Course Title:

Fundamental of Thermodynamics and Heat Transfer

Lecture 11 (Week 11):

Second Law of Thermodynamics

Lecturer: Assoc. Prof. Dr. Lila Raj Koirala

Learning Objective of Lecture:

To impart a great deal of knowledge to undergraduate students on the topics:

- ✓ Control Mass Formulation of Second Law of Thermodynamics
- ✓ Control Volume Formulation of Second Law of Thermodynamics
- ✓ Isentropic Process
- ✓ Isentropic Relations for an Ideal Gas
- ✓ Isentropic Relations for an Incompressible Substance
- ✓ Isentropic Efficiency of Thermodynamic Devices (e.g. turbine, compressor, pump, nozzle) with Steady - Flow of Fluid

5.4. Control Mass Formulation of Second Law of Thermodynamics

A control mass or closed system can interact with its surroundings in terms of energy transfer only i.e., by heat transfer and work transfer. Therefore, the second law of thermodynamics for a control mass gives the expression for the change in entropy of the control mass because of effects of heat transfer and work transfer on entropy of the closed system. Hence for control mass formulation of second of thermodynamics, at first the contributions of heat transfer and work transfer on the entropy of the control mass are separately found out and then the general expression as well as statement for the second law of thermodynamics for a control mass is developed.

As control mass exchanges energy with the surroundings by heat and work transfer, two types of idealized subsystems of the surroundings are conceived namely a reversible heat transfer reservoir and a reversible work transfer reservoir. Both reservoirs are reversible indicating that they proceed slowly through series of equilibrium states and can be restored to the original state from any intermediate state in the process. Both reservoirs are control masses and are termed as *reservoir* to indicate a very large system having infinite heat or work capacity so that its property is unchanging during heat or work transfer. Since only the change in entropy of control mass is desired, surroundings are idealized and specified. So, the only unspecified entropy changes are those associated with the control mass [1].

As we are already familiar with the fact that the change in entropy of an isolated system is greater than or equal to zero, it is appropriate to begin with an example of an isolated system. For this, let us consider an isolated system as shown in figure 5.7 consisting of a control mass interacting with multiple heat transfer reservoirs and work transfer reservoirs.

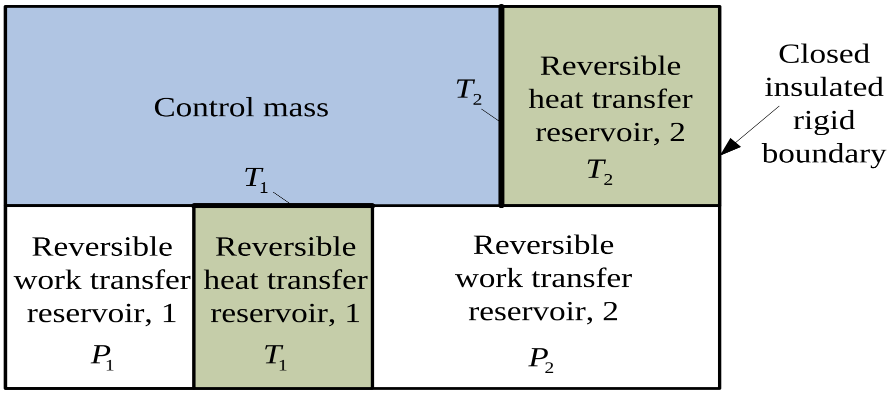

Figure 5.7. Isolated system consisting of control mass, reversible heat transfer reservoirs and reversible work transfer reservoirs [1].

Reversible Heat Transfer Reservoir and Contribution of Heat Transfer on Entropy

Let us consider a system having infinitely large heat capacity to determine the contribution of heat transfer on entropy. This system can participate in heat transfer only, but not in work transfer i.e., system boundary is rigid or incompressible (dV = 0). Hence, the system acts as a constant volume control mass. As the system has infinitely large heat capacity, its temperature is unaffected by the quantity of heat transferred to or from it. It is therefore characterized by a uniform temperature (T_i) which remains constant. Such an idealized system is called a *reversible heat transfer reservoir* [1]. The changes that do take place in the heat transfer (or thermal) reservoir as heat enters or leaves are so slow and so small that processes within it are quasi-static and are therefore regarded as reversible.

The heat is transferred to or from this reservoir. The reservoir which is at high temperature and from which heat is transferred is called *heat source*, e.g. a boiler furnace, a combustion chamber and a nuclear reactor etc. The reservoir which is at low temperature and to which heat is transferred is known as *heat sink*, e.g. atmospheric air, ocean, lake and river etc.

From Gibb's equation (5.14), for constant volume processes i.e., for heat transfer reservoir, $\delta W = PdV = 0$, we get

$$dS = \frac{dU}{T} \qquad \text{……… (5.37)}$$

Using first law of thermodynamics for a control mass that does no work is

$$\delta Q = dU + PdV = \ dU + 0 = dU \qquad \text{……… (5.38)}$$

Putting equation (5.38) in equation (5.37), an expression for the change in entropy of the reversible heat transfer reservoir can be derived as

$$dS = \frac{\delta Q}{T} \qquad \text{……… (5.39)}$$

where δQ is taken as positive as it is supplied to the reversible heat transfer reservoir from the control mass and the temperature T is uniform throughout the reservoir.

The above equation (5.39) indicates that entropy of a system increases if heat is supplied to the system from the surroundings and decreases if heat is lost by the system to the surroundings.

