

Econometrics

	Course Calendar
Week	Main Content
Week 7	Extension of Simple Regression: Functional Forms I
Week 8	Extension of Simple Regression: Functional Forms II
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Week 14	Introduction to Dummy Variables and Regression Methods
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Week 16	Application of Regression

Econometrics

Lecture 8. Extension of Regression: Functional Forms II

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Professor

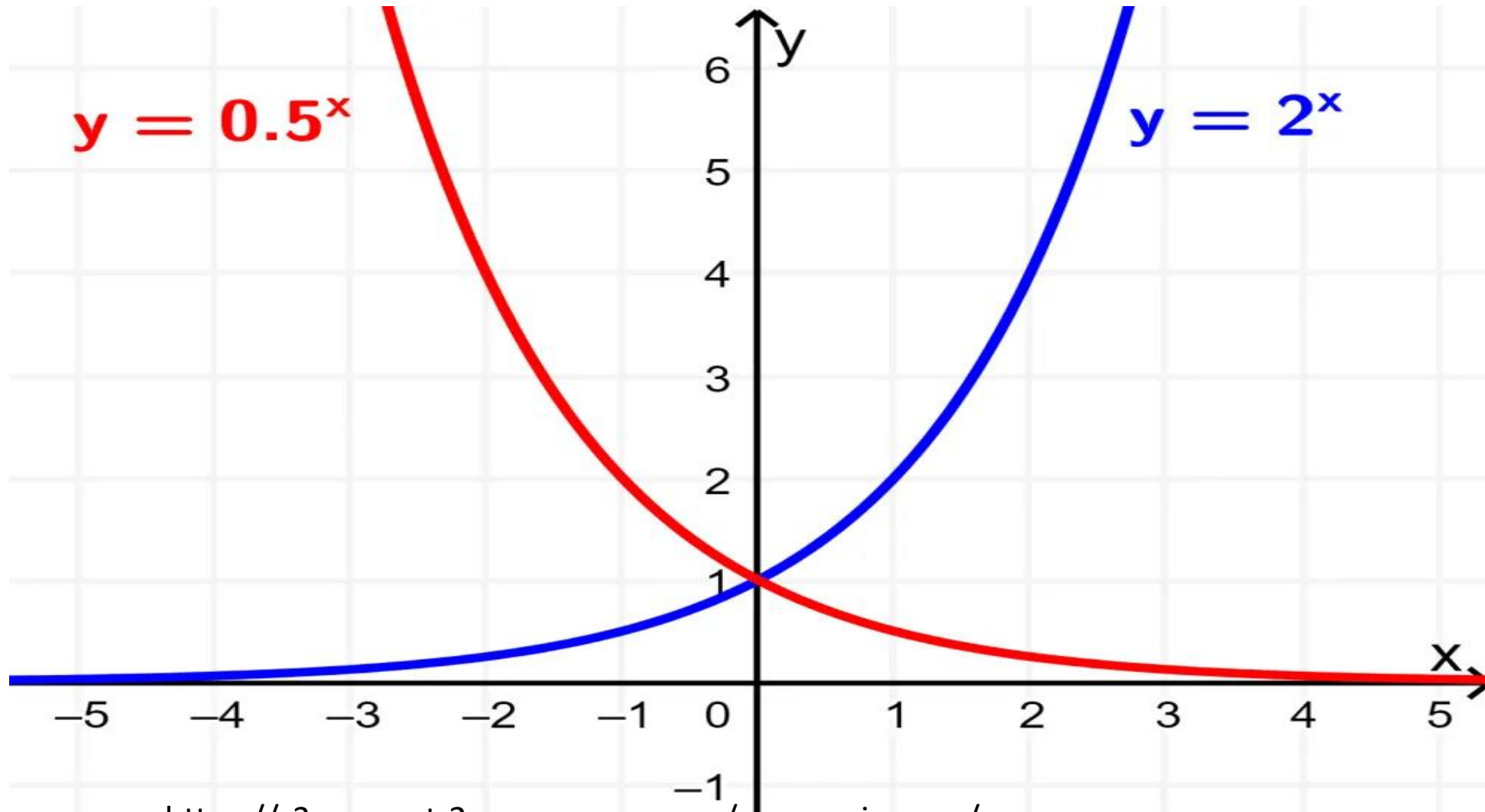
Recap

- Units of measurement
- **Regression on Standardized Variables**
- Functional forms of the linear regression model:- Regression through origin
- **Introduction to Different functional forms**

Outline

- **Logarithmic regression models**
- **Before that the Basics of Exponential functions**
- **Basics of Logarithmic transformations of variables**
- **Linear model for comparative purposes (learnt already)**
- **Semi-log model:-) lin-log model and log-lin model**

Functional Forms of Regression: Basics of Exponential Function



<https://s3-us-west-2.amazonaws.com/courses-images/wp-content/uploads/sites/1861/2017/06/23162116/0rwtncusqyjxn5rmgwee.png>

What is an exponential function?

- An example of an exponential function is the **growth of bacteria**.
- Some bacteria double every hour. If you start with 1 bacterium and it doubles every hour, you will have 2^x bacteria after x hours.
- This can be written as $f(x) = 2^x$.
- Some applications of exponential functions include population growth, compound interest, and radioactive decay.

Quadratic Functions vs. Exponential Functions

Quadratic function	Exponential function
$y = x^2$	$y = 2^x$
<p>This is a quadratic function because the base is a variable and the exponent is fixed.</p>	<p>This is an because the base is fixed and the exponent is a variable.</p>

An Exponential Function

- An exponential function is a function with the general form $y = ab^x$, $a \neq 0$, b is a positive real number and $b \neq 1$.
- In an exponential function, the base b is a constant. The exponent x is the independent variable where the domain is the set of real numbers.
- There are two types of exponential functions: **exponential growth** and **exponential decay**

In the function $f(x) = b^x$ when $b > 1$, the function represents **exponential growth**.

In the function $f(x) = b^x$ when $0 < b < 1$, the function represents **exponential decay**.

An Exponential Function: Example

$f(x)$	Growth / decay
1.) $f(x) = 5^x$ would represent exponential growth because	$ab > 1$. ($b = 5$)
2.) $f(x) = 0.83^x$ would represent exponential decay because	$0 < b < 1$. ($b = 0.83$)
3.) $f(x) = 0.5(3)^x$ would represent exponential growth because	$b > 1$. ($b = 3$)

An Exponential Function: Example

Example #1: Using a calculator and the exponential key (^), evaluate 4^x for $x = 0.5$.

Substitute the given value for x and evaluate.

