Course Title:

Fundamental of Thermodynamics and Heat Transfer

Lecture 15 (Week 15):

Heat Transfer

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Learning Objective of Lecture:

To impart a great deal of knowledge to undergraduate students on the following topics:

- ✓ Introduction to Heat Transfer
- ✓ Basic Concepts and Modes of Heat Transfer, Governing Laws of Heat Transfer
- ✓ One Dimensional Steady State Heat Conduction through a Plane Wall and Radial Steady State Heat Conduction through a Hollow Cylinder
- ✓ Heat Flow Through Composite Structures: Composite Plane Walls and Multilayered Tubes

7.0. Introduction to Heat Transfer

In chapter 2, heat is defined as the form of energy in transit that can be transferred from one system to another as a result of temperature difference. A thermodynamic analysis is concerned with the amount of heat transfer as a system undergoes a process from one equilibrium state to another, without any information concerning the time rate at which it occurs. The science that deals with the determination of the rates of such energy transfers and the temperature distribution taking place in a system is known as the *heat transfer*. The transfer of energy as heat is always from the higher-temperature medium to the lower-temperature one, and heat transfer stops when the two mediums reach the same temperature.

Heat transfer is generally a multidimensional and time dependent phenomenon, i.e., the temperature in a medium varies with position as well as time. For analysis of heat transfer problems, two types of heat transfer are considered, e.g. *steady state heat transfer* and *unsteady state or transient heat transfer*. Heat transfer in a medium is said to be a *steady state* if the temperature at any particular location on the system does not vary with time. The temperature is function of space coordinates only, but it is independent of time. During steady state conditions, the heat transfer rate is constant and there is no change of internal energy of the system. In *unsteady state or transient heat transfer* the temperature varies with time as well as position or location on the system [1]. Hence, the temperature is a function of time and space coordinates. During unsteady state heat transfer, rate of heat transfer varies with time due to change in internal energy of the system. Most of the actual heat transfer processes are unsteady in nature, but some of them are considered as steady state to simplify them.

A heat transfer analysis has applications in the design of electronic components, electric machines, transformers, bearings etc. to avoid the overheating and damage of equipment where it requires considerations of the amount of heat to be transmitted as well as the rate at which heat is to be transferred.

7.1. Basic Concepts and Modes of Heat Transfer; Governing Laws of Heat Transfer

7.1.1. Basic Concepts and Modes of Heat Transfer

According to the physical mechanism and the governing laws associated with them, heat transfer is classified into three modes namely conduction, convection and radiation. All modes of heat transfer require the existence of a temperature difference and all modes occur from the high-temperature medium to a lower-temperature one. A brief description of each mode is given below.

7.1.1.1. Conduction

Conduction is the transfer of heat from one part of a substance to another part of the same substance or from one substance to another in physical contact with it without appreciable movement of molecules or atoms forming the substance. In fact, heat conduction is due to the property of matter which allows the passage for heat energy even its parts are not in motion relative to one another. The heat conduction occurs more effectively in solids than in liquids and gases.

The heat is conducted in solids by two mechanisms namely by lattice (crystal) vibration and free electrons movement. However, there is no actual movement of molecules or atoms themselves in the heat conduction. During the lattice vibration, the faster moving molecules or atoms in the hottest part of a body transfer heat by impacting some of their energy to adjacent molecules. During the free electrons movement in good conductors like metals, a large number of free electrons move about in the lattice structure of the material which transports heat from high temperature region to the low temperature region. The amount of heat energy transported by the free electrons is larger than that by the lattice vibration. An increase in temperature causes increase in both the lattice vibration and speed of electrons. However, the increased vibration of lattice disturbs the movement of the free electrons causing reduction in transport of heat energy by the free electrons which means the overall heat conduction is reduced. In insulators and alloys, the transport of heat energy is mainly due to the lattice vibration and an increase in temperature increases the conduction of the heat [1].

The heat conduction in liquid and gases are based on the movements of atoms and molecules. In the case of gases, the kinetic energy of a molecule is a function of temperature. These molecules are in a continuous random motion exchanging energy and momentum. When a molecule from the high temperature region collides with a molecule from the low temperature region, it losses heat energy by collisions (heat conduction).

In case of liquids, the mechanism of the heat conduction is similar to that of the gases. However, the molecules are more closely spaced and intermolecular forces come into play. Random translatory motion in liquids is small and it appears that transfer of the heat energy occurs by longitudinal vibrations similar to the propagation of sound.

7.1.1.2. Convection

It is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. The convection heat transfer is possible only in a fluid medium and the heat energy is transferred from the actual movement of the molecules thereby causing molecular collisions and molecular diffusions.

Hence, the effectiveness of heat transfer by convection depends largely upon the mixing motion of the fluid. The convection heat transfer occurs while the heat energy is transferred to a flowing fluid at any solid surface over which a fluid flows. This mode of heat transfer is a combination of conduction in a very thin fluid layer so called stagnant film at the solid surface and then convection due to the mixing caused by the flow. The heat transfer by convection depends on the properties of fluid and is independent of the properties of the material of the solid surface. However, the geometry of the solid surface influences the flow of fluid and therefore the convective heat transfer [2].