Reversible Work Transfer Reservoir and Contribution of Work Transfer on Entropy

Let us consider a system having infinitely large work capacity to determine the contribution of work transfer on entropy. This system can participate in work transfer only, but not in heat transfer ($\delta Q = 0$). Hence, the system is a compressible adiabatic control mass and the work is a result of boundary expansion or contraction. As the system has infinitely large work capacity, its pressure is unaffected by the work transferred to or from it. It is therefore characterized by a uniform pressure (P_i) that remains constant. Such an idealized system is called a *reversible work transfer reservoir [1]*. Since the work interaction takes place in the reservoir very slowly, the work transfer is a quasi-static or quasi-equilibrium process and is taken as reversible. The work is done by reservoir or on the reservoir when it acts as a *work source* or *work sink* respectively. A large pressure reservoir consisting of a gas contained in a piston cylinder device that has an infinitely large weight placed on the piston is an example of work transfer (or mechanical energy) reservoir.

From the equation (5.14), the Gibb's equation for the reversible work reservoir is given as

$$dS = \frac{dU}{T} + \frac{P}{T}dV \qquad \text{... ... .} \quad (5.40)$$

Using first law of thermodynamics for a reversible work transfer reservoir where there is no heat transfer ($\delta Q = 0$), we get

$$dU = \delta Q - \delta W = 0 - PdV$$

$$\therefore \quad dU = -PdV \qquad \text{.........} \ (5.41)$$

Putting equation (5.41) in equation (5.40), an expression for the change in entropy of the reversible work transfer reservoir can be derived as

$$dS = \frac{-PdV}{T} + \frac{PdV}{T} = 0 \qquad \text{.........} \ (5.42)$$

The above equation (5.42) indicates that the reversible work transfer reservoir has no entropy change due to the work transfer. Thus, the work transfer does not have any contribution on the entropy of the system.

Now, using second law of thermodynamics for the isolated system shown in figure 5.7,

$$(dS)_{isolated} \geq 0 \qquad \text{……….(5.43)}$$

As the total change in entropy of the isolated system is the sum of the change in entropy of the control mass, the change in entropy of the reversible heat transfer reservoirs and the change in entropy of the reversible work transfer reservoirs, equation (5.43) can be rewritten as

$$[(dS)_{CM} + \sum (dS)_{RHTR} + \sum (dS)_{RWTR}] \geq 0 \qquad \text{………. (5.44)}$$

where CM, RHTR and RWTR are abbreviations for control mass, reversible heat transfer reservoir and reversible work transfer reservoir respectively.

From equation (5.42) we know that there is no effect of reversible work transfer on the entropy, so substituting $\sum (dS)_{RWTR} = 0$, in equation (5.44),

$$[(dS)_{CM} + \sum (dS)_{RHTR}] \geq 0 \qquad \text{………. (5.45)}$$

The value of entropy change for each reversible heat transfer reservoir from equation (5.39) is substituted in equation (5.45), then

$$[(dS)_{CM} + \sum \left(\frac{\delta Q_i}{T_i}\right)_{RHTR}] \geq 0 \qquad \text{………. (5.46)}$$

where $(\delta Q_i)_{RHTR}$ is positive as the heat is supplied to the reservoir by the control mass and T_i is the reservoir temperature, i.e. the boundary temperature between the reversible heat transfer reservoir and the control mass. Thus, in terms of the control mass, the heat transfer to the control mass from each reservoir, therefore

$$(\delta Q_i)_{CM} = -(\delta Q_i)_{RHTR} \qquad \text{………. (5.47)}$$

Thus, substituting equation (5.47) into equation (5.46) yields

$$[(dS)_{CM} - \sum \left(\frac{\delta Q_i}{T_i}\right)_{CM}] \geq 0$$

$$\Rightarrow \qquad (dS)_{CM} \geq \sum \left(\frac{\delta Q_i}{T_i}\right)_{CM} \qquad \text{………. (5.48)}$$

Thus, the change in entropy of a control mass can be positive (entropy increases) or negative (entropy decreases) since the heat transfer can be positive or negative. With reference to equation (5.48), second law of thermodynamics for a control mass can be stated as

The change in entropy of a control mass is greater than or equal to the sum of the heat transfers divided by the corresponding absolute temperatures of the reservoirs at the boundary [2].

Equation (5.48) is the control mass form for the second law of thermodynamics. For the reversible process it can be expressed as

$$(dS)_{CM,rev} = \sum \left(\frac{\delta Q_i}{T_i}\right)_{CM,rev} \qquad \text{……… (5.49)}$$

and for the irreversible process equation (5.48) can be expressed as

$$(dS)_{CM,irrev} > \sum \left(\frac{\delta Q_i}{T_i}\right)_{CM,irrev} \qquad \text{……… (5.50)}$$

Equation (5.49) indicates that if heat transfer to any control mass from the surroundings occurs under reversible condition, then the change in entropy of the control mass is equal to entropy contributed by the heat transfer from the surroundings. Similarly, equation (5.50) indicates that if the same amount of heat is transferred under irreversible condition then the change in entropy of the control mass is greater than the entropy contributed by the heat transfer from the surroundings. This is possible only when there is entropy generation or production within the system since the surroundings are reversible reservoirs. Therefore, the difference in values of left hand side and right hand side of equation (5.50) represents the entropy generated or entropy produced by the control mass itself.