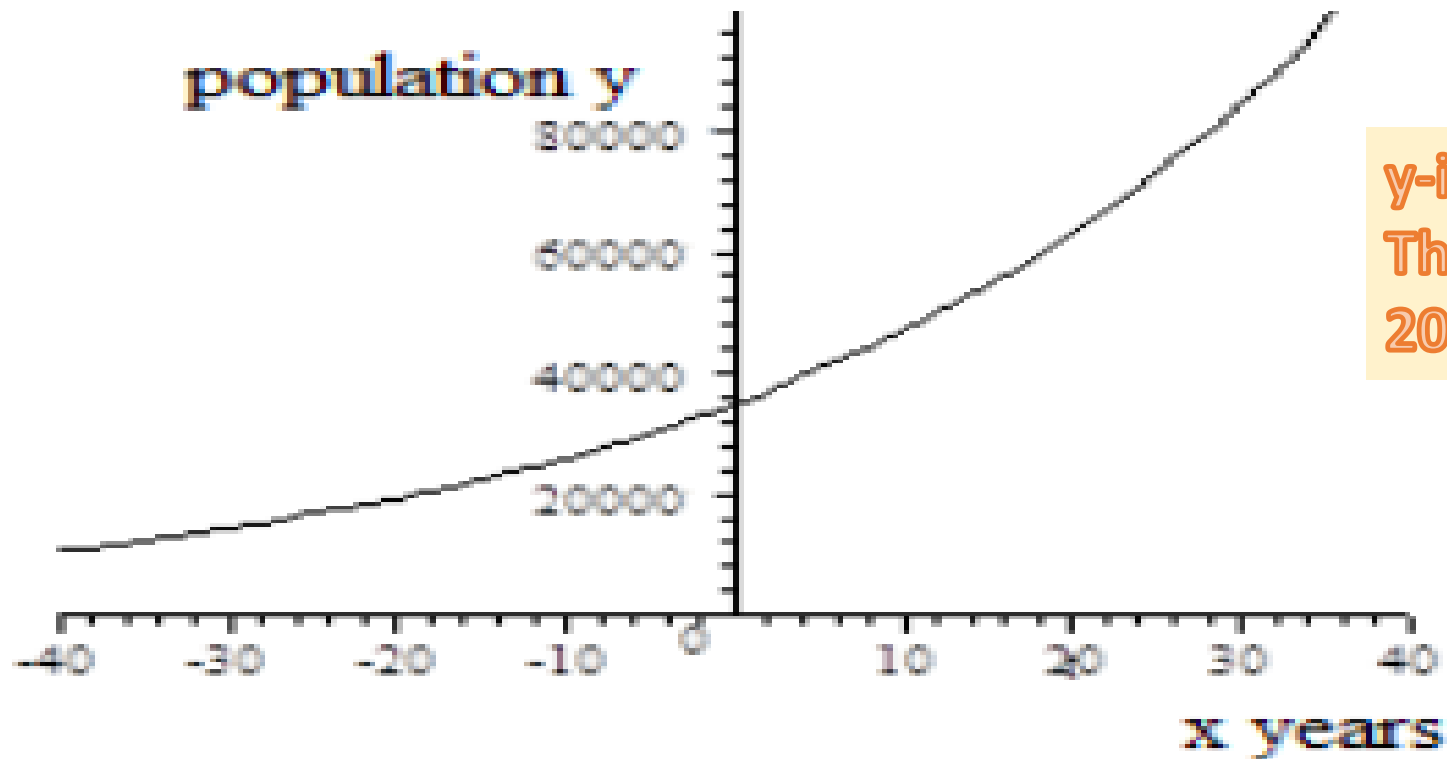
$$4^x = 4^{0.5} = 2$$

Example #2: Evaluate 5^x for $x = 4$.

Substitute the given value for x and evaluate.

$$5^x = 5^4 = 625$$

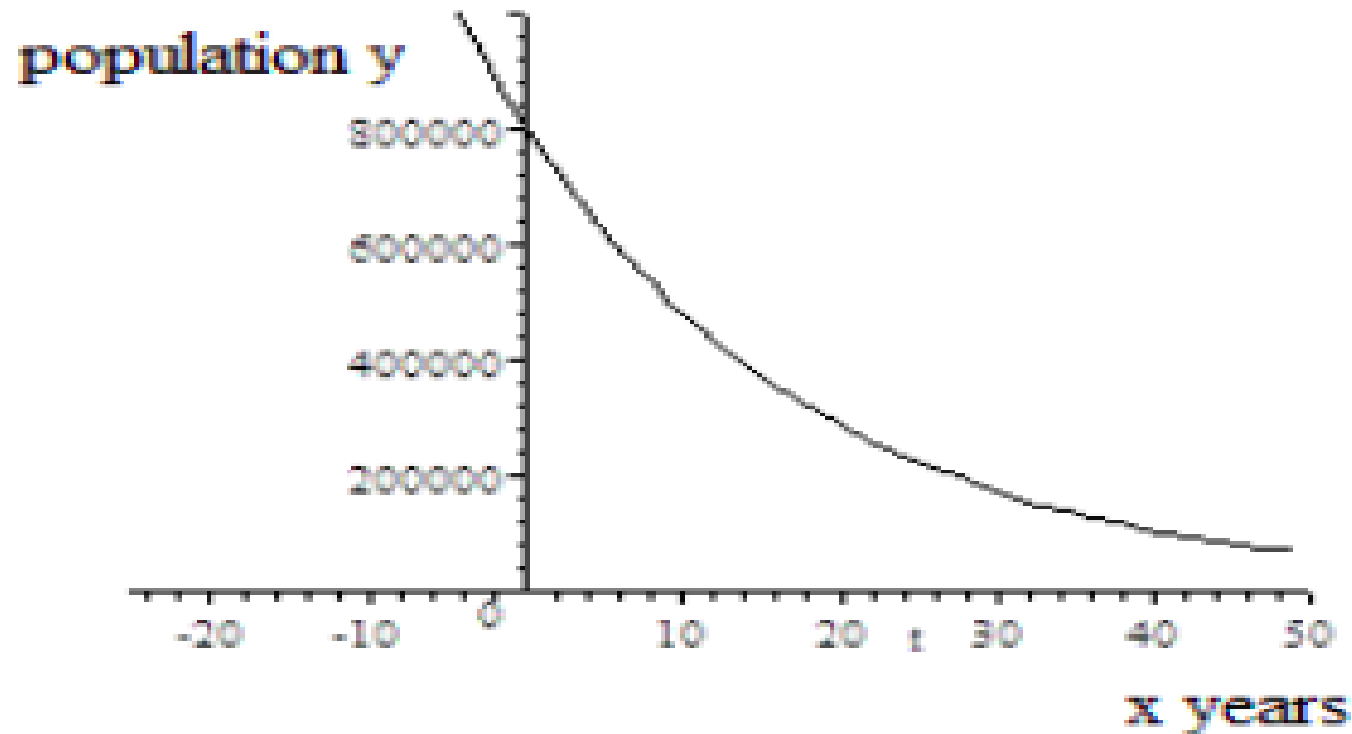
Exponential Growth



y-intercept: $(0, 35000)$
The initial population in 2015 is 35000.

Source: <https://math.libretexts.org/@api/deki/files/40280/Example5.3.1a.png?revision=1>

Exponential Decay

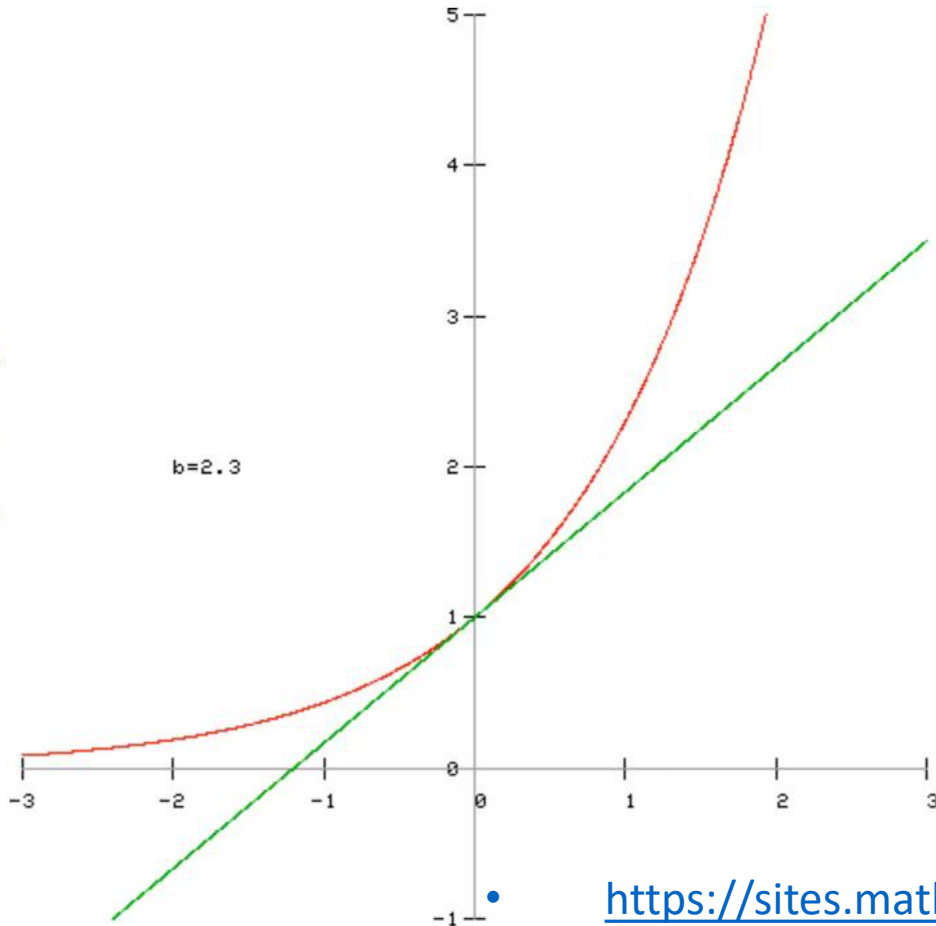


y-intercept: (0,800000)
The initial population
in 2015 is 800000.

