Course Title:

Fundamental of Thermodynamics and Heat Transfer

Lecture 16 (Week 16):

Heat Transfer

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Learning Objective of Lecture:

To impart a great deal of knowledge to undergraduate students on the following topics:

- ✓ Electrical analogy for thermal resistance
- ✓ Combined heat transfer and overall heat transfer coefficient for plane wall and tube
- ✓ Nature of Convection: Free and Forced Convection
- ✓ Heat Radiation: Types of Bodies with respect to Radiation: Black Body, White Body and Gray Body; Radiation surface properties

7.4. Electrical Analogy for Thermal Resistance

The electrical analogy for heat transfer or the thermal resistance concept is widely used in practice because it is intuitively easy to understand and it has proven to be a powerful tool in the solution of a wide range of heat transfer problems that involve parallel layers or combined series -parallel arrangements. But its use is limited to systems through which the rate of heat transfer remains constant; i.e., to systems involving steady state heat transfer with no heat generation such as resistance heating or chemical reactions within the medium.

More complex heat transfer problems involving both series and parallel arrangements can therefore be solved by using an electric analogy, i.e., by converting the given problem into an equivalent electrical circuit and applying the appropriate network theorem.

Equations derived in earlier sections for the rate of heat transfer $\dot{Q}$ are analogous to the relation for electric current I flowing through a wire which can be expressed as

$$I = \frac{\Delta V}{R_e} = \frac{(V_1 - V_2)}{L/\sigma_e A} \qquad \text{.......... (7.21)}$$

where $\Delta V = (V_1 - V_2)$ is the voltage difference across the resistance, $R_e = L/\sigma_e A$ is the electric resistance, σ_e is the electrical conductivity, L is the length and A is the cross sectional area of the wire. The relation given by equation (7.21) is called *Ohm's law* in electricity which tells that current (I) flows due to the difference in electric potential (ΔV) and the property of the substance by virtue of which it opposes the flow of current through it is called the electric resistance (R_e).

Equation for the rate of heat transfer $\dot{Q}$ can be expressed in the similar manner as

$$\dot{Q} = \frac{\Delta T}{R_{th}} = \frac{T_1 - T_2}{R_{th}} \qquad \text{.......... (7.22)}$$

where $\Delta T = (T_1 - T_2)$ is the temperature difference and R_{th} is the thermal resistance. Equation (7.22) indicates that rate of heat ($\dot{Q}$) flows due to the difference in temperature (ΔT) and the property of the substance by virtue of which it opposes the flow of heat through it is called the thermal resistance (R_{th}). Thermal resistance is expressed in K/W or $^0C/W$.

(a) Heat flow (b) Electric current flow

Figure 7.6. Analogy between thermal and electrical resistance concepts

Thus, the rate of heat transfer through a layer corresponds to the electric current, the thermal resistance corresponds to electrical resistance, and the temperature difference corresponds to voltage difference across the layer as shown in figure 7.6. In the followings, thermal resistances for different modes of heat transfer with different configurations will be determined.

7.4.1. Thermal Resistance of a Plane Wall

The rate of the heat transfer by conduction through a plane wall given by equation (7.7) can also be rearranged as

$$\dot{Q} = \frac{T_1 - T_2}{L/kA} \qquad \text{.......... (7.23)}$$

Comparing equations (7.22) and (7.23), an expression for the thermal resistance of a plane wall can be derived as

$$R_{th,wall} = \frac{L}{kA} \qquad \text{.......... (7.24)}$$

which is the resistance of the plane wall against heat conduction or simply the conduction resistance of the plane wall as shown in figure (7.7). The thermal resistance of a medium depends on the geometry and the thermal properties of the medium.