7.1.1.3. Radiation

Radiation is the transfer of heat energy in the form of electromagnetic wave from one body to another. Unlike the heat transfer by conduction and convection, radiation heat transfer is possible without any medium between the heat source and the receiver. In fact, the heat transfer by radiation or radiation exchange between two bodies occurs most effectively in vacuum where no conduction and convection heat transfer take place. A material medium present between the heat source and the receiver would either reduce or eliminate entirely the propagation of heat energy by radiation.

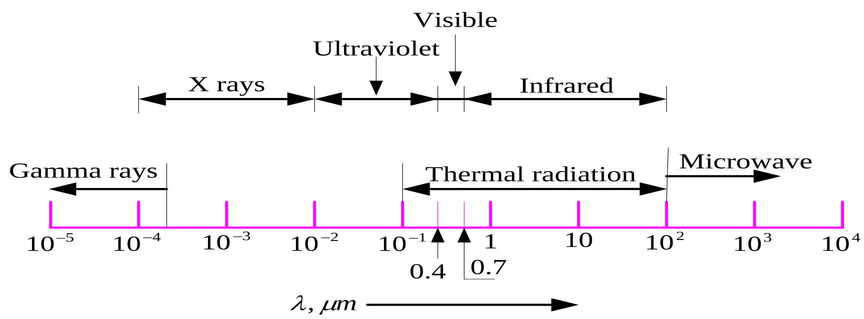


Figure 7.1. Spectrum of electromagnetic radiation [2]

All types of electromagnetic waves are classified in terms of wavelength and are propagated at the speed of light ($c = 3x10^8 m/s$). The electromagnetic spectrum is shown in figure 7.1. The emission of thermal radiation range lies between a wavelength of 0.1 μm and 100 μm depends upon the nature, temperature, and state of the emitting surface. Thermal radiations exhibits characteristics similar to those of visible light and follow optical laws [2].

All bodies having surface temperature above absolute zero temperature (above 0 Kelvin) are capable of emitting radiant heat energy. The transfer of heat by radiation occurs because a hot body emits more heat than it receives and a cold body receives more heat than it emits, so there is net heat transfer from the hot body to the cold body.

The mechanism of the heat transfer by radiation from a hot body to a cold body consists of following three phases:

(i) Conversion of heat energy of the hot body into electromagnetic waves: Energy released by rapidly oscillating molecules of a radiating surface of a hot body is not continuous but in the form of successive and discrete packets or quanta of energy called photons. The photons are propagated through the space and the movement of these photons is described as the electromagnetic waves.

(ii) Movement of photons from the hot body to the cold body through intervening space: The photons or electromagnetic waves carry energy with them and travel with unchanged frequency in straight paths with speed equal to that of light. These electromagnetic waves are identical with light waves but the wavelength of heat radiations is longer than that of light waves and therefore they are not visible to the eye.

(iii) Reconversion of electromagnetic waves into heat energy of the cold body: When the photons approach the receiving surface of a cold body, they transfer the heat energy carried with them to the relatively slow moving molecules of the cold body that is partly absorbed, reflected or transmitted through the receiving surface of the cold body. The absorbed portion of the heat energy increases the molecular energy of the cold body and results in a rise of its temperature.

Energy radiated by electromagnetic wave or energy of a photon can be given by the relation

$$E = h \, x \, f \qquad \text{……….(7.1)}$$

where h = 6.626 x 10^{-34} Js is the Planck' constant and f is the frequency of the wave.

The contribution of the radiation to heat transfer is very significant at high absolute temperature. The solar energy incident upon the earth is a well-known example of the heat transfer by radiation.

7.1.2. Governing Laws of Heat Transfer

In heat transfer, there are a number of governing laws which are associated with the different modes of heat transfer. These laws are the foundation of heat transfer and some of them are described briefly here below.

Assumptions

The calculation of rate of heat transfer $(\dot{Q})$ by using different governing laws associated with the different modes of heat transfer are based on the following assumptions [2]:

(i) The heat transfer takes place under steady state conditions.
(ii) The heat flow is unidirectional.
(iii) There is no internal heat generation.
(iv) The bounding surfaces are isothermal in character.
(v) The material is homogeneous and isotropic, i.e., the value of thermal conductivity is constant in all directions.

7.1.2.1. Fourier's Law of Heat Conduction

Magnitude of conduction heat transfer is given by Fourier's law of heat conduction which is an empirical law based on observation and states as:

The rate of flow of heat per unit area (heat flux) through a homogeneous solid is directly proportional to the temperature gradient within the solid material.