Source: <https://math.libretexts.org/@api/deki/files/40281/Example5.3.1b.png?revision=1>

Here is an animation of $y=b^x$ for a range of b values.

The graph is shown along with a tangent line to the curve at the point $(0,1)$. Notice that the line has a slope of 1 somewhere around 2.7. In fact, the tangent line to $y=b^x$ at the point $(0,1)$ has a slope of 1 precisely when $b = 2.71828182845904523...$. This value is known as e .



<https://sites.math.washington.edu/~conroy/general/expoanim/expoanim.htm>

- Basics of Logarithmic transformations of variables

What is a logarithmic function?

- Logarithmic functions are the inverses of exponential functions.
- The inverse of the exponential function $y = a^x$ is $x = a^y$.
- The logarithmic function $y = \log_a x$ is defined to be equivalent to the exponential equation $x = a^y$.
- **In other words, a function (such as $y = \log_a x$ or $y = \ln x$) that is the inverse of an exponential function (such as $y = a^x$ or $y = e^x$) so that the independent variable appears in a logarithm.**

Basics of Logarithm

- Recall that we are using natural logarithms, where the base is $e \approx 2.71828$.
- Logarithms may have other bases, for instance the decimal logarithm of base 10.
- (The base 10 logarithm is used in the definition of the Richter scale, for instance, measuring the intensity of earthquakes as $\text{Richter} = \log(\text{intensity})$).
- This is why an earthquake of magnitude 9 is 100 times more powerful than an earthquake of magnitude 7: because $10^9/10^7 = 10^2$ and $\log_{10}(10^2) = 2$.)

