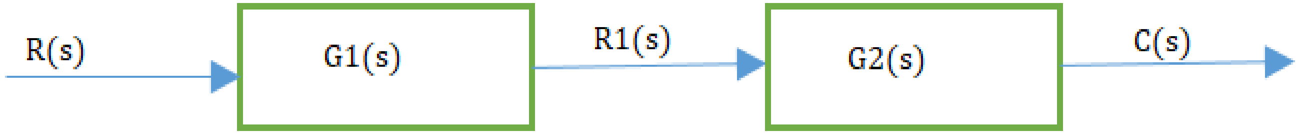


Summing points are interchanged

When the positions of summing points are interchanged, the output remain unchanged.

If any block is present in between the summing points, by interchanging the summing points the output will not be equal.

- **Rule 2: For blocks in cascade**



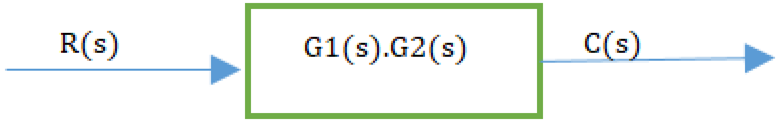
Blocks in cascade

From the figure we can write

$R_1(s) = G_1(s).R(s)$

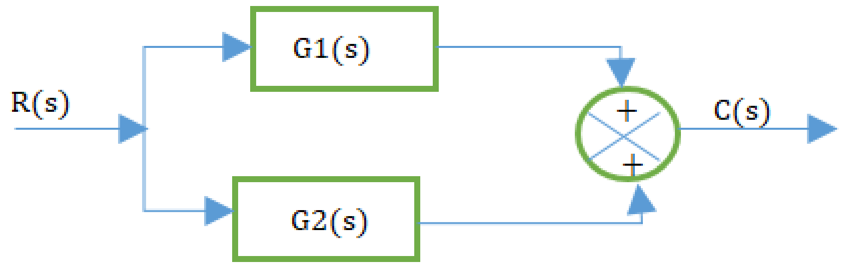
$C(s) = G_2(s).R_1(s) = G_2(s).G_1\ (s).R(s)$

The transfer function of the system=$\frac{C(s)}{R(s)} = G_1(s).G_2\ (s)$



Equivalent Block

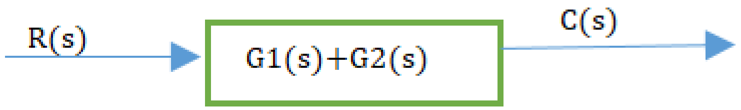
- **Rule 3: For blocks in parallel**



Parallel Blocks

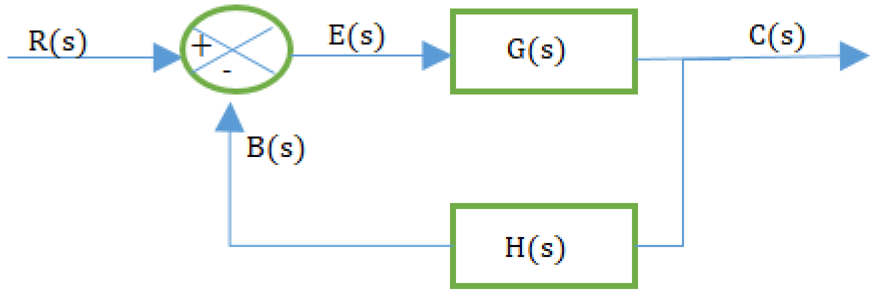
$$C(s) = R_1(s) + R_2(s) = G_1(s).R(s) + G_2(s).R(s)$$
$$= [G_1(s) + G_2(s)].R(s)$$