Mathematically, it can be represented by the equation as

$$\frac{\dot{Q}}{A} \propto \frac{dT}{dx}$$

Or,
$$\dot{Q} = -kA\frac{dT}{dx} \qquad \text{……… (7.2)}$$

where $\dot{Q}$ is the rate of heat flow through a material (W), k is the constant of proportionality and is known as the thermal conductivity of the material ($W/m.K$), A is the cross sectional area of the material perpendicular to the direction of heat flow (m^2), and dT/dx is the temperature gradient within the material (K/m).

The negative sign in equation (7.2) indicates that heat flows in the direction of decreasing temperature along with the direction of increasing thickness. The temperature gradient is always negative along the positive x direction and therefore the value of rate of heat flow becomes positive.

7.1.2.2. Newton's Law of Cooling for Convection

Magnitude of convection heat transfer is given by *Newton's law of cooling* and it states that

The rate of heat flow per unit area (heat flux) is directly proportional to the temperature difference between a solid surface and an adjacent fluid.

Mathematically, it can be expressed as

$$\frac{\dot{Q}}{A_s} \propto (T_s - T_\infty)$$

Or, $$\dot{Q} = hA_s(T_s - T_\infty) \qquad \text{………. (7.3)}$$

where, $\dot{Q}$ is the rate of convective heat transfer (W), h is the constant of proportionality and is called convective heat transfer coefficient ($W/m^2.K$), A_s is the surface area of the solid surface confining the fluid (m^2), T_s is the solid surface temperature, and T_∞ is the fluid temperature.

The value of the heat transfer coefficient h depends upon the thermodynamic and transport properties, e.g. density, viscosity, specific heat and thermal conductivity of the fluid, the geometry of the solid surface, the nature of fluid flow, e.g. laminar and turbulent flow, and the prevailing thermal conditions [2].

7.1.2.3. Stefan - Boltzmann Law of Heat Radiation

Magnitude of the radiation heat transfer by a surface is given by Stefan-Boltzmann law which states that

The maximum rate of heat radiated per unit area (heat flux) by a surface is directly proportional to the fourth power of the absolute surface temperature.

Mathematically,

$$\frac{\dot{Q}}{A_s} \propto T_s^4$$

Or, $$\dot{Q} = \sigma\, A_s\, T_s^4 \qquad \text{……. (7.4a)}$$

where $\dot{Q}$ is the maximum rate of heat transfer by radiation (W), T_s is the absolute temperature of the surface (K), A_s is the surface area exposed to the heat transfer (m^2), and σ is the constant of proportionality and is called Stefan–Boltzmann constant ($\sigma = 5.67 \times 10^{-8} W/m^2K^4$).

An idealized surface which emits the radiation heat flux at the maximum rate as per equation (7.4a) is called a black body. The heat flux emitted by all real body surfaces is less than that by a black body surface at the same temperature by a factor and is expressed as

$$\dot{Q} = \mathcal{E}\, \sigma\, A_s\, T_s^4 \qquad \text{……. (7.4b)}$$

where Ɛ is radiative property of the surface and is known as the emissivity of the radiating surface. Its value is in the range $0 \leq \varepsilon \leq 1$. It has a value of Ɛ = 1 for a black body surface and for a real body surface it is less than 1.

The difference between the rates of radiation emitted by the surface and the radiation absorbed is the net radiation heat transfer. If the rate of radiation absorption is greater than the rate of radiation emission, the surface is said to be gaining energy by radiation. Otherwise, the surface is said to be losing energy by radiation. In general, the determination of the net rate of heat transfer by radiation between two surfaces is a complicated matter since it depends on the properties of the surfaces, their orientation relative to each other, and the interaction of the medium between the surfaces with radiation [3].

When a surface of emissivity Ɛ and surface area A_s at a thermodynamic temperature T_s is completely enclosed by a much larger surface or a black body at thermodynamic temperature T_{surr} separated by a gas such as air that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by [3]

$$\dot{Q}_{net} = \varepsilon \sigma A_s \left(T_s^4 - T_{surr}^4\right) \qquad \text{……………………….(7.5)}$$

The emissivity of a surface is a measure of how it emits radiant energy in comparison with a black body surface at the same temperature. The total emissive power (rate of radiant energy per unit area) of a real body is always less than that of a black body at the same surface temperature. Hence, emissivity of a surface is defined as ratio of the total emissive power of a real body to that of black body at the same surface temperature, i.e.

$$\varepsilon = E_r / E_b \qquad \text{…………(7.6)}$$

where, E_r and E_b are the total emissive power of a real body and black body respectively. The emissivity is the property of the surface and it depends upon the characteristics (e.g. surface finish) of the surface.