Some properties of logarithms and exponential functions

1. $\log(e) = 1$

2. $\log(1) = 0$

3. $\log(x^r) = r \log(x)$

4. $\log e^A = A$

5. $e^{\log A} = A$

6. $\log(AB) = \log A + \log B$

7. $\log(A/B) = \log A - \log B$

8. $e^{AB} = (e^A)^B$

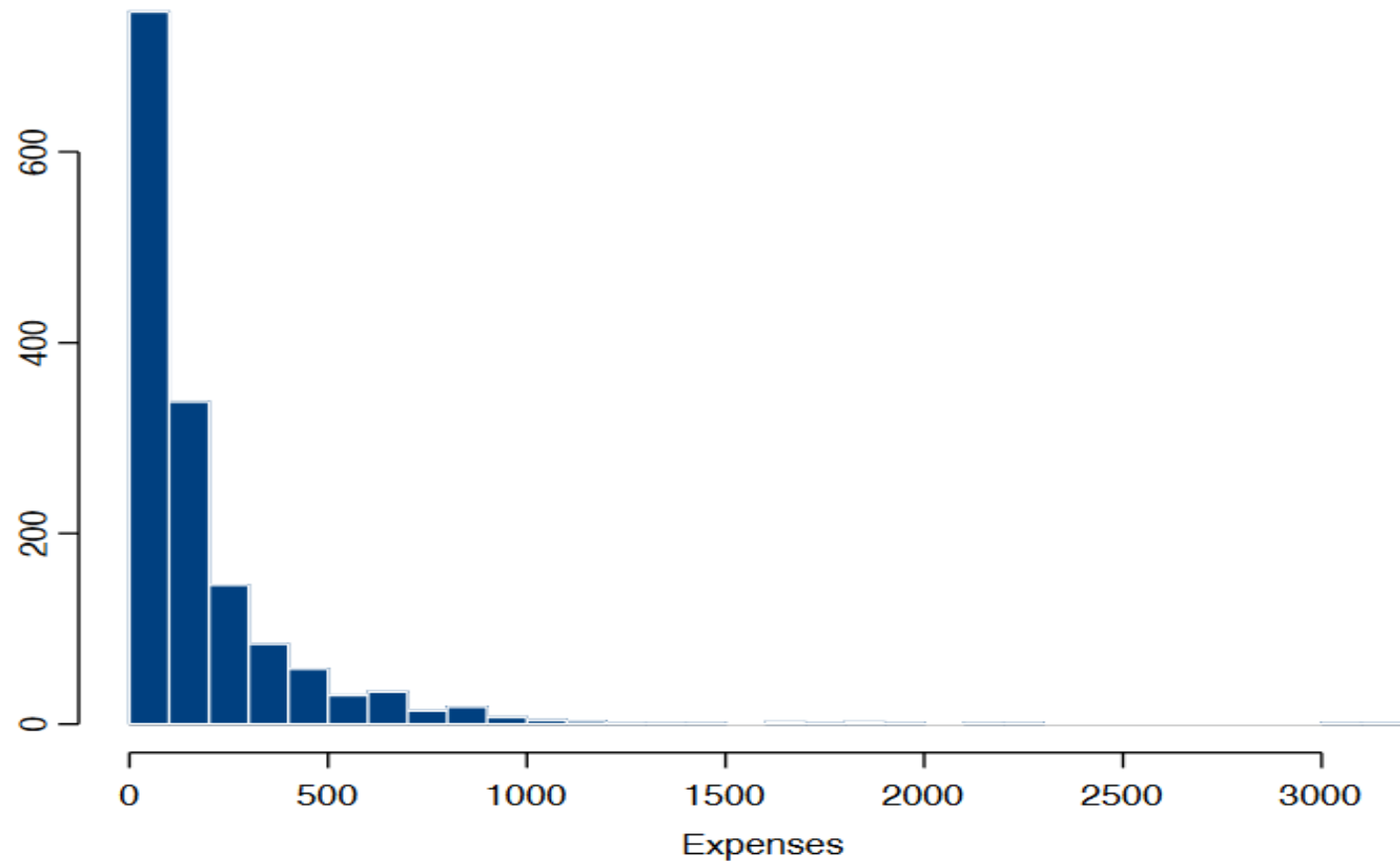
9. $e^{A+B} = e^A e^B$

10. $e^{A-B} = e^A / e^B$

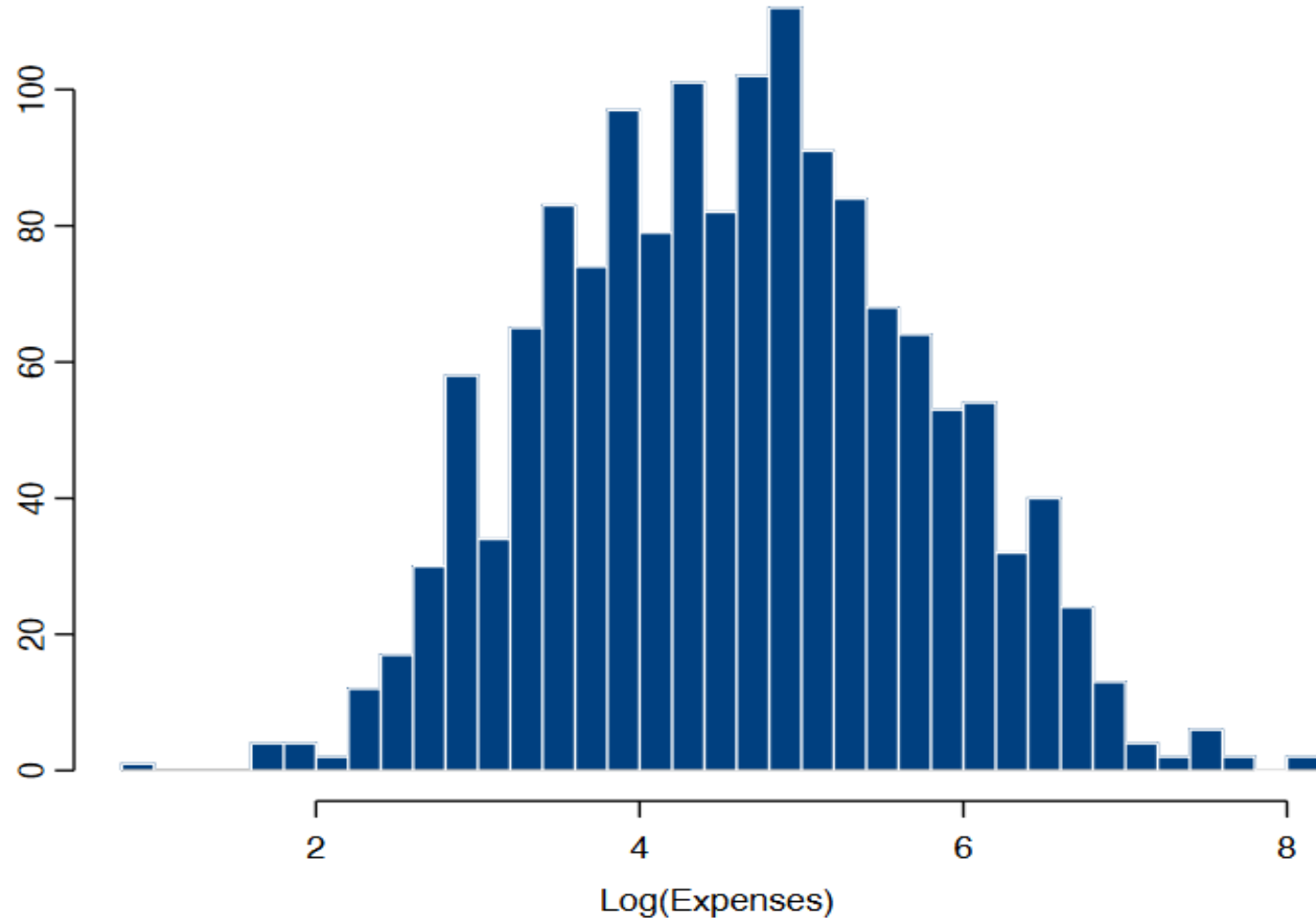
Why use logarithmic transformations of variables?

- Logarithmically transforming variables in a regression model is a very common way to handle situations where a non-linear relationship exists between the independent and dependent variables.
- Using the logarithm of one or more variables instead of the unlogged form makes the effective relationship non-linear, while still preserving the linear model.
- Logarithmic transformations are also a convenient means of transforming a highly skewed variable into one that is more approximately normal.
- (In fact, there is a distribution called the log-normal distribution defined as a distribution whose logarithm is normally distributed – but whose untransformed scale is skewed.)

For instance, if we plot the histogram of expenses, we see a significant right skew in this data, meaning the mass of cases are bunched at lower values:



If we plot the histogram of the logarithm of expenses, however, we see a distribution that looks much more like a normal distribution:



Logarithmic Regression Models

- Considering the simple bivariate linear model $Y_i = \alpha + \beta X_i + \varepsilon_i$, there are four possible combinations of transformations involving logarithms:
 - 1. the linear case with no transformations,
 - 2. the linear-log model,
 - 3. the log-linear model, and
 - 4. the log-log model or double log model.

Important terms to Remember

- Incidentally, the it is important to note these terms, which will occur frequently:
- **(1) absolute change,**
- **(2) relative or proportional change, and**
- **(3) percentage change, or percent growth rate.**
- Thus, $(X_t - X_{t-1})$ represents absolute change,
- $(X_t - X_{t-1})/X_{t-1} = (X_t/X_{t-1} - 1)$ is relative or proportional change and
- $[(X_t - X_{t-1})/X_{t-1}]100$ is the percentage change, or the growth rate.
- X_t and X_{t-1} are, respectively, the current and previous values of the variable X .

1: Linear model: comparative purpose

- $Y_i = \alpha + \beta X_i + \varepsilon_i$
- In this linear regression model, $Y_i = \alpha + \beta X_i + \varepsilon_i$, the coefficient β gives us directly the change in Y for a one-unit change in X .
- $\Delta Y / \Delta X = \text{slope}$;
- No additional interpretation is required beyond the estimate $\hat{\beta}$ of the coefficient itself.
- This same interpretation will still hold when variables have been logarithmically transformed, but it usually makes it more meaningful to interpret the changes **not in log-units but rather in percentage changes.**

2. Linear-log model:

- If we keep our dependent variable (Y) in their original scale and use natural log values for independent variable (X), the econometric specification is called a *linear-log model*.
- $Y_i = \alpha + \beta \log X_i + \varepsilon_i$
- In the linear-log model, the literal interpretation of the estimated coefficient $\hat{\beta}$ is that a one-unit increase in $\log X$ will produce an expected increase in Y of $\hat{\beta}$ units.
- Let us interpret the slope coefficient β^\wedge .

2. Linear-log model:

- As usual,
- $\beta^{\wedge} = \text{change in } Y / \text{change in } \ln X$
- $\beta^{\wedge} = \text{change in } Y / \text{relative change in } X$
- The second step follows from the fact that a change in the log of a number is a relative change.
- Please recall that $(X_t - X_{t-1})/X_{t-1} = (X_t/X_{t-1} - 1)$ is relative or proportional change.
- Symbolically, we have
- $\beta^{\wedge} \Delta Y / \Delta X / X = (6.6.12)$
- where, Δ denotes a small change.

2. Linear-log model:

- Equation (6.6.12) can be written, equivalently, as
- $Y = \beta_2(\Delta X/X)$ (6.6.13)
- This equation states that the absolute change in Y ($= \Delta Y$) is equal to slope times the relative change in X .
- If the latter is multiplied by 100, then (6.6.13) gives the absolute change in Y for a percentage change in X .
- Thus, if $(\Delta X/X)$ changes by 0.01 unit (or 1 percent), the absolute change in Y is $0.01(\beta_2)$; if in an application one finds that $\beta_2 = 500$, the absolute change in Y is $(0.01)(500) = 5.0$.

2.The lin-log models

- **The lin-log model:**
- $Y_i = \beta_1 + \beta_2 \ln X_i + u_i$ (6.5.11)
- $\beta_2 = (\text{Change in } Y) / \text{Change in } \ln X$
- $= (\text{Change in } Y) / (\text{Relative change in } X) \sim (\Delta Y) / (\Delta X / X)$
(6.5.12)
- **or $\Delta Y = \beta_2 (\Delta X / X)$** (6.5.13)
- **That is, the absolute change in Y equal to β_2 times the relative change in X.**

2. Linear-log model:

- Therefore, when regression (6.6.11) is estimated by OLS, do not forget to multiply the value of the estimated slope coefficient by 0.01, or, what amounts to the same thing, divide it by 100.
- If you do not keep this in mind, your interpretation in an application will be highly misleading.
- The practical question is: When is a lin–log model like (6.6.11) useful?
- An interesting application has been found in the Engel expenditure models, named after the German statistician Ernst Engel, 1821–1896. (See exercise 6.10.)
- Engel postulated that “the total expenditure that is devoted to food tends to increase in arithmetic progression as total expenditure increases in geometric progression.”