$$\frac{C(s)}{R(s)} = G_1(s) + G_2(s)$$



Equivalent Block

- **Rule 4: Eliminate feedback loop**



$$E(s) = R(s) - B(s)$$

$$C(s) = G(s).E(s) = G(s)[R(s) - B(S)] = G(s)R(s) - G(s)B(s)$$

$$= G(s)R(s) - \mathrm{G(s)H(s)C(s)} \qquad \therefore B(s) = H(S)B(s)$$

$$C(s) + G(s)H(s)C(s) = G(s)R(s)$$

$$C(s)[1 + H(s)G(s)] = G(s)R(s)$$

$$C(s) = \frac{G(s)}{1+H(s)G(s)}.R(s)$$

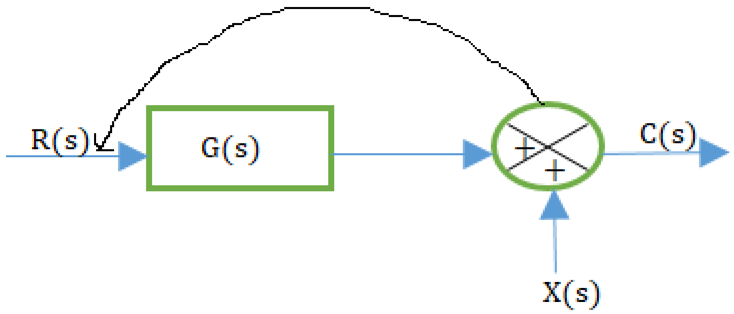
$$\frac{C(s)}{R(s)}\ The\ transfer\ function\ of\ the\ system = \frac{G(s)}{1+G(s)H(s)}$$

This equation is valid for negative feedback.

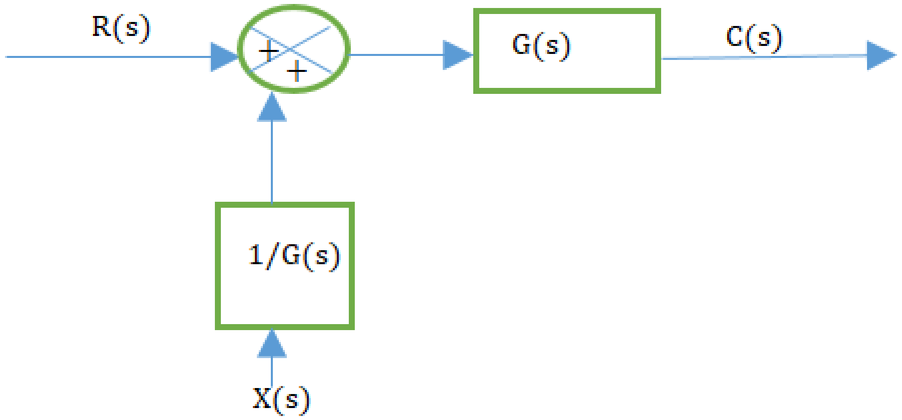
For positive feedback

$$\frac{C(s)}{R(s)} = \frac{G(s0}{1-G(s)H(s)}$$

Rule 5: Shifting a summing point before a block



Initial configuration



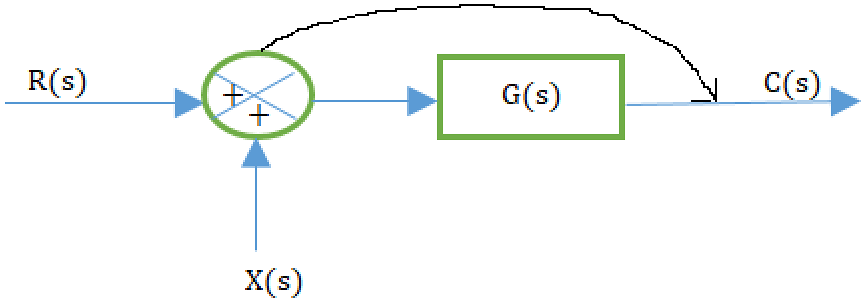
Final Configuration

$$C(s) = G(s)R(s) + X(s)$$

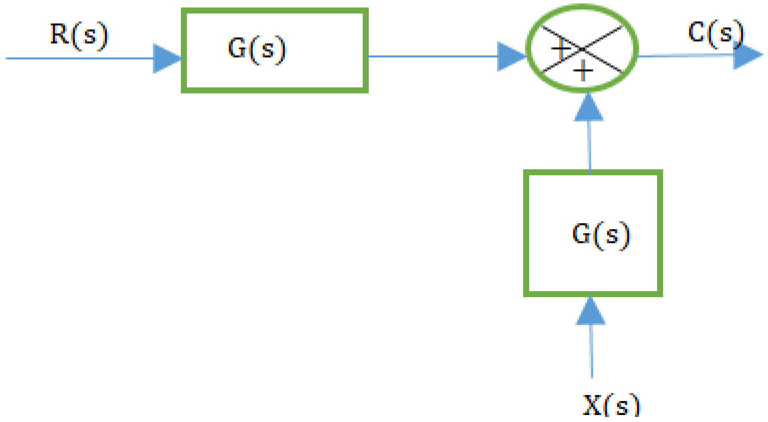
$$C(s) = G(s)\left[R(s) + \frac{X(s)}{G(s)}\right] = G(s)R(s) + X(s)$$

The output in both cases are identical.