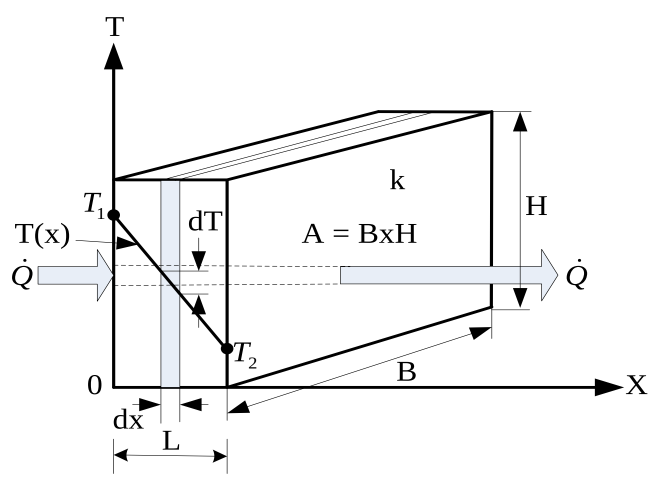
7.2. One Dimensional Steady State Heat Conduction through a Plane Wall and Radial Steady State Heat Conduction through a Hollow Cylinder

7.2.1. One Dimensional Steady State Heat Conduction through a Plane Wall

Let us consider steady heat conduction through the plane walls of a building. Heat is continuously lost to the surroundings through the wall during a winter day. It can be observed that heat transfer through the wall is in the perpendicular direction to the wall surface, and no significant heat transfer takes place in the wall in other directions as shown in figure 7.2. Heat transfer in a certain direction is driven by the temperature gradient in that direction. There is no heat transfer in a direction in which there is no change in temperature. It can be confirmed from the temperature measurements that the temperatures at the top and bottom of a wall surface as well as at the right and left ends are almost the same. Therefore, there is no heat transfer through the wall from the top to the bottom or from left to right, but there is considerable temperature difference between the inner and the outer surfaces of the wall, and thus significant heat transfer in the direction from the inner surface to the outer one.

The small thickness of the wall causes the temperature gradient in that direction to be large. Further, if the air temperatures inside and outside the building remain constant, then heat transfer through the wall of a building can be modeled as steady state and one-dimensional. The temperature of the wall in this case depends on one direction only (e.g. the *x*-direction) and can be expressed as $T(x)$.

Since there is no change in the temperature of the wall with time at any point, the rate of change of the energy of the wall is zero for steady state operation. Therefore, the rate of heat transfer into the wall must be equal to the rate of heat transfer out of it. In other words, the rate of heat transfer $(\dot{Q})$ through the wall must be constant.



7.2. One dimensional heat conduction through a plane wall [3]

Let us consider a plane wall composed of a material having an average thermal conductivity of k, thickness L, cross-sectional area of A as shown in figure 7.2. The two surfaces of the wall are maintained at constant temperatures of T_1 and T_2 where T_1 is greater than T_2. Here, the temperature gradient exist only in x −direction, i.e., temperature is a function of thickness, *T(x)*. Hence, the heat conduction through the wall can be modeled as one dimensional and steady state. By using Fourier's law from equation (7.2) and rearranging it yields

$$\dot{Q}dx = -kAdT$$

where the rate of heat transfer by conduction $\dot{Q}$ and the wall area *A* are constant. Thus, *dT/dx* is also constant which means that the temperature through the wall varies linearly with *x*. That is, the temperature distribution in the wall under steady conditions is a straight line as shown in figure 7.2.

Integrating above equation with boundary conditions, from $x = 0$, where $T(0) = T_1$ to $x = L$, where $T(L) = T_2$, we get

$$\dot{Q}\int_0^L dx = -\int_{T_1}^{T_2} kAdT$$

Assuming for a plane wall with a thermal conductivity k and uniform cross sectional area A as constant, above equation reduces to

$$\dot{Q}L = -kA(T_2 - T_1) = kA(T_1 - T_2)$$

Hence, the magnitude of the heat transfer by conduction through a plane wall is given by

$$\dot{Q} = \frac{kA}{L}(T_1 - T_2) \qquad \text{………. (7.7)}$$

Equation (7.7) indicates that the rate of heat transfer by conduction through a plane wall is proportional to the average thermal conductivity, the wall area, and the temperature difference, but is inversely proportional to the wall thickness.

When the rate of heat transfer by conduction $(\dot{Q})$ is known, the temperature *T*(*x*) at any location *x* within the wall thickness can be determined by replacing temperature T_2 by *T(x)*, and wall thickness *L* by *x* in equation (7.7), i.e.,

$$T(x) = T_1 - \frac{\dot{Q}\,x}{kA} \qquad \text{………. (7.8)}$$

7.2.2. Radial Steady State Heat Conduction through a Hollow Cylinder

Let us **c**onsider steady state heat conduction through a hollow cylinder or pipe carrying a hot fluid. Heat is continuously lost to the surroundings (e.g. cold air) through the wall of the pipe, and it can be observed that heat transfer through the pipe is in the normal direction to the pipe surface and no significant heat transfer takes place in the pipe in other directions as shown in figure 7.3. The wall of the pipe, whose thickness is rather small, separates two fluids at different temperatures, and thus the temperature gradient in the radial direction is relatively large. Further, if the fluid temperatures inside and outside the pipe remain constant, then heat transfer through the pipe is steady state. Thus heat transfer through the pipe can be modeled as steady state and one-dimensional. The temperature of the pipe in this case depends on one direction only (the radial r-direction) and can be expressed as $T = T(r)$. The temperature is independent of the axial distance. This situation is approximated in practice in long cylindrical pipes.