Thus, the inequality sign in equation (5.48) is eliminated if the concept of entropy generation or production is used as

$$(dS)_{CM} - \sum \left(\frac{\delta Q_i}{T_i}\right)_{CM} - (\delta S)_{gen} = 0 \qquad \text{……… (5.51)}$$

where

$$(\delta S)_{gen} = (dS)_{CM} - \sum \left(\frac{\delta Q_i}{T_i}\right)_{CM} \geq 0 \qquad \text{……… (5.52)}$$

indicating $(\delta S)_{gen} = 0$ for the reversible process and $(\delta S)_{gen} > 0$ for the irreversible process.

The control mass forms of the second law of thermodynamics as given by equations (5.48) and (5.51) can also be expressed on a rate basis as

$$\left(\frac{dS}{dt}\right)_{CM} \geq \sum \left(\frac{\dot{Q}_i}{T_i}\right)_{CM} \qquad \text{………. (5.53)}$$

$$or, \quad \left(\frac{dS}{dt}\right)_{CM} - \sum \left(\frac{\dot{Q}_i}{T_i}\right)_{CM} - \dot{S}_{gen} = 0 \qquad \text{………. (5.54)}$$

where

$$\dot{S}_{gen} = \left(\frac{dS}{dt}\right)_{CM} - \sum \left(\frac{\dot{Q}_i}{T_i}\right)_{CM} \geq 0 \qquad \text{………. (5.55)}$$

indicating $\dot{S}_{gen} = 0$ for the reversible process and $\dot{S}_{gen} > 0$ for the irreversible process.

The control mass forms of the second law of thermodynamics as given by equations (5.48) and (5.51) can also be expressed on a specific or unit mass basis as

$$(ds)_{CM} \geq \sum \left(\frac{\delta q_i}{T_i}\right)_{CM} \qquad \text{………. (5.56)}$$

$$or, \quad (ds)_{CM} - \sum \left(\frac{\delta q_i}{T_i}\right)_{CM} - (\delta s)_{gen} = 0 \qquad \text{………. (5.57)}$$

where

$$(\delta s)_{gen} = (ds)_{CM} - \sum \left(\frac{\delta q_i}{T_i}\right)_{CM} \geq 0 \qquad \text{………. (5.58)}$$

indicating $(\delta s)_{gen} = 0$ for the reversible process and $(\delta s)_{gen} > 0$ for the irreversible process.

For a single heat transfer reservoir undergoing reversible process, equation (5.49) reduces to

$$(dS)_{CM,rev} = \left(\frac{\delta Q}{T}\right)_{CM,rev} \qquad \text{………..(5.59)}$$

Since the reversible heat transfer occurs across an infinitesimal temperature difference, equation (5.59) becomes

$$\delta Q_{rev} = TdS \qquad \text{………. (5.60)}$$

Then, the heat transfer for a specified reversible process between states 1 and 2 is

$$Q_{12,rev} = \int_1^2 TdS \qquad \text{……….(5.61)}$$

or the same in specific form is

$$q_{12,rev} = \int_1^2 Tds \qquad \text{………..(5.62)}$$

Equation (5.61) shows that area under T - S diagram gives the magnitude of heat transfer as shown in figure 5.8.

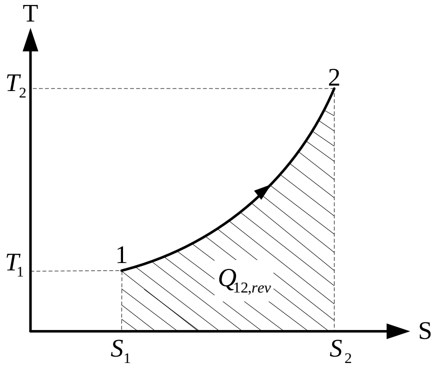

Figure 5.8. Reversible heat transfer evaluated as area under process curve on T$-S$ diagram

5.5. Control Volume Formulation of Second Law of Thermodynamics

A control volume is an open system which interacts with its surroundings in terms of mass transfer as well as energy transfer. Hence, there may be the contributions of mass transfer and energy transfer on the entropy of the control volume. The effect of mass transfer on the entropy of the control volume can be determined by evaluating properties of working substance at the inlets and outlets of the control volume. Moreover, the effects of energy transfer namely heat transfer and work transfer on the entropy of the control volume can be evaluated in a similar manner as in the case of the control mass$(dS = \delta Q/T)$. Thus, control volume formulation of second law of thermodynamics gives the expression for the change of entropy of a control volume due to mass transfer as well as energy transfer. Therefore, the second law of thermodynamics for a control volume can be stated as

The change in entropy within a control volume is greater than or equal to the net entropy transported into the control volume due to the mass transfer plus the sum of the heat transfers divided by the corresponding boundary absolute temperatures [2].

Mathematically,

$$dS_{CV} \geq S_{net} + \sum \left(\frac{\delta Q_i}{T_i}\right)_{CV} \qquad \text{……….} (5.63)$$

where S_{net} is the net entropy transported into the control volume due to mass transfer.

Alternately, on a rate basis it can also be stated as

The rate of change in entropy within a control volume is greater than or equal to the net rate of entropy transported into the control volume due to the mass transfer plus the sum of the heat transfers per unit time divided by the corresponding boundary absolute temperatures.

As explained earlier in the case of the control mass, the contribution of heat transfer on the entropy of the control volume is taken as $dS = \delta Q/T$ and there is no contribution of work transfer on the entropy of the control volume. Now, to determine the effect of mass transfer on the entropy of a control volume let us consider a control volume having three inlets and two outlets as shown in figure 5.9. The total rate of entropy transported into the control volume through all inlets is $\dot{S}_{in} = \dot{S}_1 + \dot{S}_2 + \dot{S}_3$. The total rate of entropy transported out of the control volume through all outlets is $\dot{S}_{out} = \dot{S}_4 + \dot{S}_5$. Then the net rate of entropy transported into the control volume due to the mass transfer will be $\dot{S}_{net} = \dot{S}_{in} - \dot{S}_{out}$.