- **Rule 6: Shifting a summing point after a block**



Initial Configuration



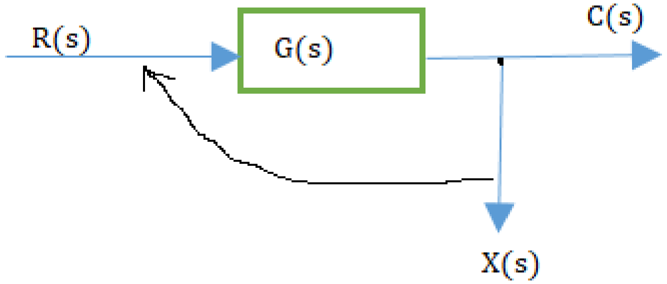
Final configuration

$$C(s) = G(s)[R(s) + X(s)]$$

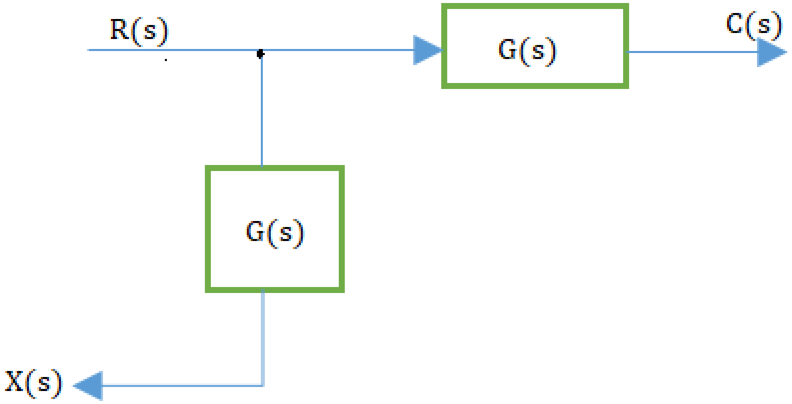
$$C(s) = R(s)G(s) + X(s)G(s)$$
$$= G(s)[R(s) + X(s)]$$

In both cases, the output is the same.