In steady state operation, there is no change in the temperature of the pipe with time at any point. Therefore, the rate of heat transfer into the pipe must be equal to the rate of heat transfer out of it. In other words, the rate of heat transfer ($\dot{Q}$) through the pipe must be constant.

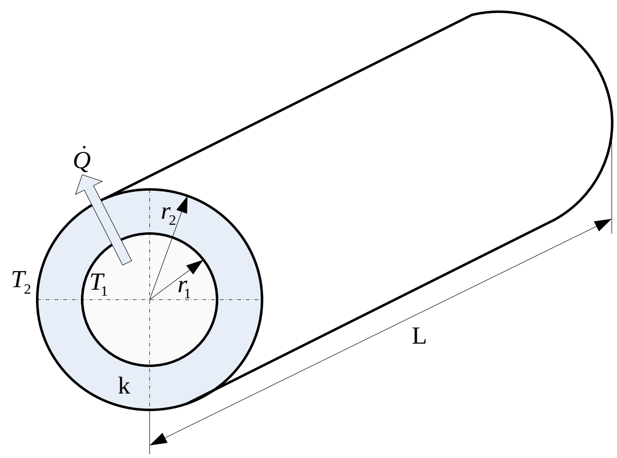


Figure 7.3: Radial heat conduction through a hollow cylinder

Let us consider a hollow cylinder or tube (pipe) of a length L and inside and outside radii of r_1 and r_2 respectively composed of a material having a thermal conductivity of k as shown in figure 7.3. Inside and outside peripheral surfaces of the cylinder are maintained at constant temperatures of T_1 and T_2 respectively, where T_1 is greater than T_2. Here, the temperature gradient exists in radial direction only, i.e., temperature is a function of radius, $T(r)$. Hence, the heat conduction through the pipe can be modeled as the steady one dimensional and Fourier's law of heat conduction can be applied as

$$\dot{Q} = -kA\frac{dT}{dr}$$

In this case, area perpendicular to the direction of heat flow is the peripheral surface area of the cylinder. Hence, the heat transfer area A depends on radius r and it varies in the direction of heat transfer. Thus, putting $A = 2\pi rL$ in above equation, we get

$$\dot{Q} = -k(2\pi r L)\frac{dT}{dr}$$

Rearranging above equation yields,

$$\dot{Q}\frac{dr}{r} = -k(2\pi L)dT$$

Integrating above equation with boundary conditions, from $r = r_1$, where $T(r_1) = T_1$ to $r = r_2$, where $T(r_2) = T_2$, we get

$$\dot{Q}\int_{r_1}^{r_2}\frac{dr}{r} = -\int_{T_1}^{T_2} k(2\pi L)dT$$

or, $$\dot{Q}\ ln\left(\frac{r_2}{r_1}\right) = -2\pi k L(T_2 - T_1) = 2\pi k L(T_1 - T_2)$$

Hence, the magnitude of radial heat conduction through a hollow cylinder is given by

$$\dot{Q} = \frac{2\pi k L(T_1 - T_2)}{ln\left(\frac{r_2}{r_1}\right)} \qquad \text{………. (7.9)}$$

When the rate of heat transfer by conduction $(\dot{Q})$ is known, the temperature *T*(r) at any location *r* within the wall thickness of the pipe can be determined by replacing temperature T_2 by *T(r)*, and outer radius r_2 by *r* in equation (7.9), i.e.,

$$T(r) = T_1 - \frac{\dot{Q}\ ln\left(\frac{r}{r_1}\right)}{2\pi k L} \qquad \text{………. (7.10)}$$

7.3. Heat Flow through Composite Structures: Composite Plane Walls and Multilayer Tubes

A composite structure is basically a combination of two or more materials each of which retains its own distinctive thermal properties. The resulting composite structure has different characteristics than that of the component materials in isolation. Thus, different arrangements of different materials with different thermal properties designed for a particular application is known as a composite structure. There are some examples of such systems (composite structures) which are made up of two or more layers of different materials, e.g. cold storage walls have a layer of bricks, a layer of thick insulation and plasters on both sides. Similarly, steam pipes have the cylindrical steel wall of the pipe, a layer of insulating material (e.g. rock wool) and then a layer of protecting plaster.

The analysis of the heat transfer by conduction in such a composite structures may be considered as an extension of the single wall structure as discussed earlier. For a composite structure the overall cumulative effect of the different values of the thermal resistance of each layer has to be accounted. Some assumptions are to be made to simplify the calculation, for example, the heat flow is steady state and one dimensional, there are no heat sources in the structures and the resistance due to interface contact is negligible as its value is small.

7.3.1. Composite Plane Wall

In practice we often encounter plane walls that consist of several layers of different materials such as a brick wall of a building with two layers of insulation plaster on both sides of the wall. The rate of steady state heat transfer through such composite wall can be determined by following the approach already used for the single layer case by noting that the rate of heat transfer by conduction $(\dot{Q})$ through a multilayer structure is constant and thus it must be the same through each layer.