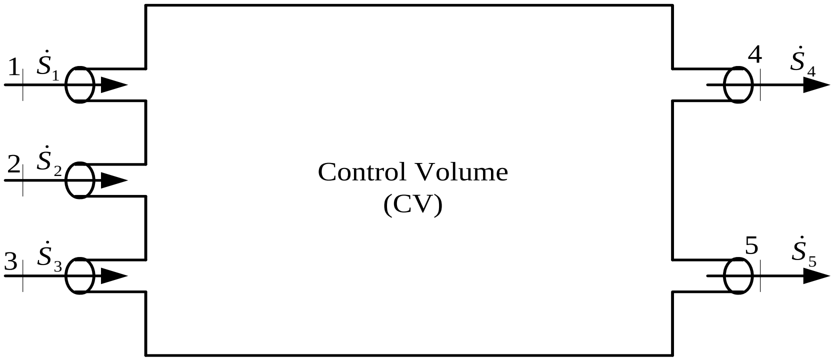

Figure 5.9. Entropy interactions due to mass transfer between a control volume and surroundings through inlets and outlets of a control volume

Thus, on a rate basis the second law for the control volume is mathematically given by

$$\left(\frac{dS}{dt}\right)_{CV} \geq \dot{S}_{net} + \sum \left(\frac{\dot{Q}_i}{T_i}\right)_{CV} \quad \text{………. (5.64)}$$

Then putting $\dot{S}_{net} = \dot{S}_{in} - \dot{S}_{out}$, equation (5.64) becomes

$$\left(\frac{dS}{dt}\right)_{CV} \geq \left(\dot{S}_{in} - \dot{S}_{out}\right) + \sum \left(\frac{\dot{Q}_i}{T_i}\right)_{CV} \quad \text{………. (5.65)}$$

If the process is reversible, equation (5.63) can be expressed as

$$\left(\frac{dS}{dt}\right)_{CV} = (\dot{S}_{in} - \dot{S}_{out}) + \sum \left(\frac{\dot{Q}_i}{T_i}\right)_{CV} \qquad \text{……. (5.66)}$$

If the process is irreversible, equation (5.65) can be expressed as

$$\left(\frac{dS}{dt}\right)_{CV} > (\dot{S}_{in} - \dot{S}_{out}) + \sum \left(\frac{\dot{Q}_i}{T_i}\right)_{CV} \qquad \text{……. (5.67)}$$

Equation (5.66) indicates that the change in entropy of a control volume during a reversible process is equal to entropy contributed by the heat transfer and mass transfer. Likewise equation (5.67) indicates that if the same amount of heat is transferred under irreversible conditions then the change in entropy of the control volume is greater than the entropy contributed by the heat transfer and mass transfer. This is possible only when there is entropy generation or production within the control volume. Hence the difference in values of left hand side and right hand side of equation (5.67) represents the entropy generated or entropy produced by the control volume itself.

Therefore, the equivalent expression corresponding to equation (5.65) in terms of entropy generation to eliminate inequality sign is

$$\left(\frac{dS}{dt}\right)_{CV} - \dot{S}_{in} + \dot{S}_{out} - \sum \left(\frac{\dot{Q}_i}{T_i}\right)_{CV} - \dot{S}_{gen} = 0 \qquad \text{………. (5.68)}$$

where $\dot{S}_{gen} = \left(\frac{dS}{dt}\right)_{CV} - \dot{S}_{in} + \dot{S}_{out} - \sum \left(\frac{\dot{Q}_i}{T_i}\right)_{CV} \geq 0$ ………. (5.69)

indicating $\dot{S}_{gen} = 0$ for the reversible process and $\dot{S}_{gen} > 0$ for the irreversible process.

On a specific basis, the rate of net entropy transported into the control volume due to the mass transfer can alternately be expressed as

$$\dot{S}_{net} = (\dot{m}s)_{in} - (\dot{m}s)_{out} \qquad \text{………(5.70)}$$

where $\dot{m}$ and s are the rate of mass flow and the specific entropy carried by the mass respectively.

Then, using equation (5.70) in equation (5.65) the control volume formulation of second law of thermodynamics is given as

$$\left(\frac{dS}{dt}\right)_{CV} \geq (\dot{m}s)_{in} - (\dot{m}s)_{out} + \sum \left(\frac{\dot{Q}_i}{T_i}\right)_{CV} \qquad \text{………. (5.71)}$$

For multiple inlets and outlets, equation (5.71) becomes

$$\left(\frac{dS}{dt}\right)_{CV} \geq \sum(\dot{ms})_{in} - \sum(\dot{ms})_{out} + \sum\left(\frac{\dot{Q}_i}{T_i}\right)_{CV} \qquad \text{………..… (5.72)}$$

The equivalent expression corresponding to equation (5.71) from using the entropy generation concept is

$$\dot{S}_{gen} = \left(\frac{dS}{dt}\right)_{CV} - (\dot{ms})_{in} + (\dot{ms})_{out} - \sum\left(\frac{\dot{Q}_i}{T_i}\right)_{CV} \geq 0 \qquad \text{……..(5.73)}$$

where $\left(\frac{dS}{dt}\right)_{CV} = 0$ and $\dot{m}_{in} = \dot{m}_{out}$ for a steady state flow of the fluid.