An Illustrative Example: Lin-log Model

- Table 2.8 gives data on expenditure on food and total expenditure, measured in rupees, for a sample of 55 rural households from India. (In early 2000, a U.S. dollar was about 40 Indian rupees.)

Table 2.8: Food and Total Expenditure (Rupees)

Observation	Food exp.	Total exp.	Observation	Food exp.	Total exp.
1	217	382	29	390	655
2	196	388	30	385	662
3	303	391	31	470	663
4	270	415	32	322	677
5	325	456	33	540	680
6	260	460	34	433	690
7	300	472	35	295	695
8	325	478	36	340	695
9	336	494	37	500	695
10	345	516	38	450	720
11	325	525	39	415	721
12	362	554	40	540	730
13	315	575	41	360	731
14	355	579	42	450	733

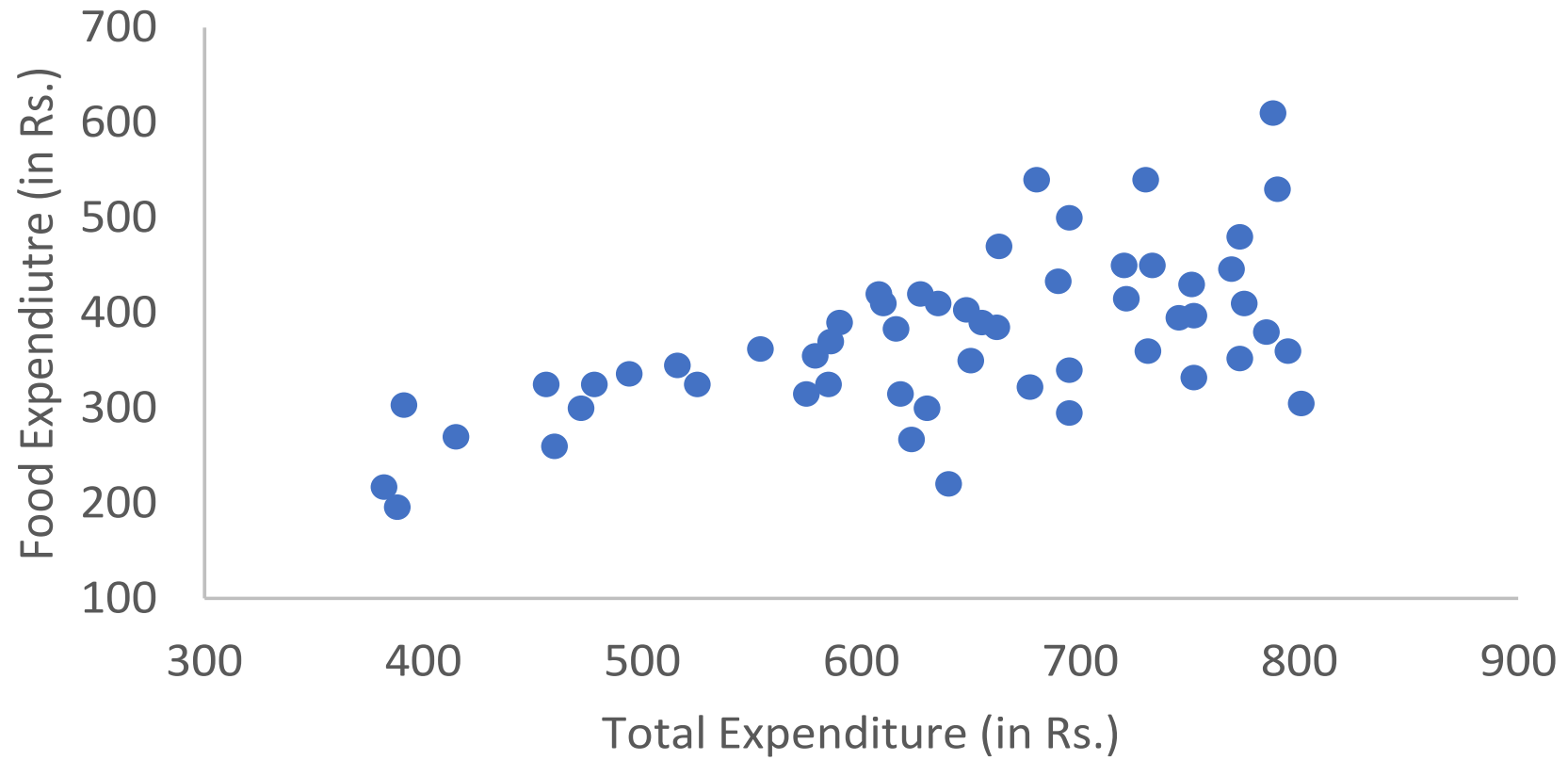
Table 2.8: Food and Total Expenditure (Rupees)

Observation	Food exp.	Total exp.	Observation	Food exp.	Total exp.
15	325	585	43	395	745
16	370	586	44	430	751
17	390	590	45	332	752
18	420	608	46	397	752
19	410	610	47	446	769
20	383	616	48	480	773
21	315	618	49	352	773
22	267	623	50	410	775
23	420	627	51	380	785
24	300	630	52	610	788
25	410	635	53	530	790
26	220	640	54	360	795
27	403	648	55	305	801
28	350	650			

An Illustrative Example: Lin-log Model

- If we plot the data given in Table 2.8 we obtain the plot in Figure 6.5.
- As this figure suggests, food expenditure increases more slowly as total expenditure increases, perhaps giving credence to Engel's law.

Scatter Plot of Food expenditure and Total Food Expenditure



An Illustrative Example: Lin-log Model

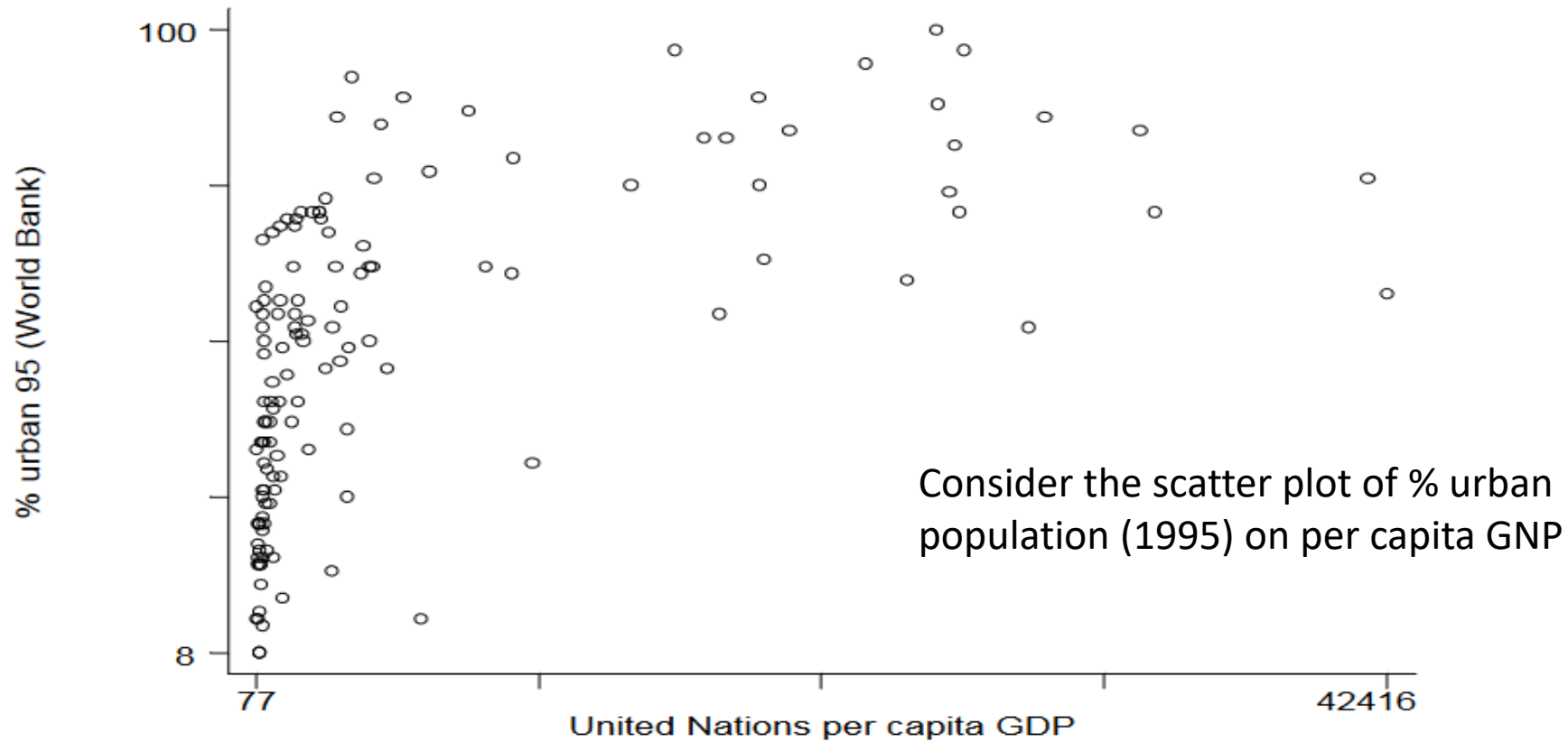
- The results of fitting the lin-log model to the data are as follows:

$$\widehat{\text{FoodExp}}_i = -1283.912 + 257.2700 \ln \text{TotalExp}_i$$
$$t = \quad (-4.3848)^* \quad (5.6625)^* \quad r^2 = 0.3769$$

(6.6.14)

- Note:
- Interpreted in the manner described earlier, the slope coefficient of about 257 means that an increase in the total food expenditure of 1 percent, on average, leads to about 2.57 rupees increase in the expenditure on food of the 55 families included in the sample. (Note: We have divided the estimated slope coefficient by 100.)

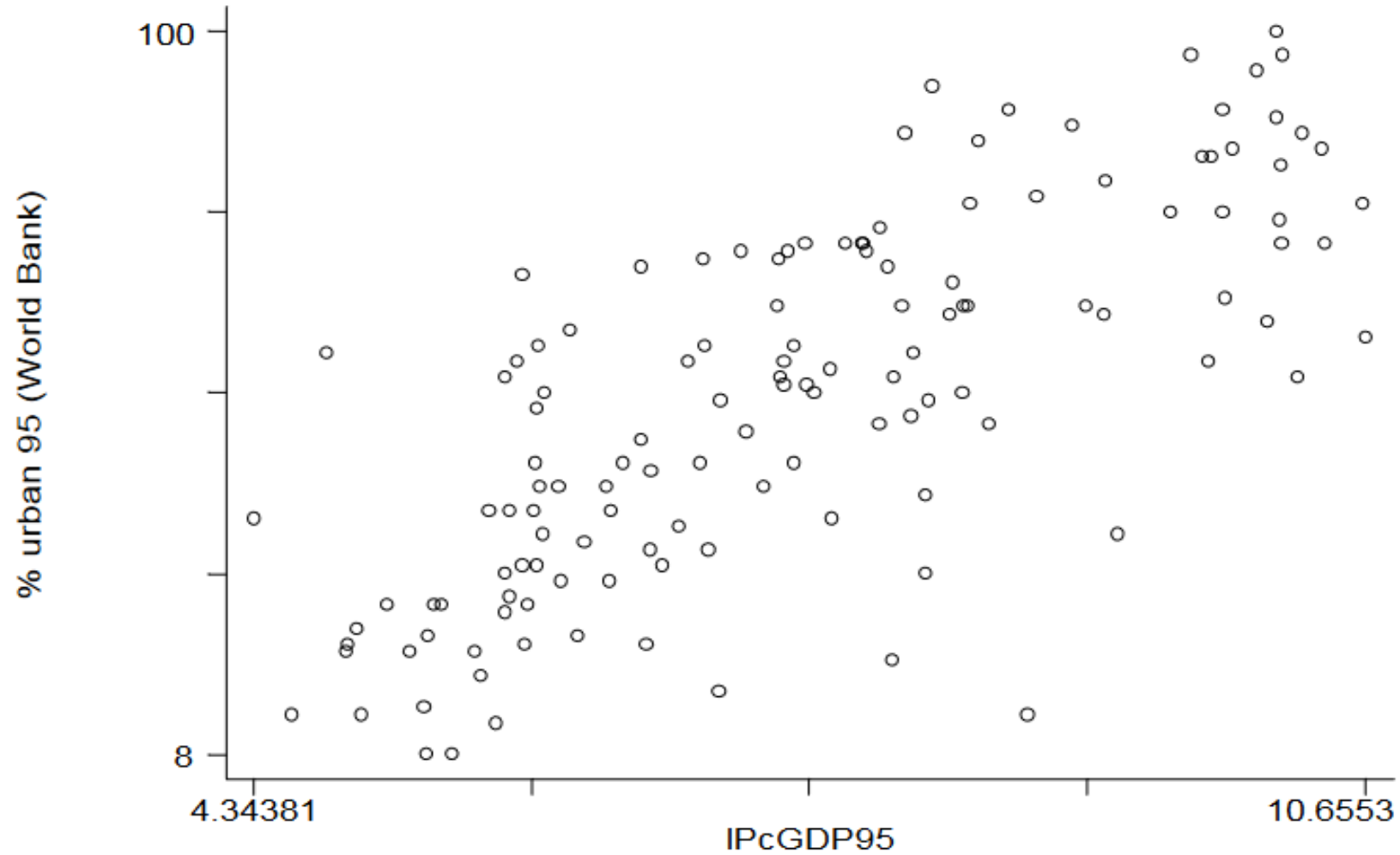
2. Linear log: One more Ex.



2. Linear log regression: Example

- The distribution of GDP is badly skewed, creating a non-linear relationship between X and Y .
- To control the skew and counter problems in heteroskedasticity, we transform GDP per capita by taking its log.
- This produces the following graph.

2. Linear log regression: Example



<https://kenbenoit.net/assets/courses/ME104/logmodels2.pdf>

3. Log linear model:

Measure the Growth Rate



<https://vazbalegal.cz/wp-content/uploads/Untitled-design-43.png>

3. Log linear model:

- If we use natural log values for dependent variable (Y) and keep our independent variables (X) in their original scale, the econometric specification is called a *log-linear model*.
- These models are used when we think the variables may have an exponential growth relationship.
- For example, if we put some cash in a saving account, we expect to see the effect of compounding interest with an exponential growth of our money!
- The original model in these types of scenarios isn't linear in parameters, but a log transformation generates the desired linearity.

3. Log linear model:

- Consider the following model of value in a savings fund that depends on initial investment, then return, and the length of time in which the funds are invested:
- $Y_t = Y_0(1 + r)^t$, where Y_t represents the value of the fund at time t , Y_0 is the initial investment in the savings fund, and r is the growth rate.
- Labor economists are also interested in similar functions because individuals usually have some initial earning power that can be supplemented with investments in skill acquisition.
- These *human capital functions* deal with the amount of money an individual can expect to earn depending on his or her initial abilities and investments in education, training, experience, and so on.

3. Log-linear Models

- Also, Economists, businesspeople, and governments are often interested in finding out the rate of growth of certain economic variables, such as population, GNP, money supply, employment, productivity, and trade deficit.
- Application of log-lin model: well-known compound interest formula
- $Y_t = Y_0(1 + r)^t$ (6.6.1)
- where r is the compound (i.e., over time) rate of growth of Y .