- **Rule 7: Shifting of take-off point before a block**



Before shifting the take-off point

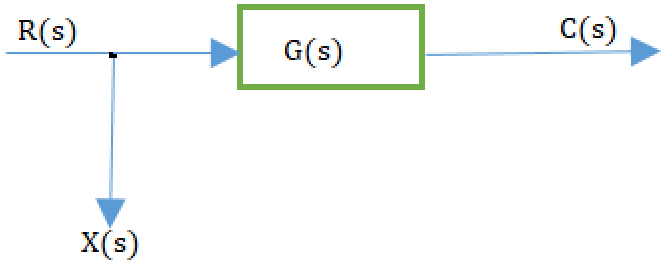


After shifting the take-off point

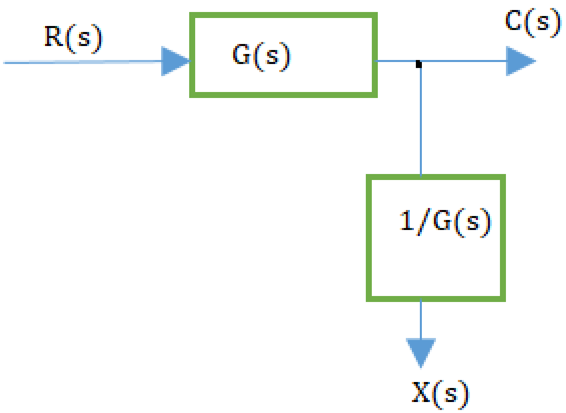
$$X(s) = C(s) = R(s)G(s)$$

$$X(s) = R(s)G(s)$$

- **Rule 8: Shifting of take-off point after a block**



Before shifting the take-off point

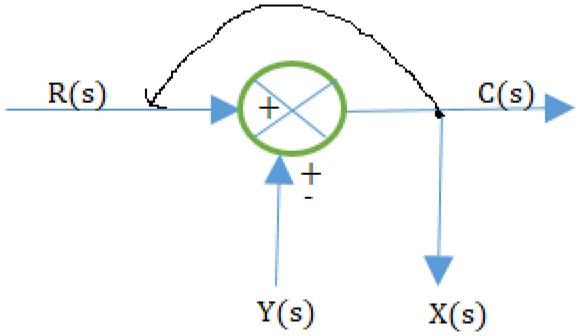


After shifting the take-off point

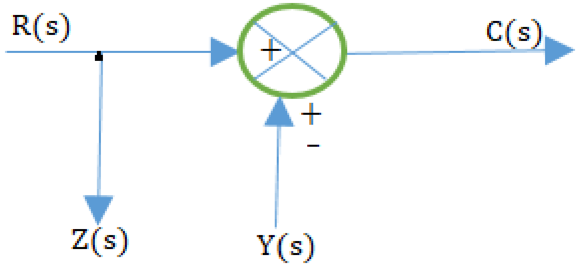
$$X(s) = R(s)$$

$$X(s) = \frac{1}{G(s)}.G(s).R(s) = R(s)$$

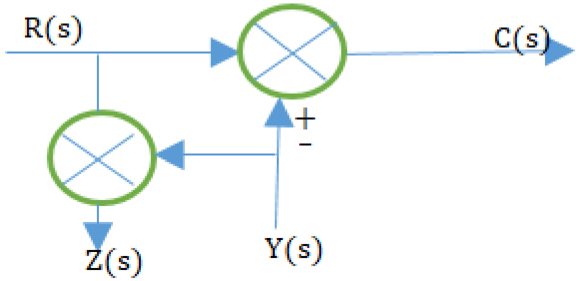
- **Rule 9: Shifting of take-off point before summing block**



Initial Situation



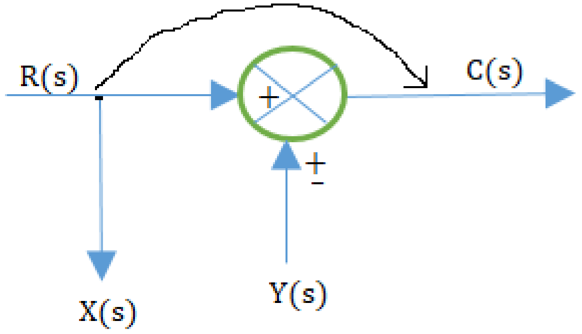
After shifting the take off point



Corrected configuration

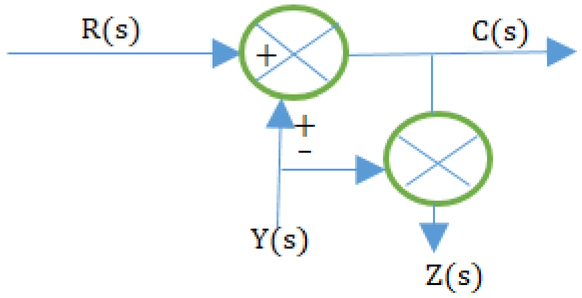
In both cases, $Z(s) = R(s) \pm Y(s)$

- **Rule 10: Shifting of take-off point after summing block**



Initial situation

$$X(s) = R(s)$$



Final configuration after shifting

$$Z(s) = R(s) \pm Y(s) \pm Y(s) = R(s)$$

Procedures for reduction of block diagram

Step 1: Reduce the cascaded blocks

Step 2: Reduce the parallel blocks

Step 3: Reduce the internal feedback loops

Step 4: It is advisable to shift take off points towards rights and summing points towards left. It is always better to avoid rule 9 and rule 10.