For the case of no mass transport into or out of the control volume, the expressions (5.65) and (5.69) reduce to

$$\left(\frac{dS}{dt}\right)_{CV} \geq \sum\left(\frac{\dot{Q}_i}{T_i}\right)_{CV} \qquad \ldots\ldots\ldots(5.74)$$

Or, $\dot{S}_{gen} = \left(\frac{dS}{dt}\right)_{CV} - \sum\left(\frac{\dot{Q}_i}{T_i}\right)_{CV} \geq 0 \qquad \ldots\ldots\ldots(5.75)$

which are the same expressions given in equations (5.53) and (5.55) for the second law of control mass.

If the system is isolated so that there are no interactions with the surroundings, the heat transfers are zero, then

$$\left(\frac{dS}{dt}\right)_{CV} \geq 0 \qquad \ldots\ldots\ldots(5.76)$$

Or, $\dot{S}_{gen} = \left(\frac{dS}{dt}\right)_{CV} \geq 0 \qquad \ldots\ldots\ldots.(5.77)$

These are the same expressions given in equations (5.5) and (5.7) for the second law of isolated system.

5.6. Isentropic Process

A thermodynamic process during which the entropy within any system remains constant or the change in entropy within the system is zero is called an isentropic process. There are different conditions for different systems where a thermodynamic process can be considered as to be an isentropic process.

We know that since an isolated system does not interact with the surroundings, the change in entropy of the isolated system is zero when the process within the system is reversible, i.e. there is no entropy generation within the system and therefore, the process is an isentropic.

As mentioned earlier in section 5.4 from equation (5.39) that the change in entropy of a control mass undergoing a reversible process is given by

$$dS_{CM} = \frac{\delta Q}{T} \qquad \text{……….} \ (5.78)$$

If the control mass undergoing a process which is both reversible as well as adiabatic ($\delta Q = 0$), then above equation (5. 78) reduces to

$$dS_{CM} = 0 \qquad \text{……….} \ (5.79)$$

Thus, the change in entropy of the control mass undergoing a reversible and adiabatic process is zero i.e., the entropy of the control mass remains constant and the process is an isentropic.

Similarly, from equation (5.63) the change in entropy of a control volume undergoing a reversible process is given by

$$dS_{CV} = S_{net} + \sum \left(\frac{\delta Q_i}{T_i}\right)_{CV} \qquad \text{……….} \ (5.80)$$

Or,

$$dS_{CV} = (ms)_{in} - (ms)_{out} + \sum \left(\frac{\delta Q_i}{T_i}\right)_{CV} \qquad \text{……….} \ (5.81)$$

If the control volume undergoing a reversible process which has a steady reversible flow of fluid $[(ms)_{in} = (ms)_{out}]$ and has an adiabatic boundary ($\delta Q = 0$), then from equation (5.81) it has no entropy change ($dS_{CV} = 0$) and the process is an isentropic.

Many engineering systems or devices such as turbines, compressors, pumps, nozzles, and diffusers are essentially adiabatic in their operation, and they perform best or have high efficiency when the irreversibilities, such as the friction associated with the process, are minimized. Therefore, an isentropic process can serve as an appropriate model for actual processes. Also, isentropic processes enable us to define efficiency so called isentropic efficiency for processes to compare the actual performance of these devices to the performance under idealized conditions.

5.6.1. Isentropic Relations for an Ideal Gas

Several relations for the isentropic processes of ideal gases can be obtained by setting the entropy-change relations developed previously equal to zero. So, for an isentropic process, putting $S_2 - S_1 = 0$, in equations (5.26) and (5.30), we get

$$0 = mc_V ln\left(\frac{T_2}{T_1}\right) + mRln\left(\frac{V_2}{V_1}\right) \qquad \text{……… (5.82)}$$

$$0 = mc_P ln\left(\frac{T_2}{T_1}\right) - mRln\left(\frac{P_2}{P_1}\right) \qquad \text{……… (5.83)}$$

Equation (5.82) can be expressed as,

$$c_V \; ln\left(\frac{T_2}{T_1}\right) = -R \; ln\left(\frac{V_2}{V_1}\right)$$

or, $$ln\left(\frac{T_2}{T_1}\right)^{C_V} = ln\left(\frac{V_2}{V_1}\right)^{-R}$$

Taking antilog on both sides of above equation, we get

$$\left(\frac{T_2}{T_1}\right)^{C_V} = \left(\frac{V_2}{V_1}\right)^{-R} = \left(\frac{V_1}{V_2}\right)^{R}$$

or, $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{R}{C_V}} = \left(\frac{V_1}{V_2}\right)^{\frac{C_P - C_V}{C_V}} = \left(\frac{V_1}{V_2}\right)^{\frac{C_P}{C_V} - 1}$$

∴ $$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma - 1} \qquad \text{……… (5.84)}$$

where $\gamma = \frac{C_P}{C_V}$ is the specific heat ratio of an ideal gas. This equation (5.84) provides the temperature – volume relationship for an ideal gas undergoing an isentropic process.