Application of log-lin model:

- well-known compound interest formula $Y_t = Y_0(1 + r)^t$ (6.6.1)
- where r is the compound (i.e., over time) rate of growth of Y ,
- $Y_t = Y$ in current period;
- $Y_0 = Y$ at the initial period,
- t refers to time period.
- Ex: $Y_0 = 5000$; t - two years; $r = 10\%$
- Applying the values in the formula given in 6.6.1. will result in = 5000
 $(1 + 10/100)^2 = 5000 * (1.1)^2$
 $= 5000 * (1.21) = 6050.$

3. Log-lin Models: Application

- Taking the natural logarithm of (6.6.1), we can write:
- $\ln Y_t = \ln Y_0 + t \ln (1 + r)$ (6.6.2)
- Now letting
- $\beta_1 = \ln Y_0$ (6.6.3)
- $\beta_2 = \ln (1 + r)$ (6.6.4)
- we can write (6.6.2) as
- $\ln Y_t = \beta_1 + \beta_2 t$ (6.6.5)
- Adding the disturbance term to (6.6.5), we obtain
- $\ln Y_t = \beta_1 + \beta_2 t + u_t$ (6.6.6)
- We add the error term because the compound interest formula will not hold exactly.

3. Log-lin Models: Application

- This model is like any other linear regression model in that the parameters β_1 and β_2 are linear.
- The only difference is that the regressand is the logarithm of Y and the regressor is “time,” which will take values of 1, 2, 3, etc.
- Let us examine the properties of model
- In this model the slope coefficient measures the constant proportional or relative change in Y for a given absolute change in the value of the regressor (in this case the variable t), i.e.,

3. Log-lin Models

- $\beta_2 = \text{relative change in } Y \text{ regressand} / \text{absolute change in regressor } X$
(6.6.7)
- $\beta_2 = \Delta Y / Y / \Delta X$
- $(Y_t - Y_{t-1}) / Y_{t-1} = (Y_t / Y_{t-1} - 1)$ is relative or proportional change
- If we multiply the relative change in Y by 100, (6.6.7) will then give the percentage change, or the growth rate, in Y for an absolute change in X , the regressor.
- That is, 100 times β_2 gives the growth rate in Y ; 100 times β_2 is known in the literature as the semielasticity of Y with respect to X .

An Illustrative Example: The Rate of Growth Expenditure on Services

- To illustrate the growth model (6.6.6), consider the data on expenditure on services given in Table 6.3.
- Table 6.3 presents data on total personal consumption expenditure (PCEXP), expenditure on durable goods (EXPDUR), expenditure on nondurable goods (EXPNONDUR), and expenditure on services (EXPSERVICES), all measured in 1992 billions of dollars.

Table 6.3: Expenditure on Durable Goods in Relation to Total Personal Consumption Expenditure

Observation	EXPSERVICES	EXPNONDUR	EXPDUR	PCONSEXP
1993-I	2445.3	1337.5	504	4286.8
1993-II	2455.9	1347.8	519.3	4322.8
1993-III	2480	1356.8	529.9	4366.6
1993-IV	2494.4	1361.8	542.1	4398
1994-I	2510.9	1378.4	550.7	4439.4
1994-II	2531.4	1385.5	558.8	4472.2
1994-III	2543.8	1393.2	561.7	4498.2
1994-IV	2555.9	1402.5	576.6	4534.1
1995-I	2570.4	1410.4	575.2	4555.3
1995-II	2594.8	1415.9	583.5	4593.6
1995-III	2610.3	1418.5	595.3	4623.4
1995-IV	2622.9	1425.6	602.4	4650

Table 6.3: Expenditure on Durable Goods in Relation to Total Personal Consumption Expenditure

Observation	EXPSERVICES	EXPNONDUR	EXPDUR	PCONSEXP
1996-I	2648.5	1433.5	611	4692.1
1996-II	2668.4	1450.4	629.5	4746.6
1996-III	2688.1	1454.7	626.5	4768.3
1996-IV	2701.7	1465.1	637.5	4802.6
1997-I	2722.1	1477.9	656.3	4853.4
1997-II	2743.6	1477.1	653.8	4872.7
1997-III	2775.4	1495.7	679.6	4947
1997-IV	2804.8	1494.3	648.8	4981
1998-I	2829.3	1521.2	710.3	5055.1
1998-II	2866.8	1540.9	729.4	5130.2
1998-III	2904.8	1549.1	733.7	5181.8

Source: Economic Report of the President, 1999, Table B-17, p. 347
 taken from Basic Econometrics by Domadar Gujrati, page, 178

An Illustrative Example: The Rate of Growth Expenditure on Services

- The regression results are as follows:

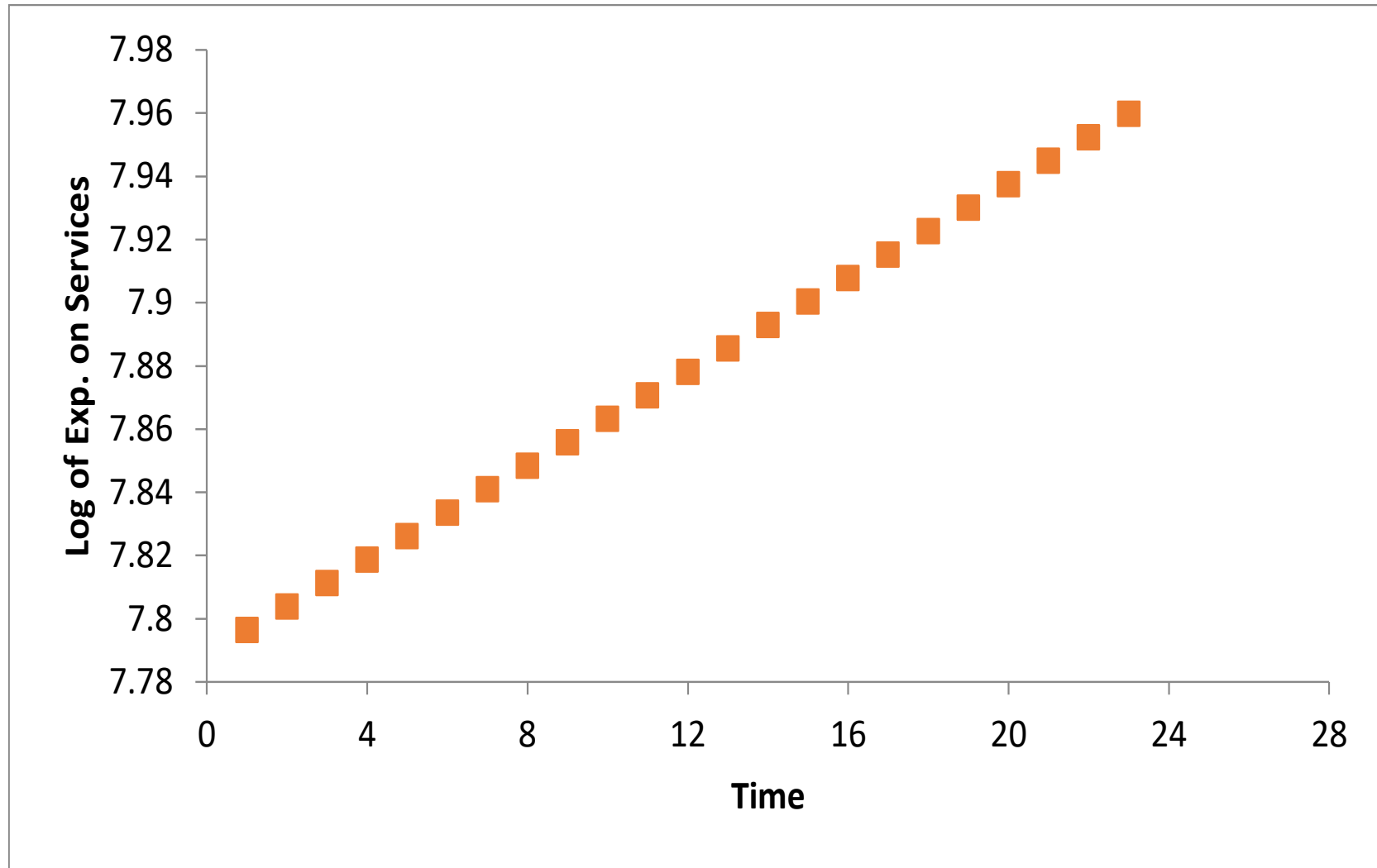
$$\begin{aligned} \widehat{\ln EXS}_t &= 7.7890 + 0.00743t \\ \text{se} &= (0.0023) \quad (0.00017) \quad (6.6.8) \\ t &= (3387.619)^* \quad (44.2826)^* \quad r^2 = 0.9894 \end{aligned}$$

- Note: EXS stands for expenditure on services and * denotes that the p value is extremely small. The interpretation of Eq. (6.6.8) is that over the quarterly period 1993:1 to 1998:3, expenditure on services increased at the (quarterly) rate of 0.743 percent. Roughly, this is equal to an annual growth rate of 2.97 percent.