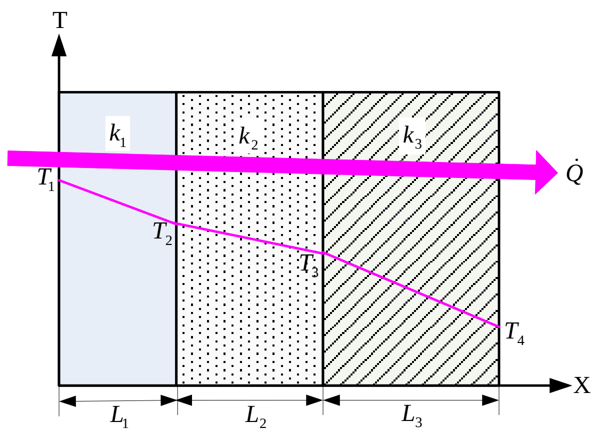


Figure 7.4. Heat conduction through a composite plane wall with three layers placed in series

Let us consider a multilayer plane wall consisting of three layers of three different materials placed in series so as to make a composite plane structure as shown in figure 7.4. Thicknesses of first, second and third layers are L_1, L_2 and L_3 respectively and thermal conductivities of each layer are k_1, k_2 and k_3 respectively. Inner and outer faces of the composite wall subjected to temperatures of T_1 and T_4 respectively. Interface temperature T_2 is between first and second layer as well as interface temperature T_3 is between second and third layer.

For steady state and one dimensional heat transfer by conduction, heat flowing through each layer of plane wall should be same, i.e.,

$$\dot{Q} = \frac{k_1 A(T_1 - T_2)}{L_1}$$

$$\dot{Q} = \frac{k_2 A(T_2 - T_3)}{L_2}$$

$$\dot{Q} = \frac{k_3 A(T_3 - T_4)}{L_3} \qquad \text{……….} \; (7.11)$$

The subscripts 1, 2 and 3 in the above relations indicate the first, second and third layer respectively. Rearranging the above relations, the temperature drop across each layer is easily determined as

$$T_1 - T_2 = \frac{\dot{Q}}{A}\frac{L_1}{k_1}$$

$$T_2 - T_3 = \frac{\dot{Q}}{A}\frac{L_2}{k_2}$$

$$T_3 - T_4 = \frac{\dot{Q}}{A}\frac{L_3}{k_3} \qquad \text{……….} \; (7.12)$$

Adding above equations yields

$$T_1 - T_4 = \frac{\dot{Q}}{A}\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right) \qquad \text{…...…….} \; (7.13)$$

Rearranging equation (7.13), an expression for conduction heat transfer through a composite plane wall can be derived as

$$\dot{Q} = \frac{A(T_1 - T_4)}{\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)} \qquad \text{……….} \; (7.14)$$

This result can be extended to plane walls that consist of four or more layers by adding an additional term L_i/K_i in denominator for each additional layer *i*.

Once $\dot{Q}$ is known from equation (7.14), an unknown surface temperature T_i at any surface or interface *i* can be determined by using equation (7.12) as

$$T_i = T_{i+1} + \frac{\dot{Q}}{A}\frac{L_i}{k_i} \qquad \text{... ... ... (7.15)}$$

7.3.2. Multilayered Tubes (Hollow Cylinder)

Steady state heat transfer through multilayered tubes or composite hollow cylinders can be handled just like composite or multilayered plane walls discussed earlier by simply considering as an extension of the single layers placed in series. For example, let us consider a composite hollow cylinder or pipe of length L consisting of three cylindrical layers of three different materials as shown in figure 7.5. First layer with inner and outer radii of r_1 and r_2 has a thermal conductivity of k_1, second layer with inner and outer radii of r_2 and r_3 has a thermal conductivity of k_2 and third layer with inner and outer radii of r_3 and r_4 has a thermal conductivity of k_3. Inner and outer surfaces of the composite layers are subjected to temperatures of T_1 and T_4 respectively. Interface temperature T_2 is between first and second layer as well as interface temperature T_3 is between second and third layer.

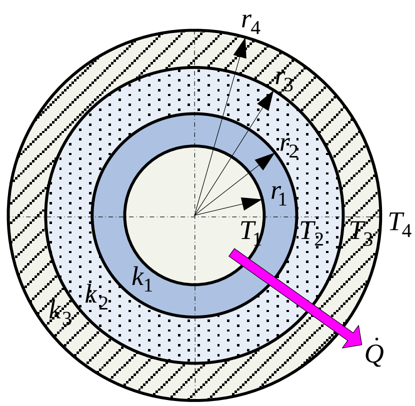


Figure 7.5: Heat conduction through a three - layered composite hollow cylinder (pipe)

For steady state radial (one dimensional) heat transfer by conduction heat flowing through each layer of hollow cylinder should be same, i.e.