Similarly, equation (5.83) can be expressed as,

$$c_P\ ln\left(\frac{T_2}{T_1}\right) = R\ ln\left(\frac{P_2}{P_1}\right)$$

or, $$ln\left(\frac{T_2}{T_1}\right)^{c_P} = ln\left(\frac{P_2}{P_1}\right)^R$$

Taking antilog on both sides of above equation, we get

$$\left(\frac{T_2}{T_1}\right)^{c_P} = \left(\frac{P_2}{P_1}\right)^R$$

or, $$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{R}{c_P}} = \left(\frac{P_2}{P_1}\right)^{\frac{c_P - c_V}{c_P}} = \left(\frac{P_2}{P_1}\right)^{1-\frac{c_V}{c_P}} = \left(\frac{P_2}{P_1}\right)^{1-\frac{1}{\gamma}}$$

∴ $$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \qquad \text{………. (5.85)}$$

This equation (5.85) provides the temperature – pressure relationship for an ideal gas undergoing an isentropic process.

Equating equations (5.84) and (5.85) the pressure – volume relationship for an ideal gas undergoing an isentropic process can be obtained as

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \qquad \text{………. (5.86)}$$

which is same as in the case of the adiabatic process.

5.6.2. Isentropic Relations for an Incompressible Substance

An incompressible substance is basically characterized by the property that it has constant volume. The change in entropy of an incompressible substance as given by equation (5.32) is

$$dS = \frac{dU}{T} = \frac{m\ c\ dT}{T} \qquad \text{………. (5.87)}$$

For an isentropic process, putting $dS = 0$ in equation (5.87), we get the relation for an incompressible substance undergoing an isentropic process as

$$dS = \frac{dU}{T} = \frac{m\,c\,dT}{T} = 0$$

$\therefore \quad dU = 0$

Also, $\quad dT = 0$ ………. (5.88)

Thus, during an isentropic process there is no change in the internal energy of the incompressible substance. Moreover, an isentropic process is also an isothermal process for an incompressible substance.

Isentropic relation for an incompressible substance can also be expressed in terms of specific enthalpy. For this let us start with the definition of the enthalpy as

$$H = U + PV \qquad \text{………. (5.89)}$$

Then the change in enthalpy during any process is given by

$$dH = dU + PdV + VdP \qquad \text{………. (5.90)}$$

Putting $dV = 0$ for an incompressible substance and $dU = 0$ for an isentropic process from equation (5.88) in equation (5.90), we get

$$dH = VdP \qquad \text{………. (5.91)}$$

Integrating equation (5.91) for any process 1 - 2 between state 1 and state 2, an isentropic relation for an incompressible substance can be derived as

$$H_2 - H_1 = V(P_2 - P_1) \qquad \text{………. (5.92)}$$

Dividing both sides of equation (5.92) by mass of incompressible substance the equation (5.92) becomes

$$h_2 - h_1 = v(P_2 - P_1) \qquad \text{………. (5.93)}$$

This equation (5.93) is frequently used to determine specific enthalpy of an incompressible substance e.g. of a sub-cooled or compressed liquid of working substance in cycle problems.

5.7. Isentropic Efficiency of Thermodynamic Devices with Steady - Flow of Fluid

As explained in previous section a thermodynamic process occurring in a control volume or engineering devices working under steady - flow conditions, such as turbines, compressors, and nozzles, will be isentropic if it does not involve any kind of irreversibilities e.g. frictional losses, heat losses, etc. An isentropic process is therefore an ideal process which should involve no irreversibilities since the effect of irreversibilities is always to downgrade the performance of engineering devices. Thus, the ideal process that can serve as a suitable model for adiabatic steady-flow devices is the isentropic process. But the process occurring in any actual device involves irreversibilities or losses and then the actual process differs from the idealized isentropic process. The more closely the actual process approximates the idealized isentropic process, the better the device performs. Thus, it would be desirable to have a parameter that expresses quantitatively how efficiently an actual device approximates an idealized one. This parameter is the isentropic efficiency**,** which is a measure of the deviation of actual processes from the corresponding idealized ones.

5.7.1. Isentropic Efficiency of a Turbine

For a turbine working under steady condition, the inlet state of the working fluid and the exhaust pressure are fixed. Therefore, the ideal process for an adiabatic turbine is an isentropic process between the inlet state and the exhaust pressure. The desired output of a turbine is the work produced, and the isentropic efficiency of a turbine is defined as the ratio of the actual work output of the turbine to the work output that would be achieved if the process between the inlet state and the exit pressure were isentropic:

$$\eta_{sT} = \frac{Actual\ turbine\ work}{Isentropic\ turbine\ work} = \frac{W_a}{W_s} \qquad \text{………. (5.94)}$$

where η_{sT}, W_a and W_s are the isentropic efficiency, actual work output and isentropic work output of a turbine respectively.

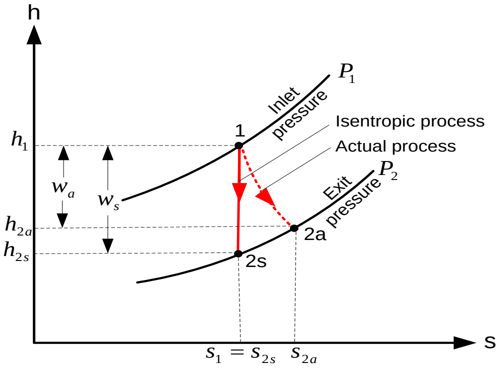

Figure 5.10. The *h-s* diagram for the actual and isentropic processes of an adiabatic turbine [3]

Usually the changes in kinetic and potential energies associated with a fluid stream flowing through a turbine are small relative to the change in enthalpy and can be neglected. Then the work output of an adiabatic turbine simply becomes the change in specific enthalpy, and equation (5.94) becomes

$$\eta_{sT} = \frac{h_1 - h_{2a}}{h_1 - h_{2s}} \qquad \text{……….. (5.95)}$$

where $h_{1,}$ h_{2a} and h_{2s} are the specific enthalpy values at the inlet state, at the exit state for actual process and isentropic process respectively as shown in figure (5.10).