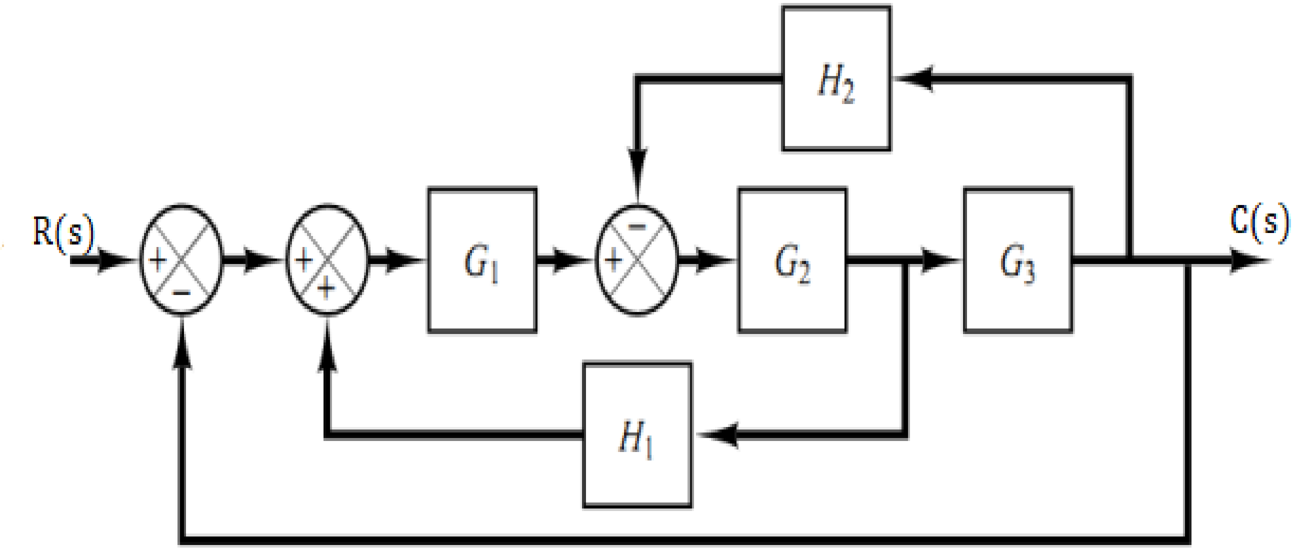
Procedures for reduction of block diagram (cont’d)

Step 5: Repeat steps 1 to 4 until the simple form is obtained

Step 6: Find the transfer function of the overall system using the formula $\frac{C(s)}{R(s)}$

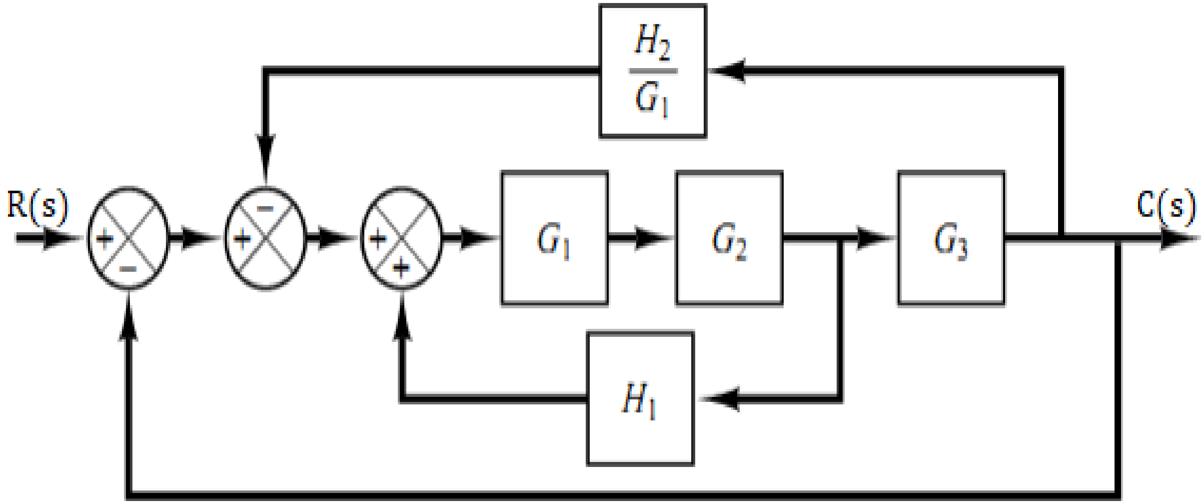
Solved Example

Consider the system shown in the following figure, simplify it and determine the ratio $\frac{C(s)}{R(s)}$.