$$\dot{Q} = \frac{2\pi k_1 L(T_1 - T_2)}{ln\left(\frac{r_2}{r_1}\right)}$$

$$\dot{Q} = \frac{2\pi k_2 L(T_2 - T_3)}{ln\left(\frac{r_3}{r_2}\right)}$$

$$\dot{Q} = \frac{2\pi k_3 L(T_3 - T_4)}{ln\left(\frac{r_4}{r_3}\right)} \qquad \text{………. (7.16)}$$

Rearranging the above relations, the temperature drop across each layer is easily determined as

$$T_1 - T_2 = \frac{\dot{Q}}{2\pi L}\frac{ln\left(\frac{r_2}{r_1}\right)}{k_1}$$

$$T_2 - T_3 = \frac{\dot{Q}}{2\pi L}\frac{ln\left(\frac{r_3}{r_2}\right)}{k_2}$$

$$T_3 - T_4 = \frac{\dot{Q}}{2\pi L}\frac{ln\left(\frac{r_4}{r_3}\right)}{k_3} \qquad \ldots\ldots\ldots (7.17)$$

Adding above equations yields

$$T_1 - T_4 = \frac{\dot{Q}}{2\pi L}\left(\frac{ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{ln\left(\frac{r_4}{r_3}\right)}{k_3}\right) \qquad \ldots\ldots\ldots (7.18)$$

Rearranging equation (7.18), an expression for conduction heat transfer through a composite hollow cylinder can be expressed as

$$\dot{Q} = \frac{2\pi L(T_1 - T_4)}{\left(\frac{ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{ln\left(\frac{r_4}{r_3}\right)}{k_3}\right)} \qquad \ldots\ldots\ldots\ldots (7.19)$$

This result can be extended to hollow cylinders that consist of four or more layers by adding an additional term $\ln(\frac{r_{i+1}}{r_i})/k_i$ in denominator for each additional layer *i*.

Once $\dot{Q}$ has been calculated from equation (7.19), an unknown surface temperature $T_{\boldsymbol{i}}$ at any peripheral surface or interface *i* can be determined by using equation (7.17) as

$$T_{\boldsymbol{i}} = T_{i+1} + \frac{\dot{Q}}{2\pi L}\frac{\ln(\frac{r_{i+1}}{r_i})}{k_i} \qquad \ldots \ldots \ldots (7.20)$$

Lecture Highlights

- *Heat transfer:* It is the transfer of energy without transfer of mass from one region to another region as a result of temperature difference. Heat transfer takes place from high temperature to low temperature region by three modes, namely conduction, convection and radiation.
- *Applications of heat transfer:* It is widely used in most of the engineering disciplines such as
  - Design of thermal and nuclear power plants, heat engines, boilers, condensers, heat exchangers etc.
  - Heat pump, refrigeration and air conditioning units.
  - Design of cooling systems for electric motors, generators and transformers.
  - Construction of structures and minimization of building heat losses.
  - Thermal control of space vehicles.
- *Differences between thermodynamics and heat transfer are:*

Thermodynamics	Heat transfer
1. It deals with the equilibrium states of a system. 2. There should not be a temperature gradient. 3. It helps to determine the amount of work and heat transfers when a system undergoes from one equilibrium state to another. However, it cannot help to find the temperature of the system at any interval of time before the equilibrium condition is attained.	1. It occurs only when a system is in a non-equilibrium state. 2. Heat transfer takes place only when there is temperature gradient. 3. It helps to predict the distribution of temperature as a function of time and determine the heat energy transferred per unit time from one region to another due to temperature difference.

- *Conduction:* It is the mode of heat transfer due to temperature gradient from one part of a substance to other part or one substance to another in perfect physical contact without actual movement of molecules.
- In solids, heat is transferred through conduction by the following two mechanisms: by movement of free electrons and lattice (crystal) vibrations. Whereas heat conduction in liquids and gases occurs comparatively in negligible amount and is based on the movements of atoms and molecules.
- *Thermal conductivity* of the good conductors decreases when temperature increases, whereas that of the insulators and alloys increases with increase in temperature.
- *Assumptions:* The following assumptions are made for calculation of rate of heat flow by using different governing laws of heat transfer:
  - Conduction of heat occurs under steady state conditions.
  - Heat flow is unidirectional.
  - No heat source within medium.