5.7.2. Isentropic Efficiency of a Compressor and Pump

In case of work consuming devices such as compressor and pump actual work input is always more than the isentropic work input. This is because work input has to be increased to overcome the losses to get the same desired output effect e.g. to raise the pressure of a gas to a specified value in a compressor. Hence, the isentropic efficiency of a compressor or pump is defined as the ratio of the work input that would have been required when the compressor or pump operates under isentropic condition to work input required for the actual compressor or pump, i.e.

$$\eta_{sC,}\, \eta_{sP} = \frac{Isentropic\ compressor/pump\ work}{Actual\ compressor/pump\ work} = \frac{W_s}{W_a} \qquad \text{……..... (5.96)}$$

where η_{sC}, and η_{sP} are the isentropic efficiencies of a compressor and a pump respectively. The inlet conditions and the exit pressure of the working substances are the same for both the actual and the isentropic compressor or pump.

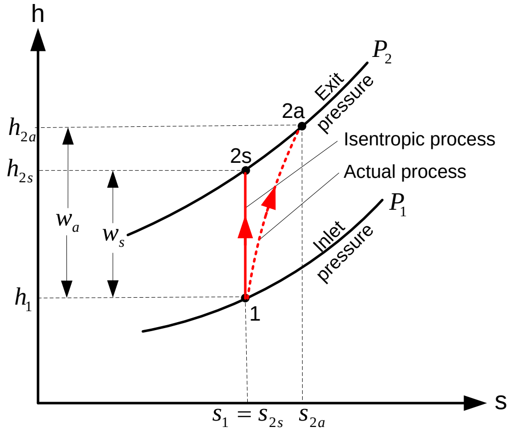

Figure 5.11. The *h-s* diagram of the actual and isentropic processes of an adiabatic compressor[3]

When the changes in kinetic and potential energies of the gas being compressed are negligible, the work input to an adiabatic compressor becomes equal to the change in specific enthalpy, and equation (5.96) for this case becomes

$$\eta_{sC} = \frac{h_{2s} - h_1}{h_{2a} - h_1} \qquad \text{……… (5.97)}$$

where h_{2a} and h_{2s} are the specific enthalpy values at the exit state for actual and isentropic compression processes, respectively, as illustrated in figure (5.11).

When the changes in potential and kinetic energies of a liquid are negligible, the isentropic efficiency of a pump is defined similarly as

$$\eta_{sP} = \frac{h_{2s} - h_1}{h_{2a} - h_1} = \frac{v(P_2 - P_1)}{h_{2a} - h_1} \qquad \text{……… (5.98)}$$

where v, P_1 and P_2 are the specific volume, inlet and exit pressures of the liquid in a pump respectively.

5.7.3. Isentropic Efficiency of a Nozzle

Nozzles are essentially adiabatic devices and are used to accelerate a fluid (to increase exit velocity of a fluid). Therefore, the isentropic process serves as a suitable model for nozzles. The isentropic efficiency of a nozzle is defined as the ratio of the actual kinetic energy of the fluid at the nozzle exit to the kinetic energy value at the exit of an isentropic nozzle for the same inlet state and exit pressure. That is,

$$\eta_{sN} = \frac{Actual\ KE\ at\ \ exit}{Isentropic\ KE\ at\ exit} = \frac{m(\bar{V}_{2a})^2/2}{m(\bar{V}_{2s})^2/2} = \frac{(\bar{V}_{2a})^2}{(\bar{V}_{2s})^2} \qquad \text{………… (5.99)}$$

where $\bar{V}_{2a}$ and $\bar{V}_{2s}$ are the velocities of the fluid at nozzle exit for a actual and isentropic nozzle respectively as shown in figure (5.12). The exit pressure is the same for both the actual and isentropic processes, but the exit state is different.

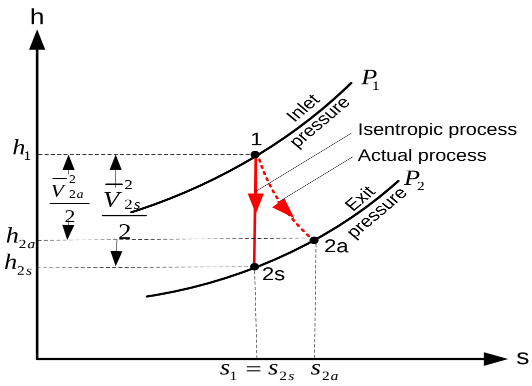

Figure 5.12. The *h-s* diagram of the actual and isentropic processes of an adiabatic nozzle [3]