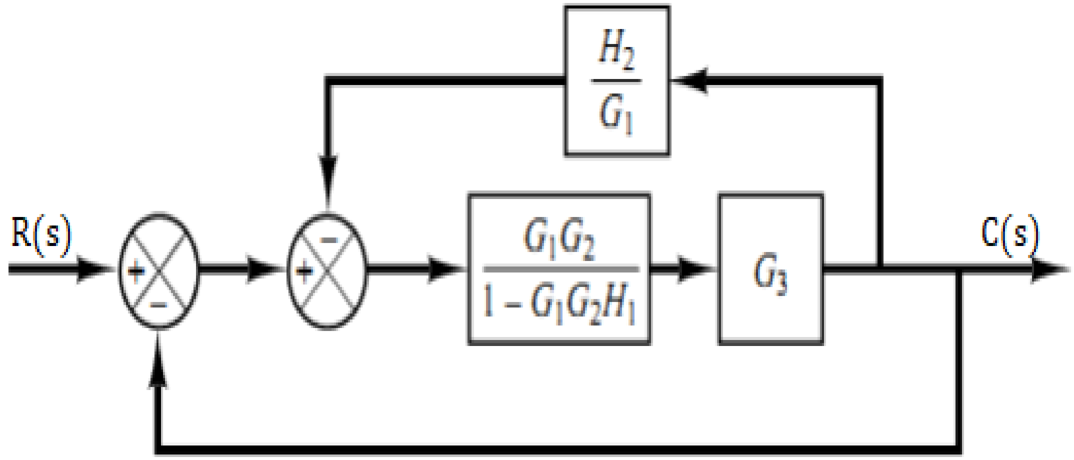


Solution:

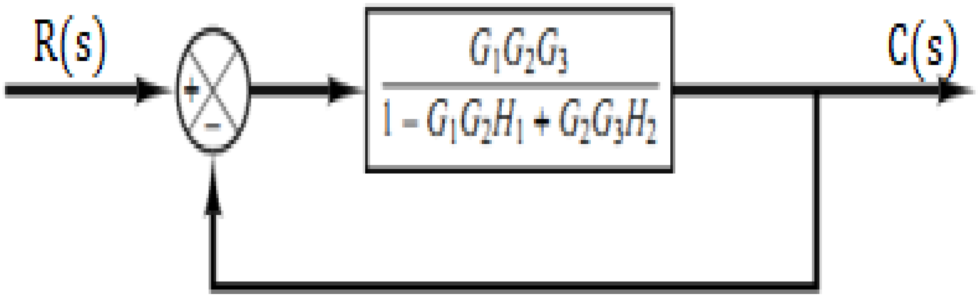
Step 1: Moving the summing point of H2 and G1 towards left, the figure becomes



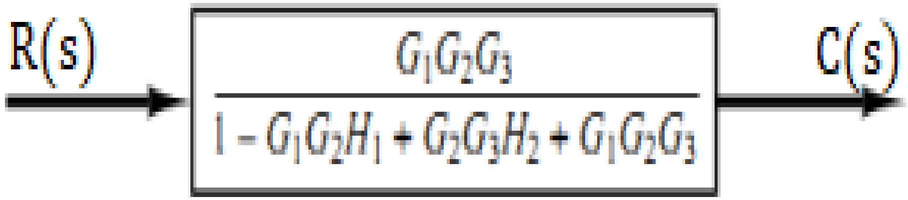
Step 2: By combining series connected blocks G1 and G2 and eliminate the feedback H1, we get



Step 3: By combining series connected blocks $\frac{G_1 G_2}{1-G_1 G_2 H_1}$ and G_3 and eliminate the feedback $\frac{H_2}{G_1}$,we get



Step 4: By eliminating the unit feedback, we get



The overall system transfer function

$$\frac{C(s)}{R(s)} = \frac{G_1G_2G_3}{1 - G_1G_2H_1 + G_2G_3H_2 + G_1G_2G_3}$$

References

1. Smarajit Gosh (2007), Control systems, Pearson Education
2. Manirakiza and Kanyarwanda (2020), ELT 303Lecture Note, IPRC Gishari, pages 26-31.
3. K. Ogata (1997), Modern Control Engineering, 3rd Edition, Prentice Hall.

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