- *Fourier law:* It is the governing law of heat conduction which states that "The rate of heat flow per unit area normal to the direction of heat flow through a body is directly proportional to the temperature gradient in that direction". Mathematically,

  - $$\frac{\dot{Q}}{A} \propto \frac{dT}{dx} \qquad \Rightarrow \qquad \dot{Q} = -kA\frac{dT}{dx}$$

  where k is *thermal conductivity* of the material and the negative sign indicates that heat flows in the direction of decreasing temperature. The thermal conductivity depends essentially upon the factors such as material structure, moisture content, density of the material and operating conditions such as pressure and temperature.
- *Convection*: It is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion. It occurs by the actual movement of the molecules thereby causing molecular collisions and molecular diffusions.
- *Newton's law of cooling*: It is the governing law of convection heat transfer and states that "The rate of heat flow per unit surface area is directly proportional to the temperature difference between a solid surface and an adjacent fluid".

  $$\frac{\dot{Q}}{A_s} \propto \left(T_s - T_\infty\right) \qquad \Rightarrow \qquad \dot{Q} = hA_s\left(T_s - T_\infty\right)$$

  where h is the *heat transfer coefficient of convection* and it depends upon the thermodynamic and transport properties such as density, viscosity, specific heat and thermal conductivity of the fluid, the geometry of the surface, the nature of the fluid flow, and the prevailing thermal conditions.
- *Rate of heat flow through a plane wall* given by

  $$\dot{Q} = \frac{kA}{L}(T_1 - T_2) = \frac{\Delta T_{overall}}{R_{th,wall}}$$

  where $R_{th,wall} = \dfrac{L}{kA}$
- *Rate of heat flow through a plane wall with three layers* is given by

  $$\dot{Q} = \frac{A(T_1 - T_4)}{\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)} = \frac{\Delta T_{overall}}{R_{total}}$$

  where $R_{total} = \dfrac{1}{A}\left(\dfrac{L_1}{k_1} + \dfrac{L_2}{k_2} + \dfrac{L_3}{k_3}\right)$
- *Rate of heat flow through a hollow cylinder* is given by

  $$\dot{Q} = \frac{2\pi kL(T_1 - T_2)}{ln\left(\frac{r_2}{r_1}\right)} = \frac{\Delta T_{overall}}{R_{th,cyl}}$$

  where $R_{th,cyl} = \dfrac{\ln(r_2/r_1)}{2\pi kL}$

- *Rate of heat flow through a hollow cylinder with three layers* is given by

$$\dot{Q} = \frac{2\pi L(T_1 - T_4)}{\left(\frac{ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{ln\left(\frac{r_4}{r_3}\right)}{k_3}\right)} = \frac{\Delta T_{overall}}{R_{total}}$$

$$\text{where } R_{total} = \frac{1}{2\pi L}\left[\frac{\ln(r_2/r_1)}{k_1} + \frac{\ln(r_3/r_2)}{k_2} + \frac{\ln(r_4/r_3)}{k_3}\right]$$

- *Radiation:* It is the mode of heat transfer which does not require any medium, i.e. heat transfer takes place in vacuum in the form of electromagnetic wave.
- *Stefan – Boltzmann law:* It states that "The maximum rate of heat radiated per unit area (heat flux) by a surface is directly proportional to the fourth power of the absolute surface temperature". i.e.

$$\frac{\dot{Q}}{A_s} \propto T_s^4 \qquad \Rightarrow \qquad \dot{Q} = \sigma\, A_s\, T_s^4$$

where σ = $5.67 \times 10^{-8} W/m^2K^4$ is the Stefan – Boltzmann constant and this maximum rate of heat is radiated by a black body surface.
- *The rate of heat radiation from a real surface is given by*

$$\dot{Q} = \varepsilon\, \sigma\, A_s\, T_s^4$$

where Ɛ is the emissivity of a real surface and is defined as the ratio of the rate of heat radiated by a real body to that by a black body at the same surface temperature.

- When a surface of emissivity Ɛ and surface area A_s at a thermodynamic temperature T_s is completely enclosed by a much larger surface or a black body at thermodynamic temperature T_{surr} separated by a gas such as air that does not intervene with radiation, the net rate of radiation heat transfer between these two surfaces is given by

$$\dot{Q}_{net} = \varepsilon \sigma A_s \left(T_s^4 - T_{surr}^4\right)$$

- *Differences between different modes of heat transfer:*

Conduction	**Convection**	**Radiation**
1. It is transfer of heat mainly in solid medium and not in vacuum. 2. It involves free electron movement and lattice vibration and does not involve actual movement of atoms and molecules. 3. It occurs in short range of distance.	1. It is transfer of heat mainly in liquid or gas medium and not in vacuum. 2. It involves actual movement of atoms and molecules. 3. It occurs in medium range of distance.	1. It is transfer of heat in all mediums but in vacuum it is most efficient. 2. t involves the movement of photons as electro-magnetic wave. 3. Wave can travel with speed of light, so it occurs in long range of distance.

4. It depends on thermal conductivity *k* of material. 5. Heat transfer is calculated by using Fourier's law as $\dot{Q} = -kA\frac{dT}{dx}$ 6. For example, heat transfer in solid, plain wall, hollow cylinder etc.	4. It depends on heat transfer coefficient *h* of convective surface. 5. Heat transfer is calculated by using Newton's law of cooling as $\dot{Q} = hA_s\Delta T$ 6. For example, heat transfer across the room air from heater surface.	4. It depends on emissivity of radiating surface. 5. Heat radiated by a surface is calculated by using Stefan Bolzmann law as $\dot{Q} = \in \sigma A_s T^4$ 6. For example, heat transfer from the sun to the earth.

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