Lecture Highlights

- *Reversible heat transfer reservoir:* It is an idealized system having infinite heat capacity (i.e., no change of its temperature by the heat transfer) which can interact with its surroundings only by heat transfer but not by work transfer.
- *Reversible work transfer reservoir:* It is an idealized system having infinite work capacity (i.e., no change of its pressure by the work transfer) which can interact with its surroundings only by work transfer but not by heat transfer.
- *Change in entropy for a reversible heat transfer reservoir* is given by $dS = \frac{\delta Q}{T}$.
- *Change in entropy for a reversible work transfer reservoir* is given by $dS = 0$.
  This means that work transfer does not have any contribution on the entropy of the system.
- *Second law of thermodynamics for control mass:* "The change in entropy of a control mass is greater than or equal to the sum of heat transfers divided by the corresponding boundary absolute temperatures". It is expressed as:

$$(dS)_{CM} \geq \sum \left(\frac{\delta Q_i}{T_i} \right)_{CM}$$

Or, $$(dS)_{CM} - \sum \left(\frac{\delta Q_i}{T_i} \right)_{CM} - \delta S_{gen} = 0$$

where $$\delta S_{gen} = (dS)_{CM} - \sum \left(\frac{\delta Q_i}{T_i} \right)_{CM} \geq 0$$

On rate basis:

$$\left(\frac{dS}{dt}\right)_{CM} \geq \sum\left(\frac{\delta\dot{Q}_i}{T_i}\right)_{CM}$$

Or, $$\left(\frac{dS}{dt}\right)_{CM} - \sum\left(\frac{\delta\dot{Q}_i}{T_i}\right)_{CM} - \dot{S}_{gen} = 0$$

where $\dot{S}_{gen} = \left(\frac{dS}{dt}\right)_{CM} - \sum\left(\frac{\delta\dot{Q}_i}{T_i}\right)_{CM} \geq 0$.

- *For a single thermal reservoir undergoing reversible process*, the change in entropy of a control mass is given by

  $$(dS)_{CM} = \frac{\delta Q}{T} \quad \Rightarrow \quad \delta Q = T.dS$$

  This means the area under the *T* - *S* diagram gives the magnitude of heat transfer.
- *Second law of thermodynamics for a control volume:* "The change in entropy of a control volume is greater than or equal to the net entropy transported into the control volume due to mass transfer plus the sum of heat transfers divided by the corresponding boundary absolute temperatures". It can be expressed as:

  $$(dS)_{CV} \geq S_{net} + \sum\left(\frac{\delta Q_i}{T_i}\right)_{CV}$$

  where $S_{net} = S_{in} - S_{out} = m_{in}s_{in} - m_{out}s_{out}$.

  Or, $$(dS)_{CV} - S_{net} - \sum\left(\frac{\delta Q_i}{T_i}\right)_{CV} - \delta S_{gen} = 0$$

  where $\delta S_{gen} = (dS)_{CV} - S_{net} - \sum\left(\frac{\delta Q_i}{T_i}\right)_{CV} \geq 0$

  On rate basis,

  $$\left(\frac{dS}{dt}\right)_{CV} \geq \dot{S}_{net} + \sum\left(\frac{\delta\dot{Q}_i}{T_i}\right)_{CV}$$

  where $\dot{S}_{net} = \dot{S}_{in} - \dot{S}_{out} = \dot{m}_{in}s_{in} - \dot{m}_{out}s_{out}$

  Or, $$\left(\frac{dS}{dt}\right)_{CV} - \dot{S}_{net} - \sum\left(\frac{\delta\dot{Q}_i}{T_i}\right)_{CV} - \dot{S}_{gen} = 0$$

  where $\dot{S}_{gen} = \left(\frac{dS}{dt}\right)_{CV} - \dot{S}_{net} - \sum\left(\frac{\delta\dot{Q}_i}{T_i}\right)_{CV} \geq 0$

- *Isentropic process*: The process during which the entropy remains constant is known as isentropic process. A process must be a reversible as well as adiabatic to be an isentropic process.
- *Isentropic relations for an ideal gas:*
    - Pressure-volume relation:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

    - Temperature-volume relation:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

    - Temperature-pressure relation:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

- *Isentropic relation for an incompressible substance:*

$$h_2 - h_1 = v(P_2 - P_1)$$

- *Isentropic efficiency of thermodynamic devices with steady flow of fluid:*
  A process occurring in any thermodynamic devices with steady flow of fluid will be an isentropic or ideal process if it does not involve any kind of losses (frictional loss, heat loss etc.).
    - *Isentropic efficiency of a turbine:* It is defined as the ratio of work output from a real turbine to work that would have been produced when the turbine operates under isentropic condition, i.e.

$$\eta_{i,T} = \frac{W_{real}}{W_{isen}}$$

      This is because in case of work producing devices, real work is always less than the isentropic work output due to losses.
    - *Isentropic efficiency of a compressor/pump:* It is defined as the ratio of work that would have been required when a compressor/pump operates under isentropic condition to work input to the real compressor/pump, i.e.

$$\eta_{i,C} \text{ or } \eta_{i,P} = \frac{W_{isen}}{W_{real}}$$

      This is because in case of work consuming devices, real work is always more than isentropic work input. Work input has to be increased to overcome the losses to get the same desired output effects.

- *Isentropic efficiency of a nozzle:* It is defined as the ratio of the kinetic energy of the fluid at the exit of a real nozzle to the fluid kinetic energy at the exit of an isentropic nozzle for the same inlet condition, i.e.

$$\eta_{i,N} = \frac{\bar{V}_{real}^{2}}{\bar{V}_{isen}^{2}}$$

References

[1] *Fundamentals of Engineering Thermodynamics*: Howell J. R. and Buckius R. O., McGraw-Hill, New York, 1992.

[2] *Fundamentals of Thermodynamics & Heat Transfer*: Luintel M.C., Heritage Publishers & Distributors Pvt. Ltd., Kathmandu, Nepal, 2016.

[3] *Thermodynamics: An Engineering Approach:* Cengel Y. A. and Boles M. A., McGraw-Hill Education, New York, 2018.