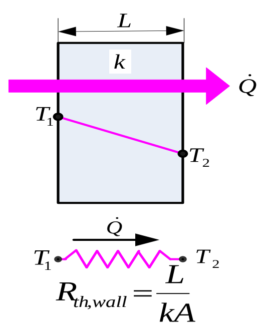

Figure 7.7. The thermal resistance circuit for conduction heat transfer through a plane wall [1]

7.4.2. Thermal Resistance of a Hollow Cylinder

The rate of the heat transfer through a hollow cylinder or pipe given by equation (7.9) can also be rearranged as

$$\dot{Q} = \frac{T_1 - T_2}{\frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi kL}} \qquad \text{.......... (7.25)}$$

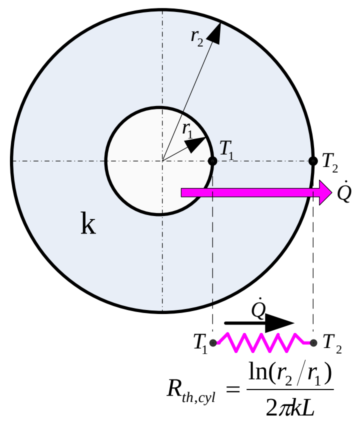

Figure 7.8. The thermal resistance circuit for conduction heat transfer through a hollow cylinder [1]

Comparing equations (7.22) and (7.25), an expression for the thermal resistance of a hollow cylinder can be derived as

$$R_{th,cyl} = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi kL} \quad \text{………. (7.26)}$$

which is the resistance of the pipe or cylindrical layer against heat conduction, or simply the conduction resistance of the pipe or cylindrical layer as shown in figure 7.8.

7.4.3. Thermal Resistance of a Convective Layer

The rate of the heat transfer by convection between a solid surface and a fluid layer given by equation (7.3) can also be rearranged as

$$\dot{Q} = \frac{T_s - T_\infty}{\frac{1}{hA_s}} \quad \text{………. (7.27)}$$

Comparing equations (7.22) and (7.27), an expression for the thermal resistance of a convective layer can be derived as

$$R_{th,conv} = \frac{1}{hA_s} \quad \text{………. (7.28)}$$

which is the resistance of the solid surface against the heat convection, or simply the convection resistance of the solid surface as shown in figure 7.9. When the convection heat transfer coefficient is very large (h → ∞), the convection resistance becomes zero and $T_s \approx T_\infty$. That means the solid surface offers no resistance to convection, and thus it does not slow down the heat transfer process. This situation is approached in practice at surfaces where boiling and condensation occur. Equation (7.28) for convection resistance is valid for surfaces of any shape provided that the assumption of the convection heat transfer coefficient, h is constant and uniform is reasonable.

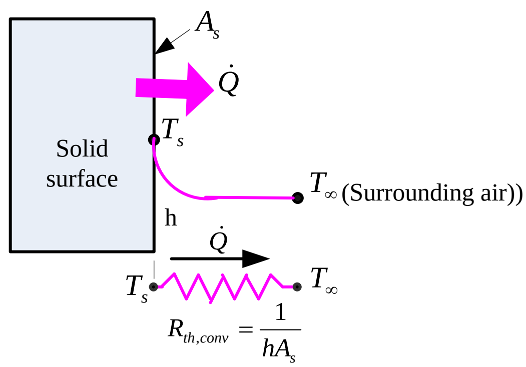

Figure 7.9. Schematic diagram for thermal resistance of convection heat transfer at a solid surface [1]

7.4.4. Thermal Resistance of a Radiation Layer

The rate of the heat transfer by radiation between a surface of emissivity $\in$ and area A_s at temperature T_s and the surrounding surfaces at some average temperature T_∞ given by equation (7.6) can also be rearranged as

$$\dot{Q} = \frac{(T_s^4 - T_\infty^4)}{\frac{1}{\in \sigma A_s}} = \frac{T_s - T_\infty}{\frac{1}{\in \sigma A_s (T_s + T_\infty)(T_s^2 + T_\infty^2)}} \qquad \text{........(7.29)}$$

Comparing equations (7.22) and (7.29), an expression for the thermal resistance of a radiation layer can be derived as

$$R_{th,rad} = \frac{1}{\in \sigma A_s (T_s + T_\infty)(T_s^2 + T_\infty^2)} \qquad \text{.......... (7.30)}$$

which is the resistance of a solid surface against the heat radiation, or simply the radiation resistance of the solid surface as shown in figure 7.10. Both T_s and T_∞ must be in Kelvin in the evaluation of $R_{th,rad}$ by using equation (7.30). The definition of the thermal resistance for the radiation heat transfer enables us to express radiation conveniently in an analogous manner to convection in terms of a temperature difference. But the thermal resistance for the radiation heat transfer $R_{th,rad}$ depends strongly on Kelvin temperature of the solid surface while that for the convection heat transfer $R_{th,conv}$ usually does not.