An Illustrative Example: The Rate of Growth Expenditure on Services

- Since $7.7890 = \log$ of EXS at the beginning of the study period, by taking its antilog we obtain 2413.90 (billion dollars) as the beginning value of EXS (i.e., the value at the end of the fourth quarter of 1992). The regression line obtained in Eq. (6.6.8) is sketched in Figure 6.4

FIG. 6.4: Regression Line using 6.6.8



3. Log-linear Models

- Economic Application **of the log-lin model**: measure the growth rate.
- Basically, the growth rate is a metric - to express the annual change in a variable, represented as a percentage.
- For example, an economy's growth rate is the annual rate change of the country's GDP—increase or decrease. This rate represents the economy's expansion or recession, respectively.
- $Y_t = Y_0 (1+r)^t$ (6.5.1)
- $\ln Y_t = \ln Y_0 + t \ln(1+r)$ (6.5.2)
- $\ln Y_t = \beta_1 + \beta_2 t$, called constant growth model (6.5.5),
- where $\beta_1 = \ln Y_0$; $\beta_2 = \ln(1+r)$

To Remember in 3.Log linear model:



A generic exponential growth function can be written as $Y = Y_0(1 + r)^X$, where the value of Y for a given X can be derived only if the growth rate (r) is known. The growth rate can be estimated, but a log transformation must be used to estimate using OLS with an error term.


Instantaneous Vs. compound rate of growth

- The coefficient of the trend variable in the growth model $\ln Y_t = \beta_1 + \beta_2 t + u_t$ as in (6.6.6), β_2 , gives the instantaneous (at a point in time) rate of growth and not the compound (over a period of time) rate of growth.
- But the latter can be easily found from $\beta_2 = \ln(1 + r)$ as in (6.6.4) by taking the antilog of the estimated β_2 and subtracting 1 from it and multiplying the difference by 100.
- For example, the estimated slope coefficient is 0.00743.

Instantaneous Vs. compound rate of growth

- Therefore, $[\text{antilog}(0.00743) - 1] = 0.00746$ or 0.746 percent.
- Thus, in the illustrative example, the compound rate of growth on expenditure on services was about 0.746 percent per quarter, which is slightly higher than the instantaneous growth rate of 0.743 percent.
- This is of course due to the compounding effect.
- β_2 is **instantaneous rate of growth**: The instantaneous rate of growth represents the fraction by which a population would grow during a very short period of time.
- $\text{antilog}(\beta_2) - 1$ is **compound rate of growth**: the mean annual growth rate of an investment over a specified period of time longer than one year.

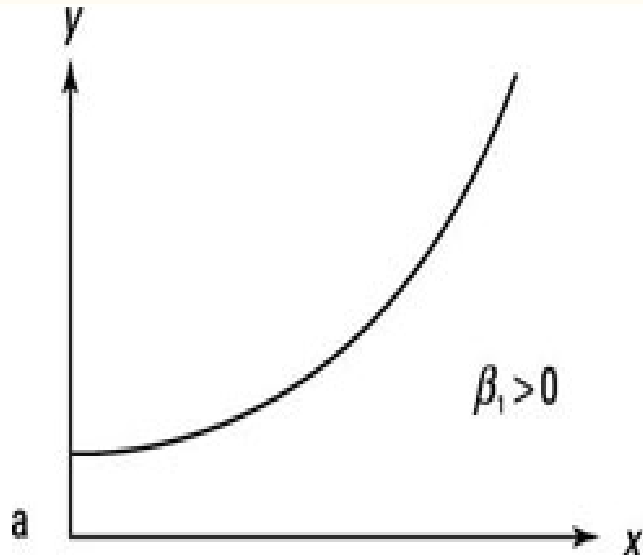
3. Log linear Model

- If we estimate a log-linear regression, a couple outcomes for the coefficient on X produce the most likely relationships:
- (i) $\beta_1 > 0$ and (ii) $\beta_1 < 0$
- This is shown in the next slide with a graph:
- 
- Regression coefficients in a log-linear model don't represent the slope.

3. Log linear Model

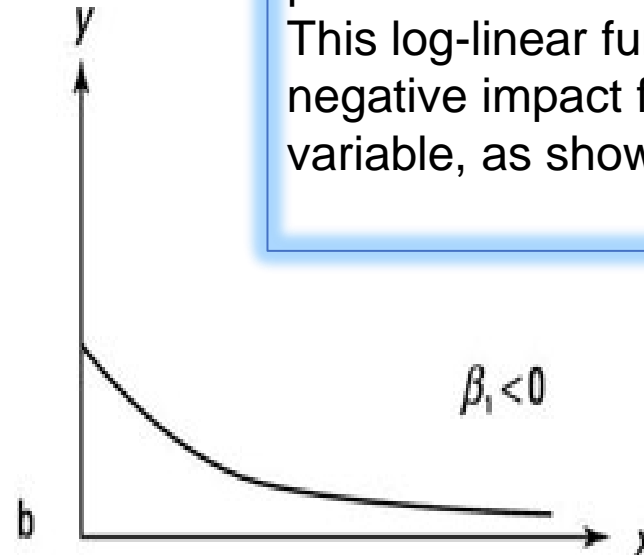
$$\beta_1 > 0$$

This log-linear function illustrates a positive impact from the independent variable, as shown in part (a).



$$\beta_1 < 0$$

This log-linear function depicts a negative impact from the independent variable, as shown in part (b).



<https://www.dummies.com/wp-content/uploads/415143.image10.jpg>

Summary and conclusions

- Basics of exponential and logarithmic functions.
- Important terms to remember: **(1) absolute change; (2) relative or proportional change, and (3) percentage change, or percent growth rate.**
- Important is the functional form of the relationship between the regressand and the regressor(s).
- Some of the important functional forms discussed in this Lecutre are **(a) semi-log model:-lin-log regression and log-lin regression models.**

Summary and conclusions

- In the log-lin model where the dependent variable (Y regressand) is logarithmic and the regressor X is time, the estimated slope coefficient (multiplied by 100) measures the (instantaneous) rate of growth of the regressand.
- These log-lin models are often used to measure the growth rate of many economic phenomena such as GDP, Money supply, etc.
- In the lin-log model where the regressor is logarithmic, its coefficient measures the absolute rate of change in the regressand for a given percent change in the value of the regressor.
- Slope measures = $\Delta Y / Y / \Delta X$ = relative change in Y / absolute change in X

Basic Econometrics

Chapter 6: EXTENSIONS OF THE TWO-VARIABLE LINEAR REGRESSION MODEL in Basic Econometrics by Domodar Gujarati.

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What next?

- Double log model
- Reciprocal Models
- Log Hyperbola or Logarithmic Reciprocal Model
- Choice of Functional Form