If a surface exposed to the surrounding air involves the convection and the radiation simultaneously, then the total heat transfer at the surface is determined by adding the radiation and convection components.

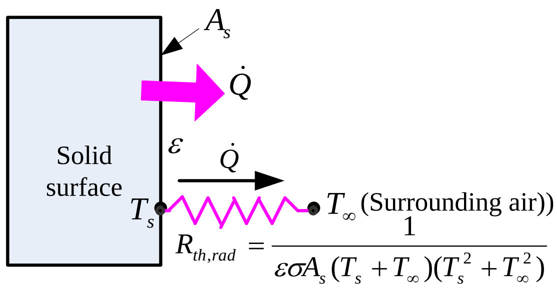

Figure 7.10. Schematic diagram for thermal resistance of radiation heat transfer from a solid surface [1]

7.4.5. Applications of Electrical Analogy Approach in Heat Transfer through Composite Structures

The electrical analogy or the thermal resistance concept can also be used to solve steady heat transfer problems that involve series layers, parallel layers or combined series-parallel arrangements. Although such problems are often two- or even three-dimensional, approximate solutions can be obtained by assuming one dimensional heat transfer and using the thermal resistance circuit.

7.4.5.1. Heat Transfer through a Composite Plane Wall in Series Arrangement

Let us consider a composite plane wall shown in figure 7.11 which consists of three layers in series arrangement. The thermal resistance circuit which consists of three series resistances can be represented as shown in the same figure.

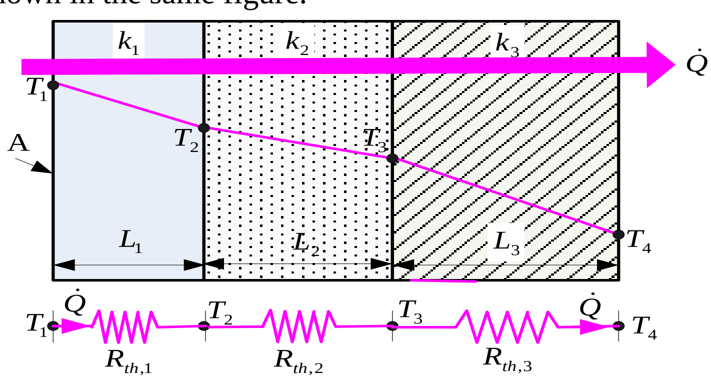

Figure 7.11: The thermal resistance circuit for heat transfer through a three-layered composite plane wall in series arrangement

In this arrangement same amount of heat flows through each layer of the plane wall. All three thermal resistances are therefore placed in series. Analogous to a voltage drop in case of current flow through an electric resistance, there is a temperature drop when heat flows through a thermal resistance. As mentioned earlier, thermal resistances for each layer of the plane wall are given as

$$R_{th,1} = \frac{L_1}{Ak_1} \quad , \quad R_{th,2} = \frac{L_2}{Ak_2} \quad \text{and} \quad R_{th,3} = \frac{L_3}{Ak_3}$$

The resistances are in series in the thermal resistance circuit, and thus the total thermal resistance is simply the arithmetic sum of the individual thermal resistances in the path of heat transfer. Hence, the total thermal resistance is derived as

$$R_{total} = R_{th,1} + R_{th,2} + R_{th,3} = \frac{1}{A}\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right) \qquad \text{……… (7.31)}$$

Then rate of overall heat transfer for a three layered composite plane wall in series arrangement is given by

$$\dot{Q} = \frac{\Delta T_{overall}}{R_{total}} = \frac{T_1 - T_4}{\frac{1}{A}\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)} = \frac{A(T_1 - T_4)}{\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)} \qquad \text{……… (7.32)}$$

Equation (7.32) is identical to equation (7.14) obtained previously for a composite plane wall in section 7.6.1.

7.4.5.2. Heat Transfer through a Composite Hollow Cylinder in Series Arrangement

Let us consider a composite hollow cylinder or pipe which consists of three layers in series arrangement as shown in figure 7.12. The thermal resistance circuit which consists of three series resistances can be represented as shown in the same figure.

In this arrangement same amount of heat flows through each layer of the hollow cylinder or pipe in radial direction. All three thermal resistances are therefore placed in series. As mentioned earlier, thermal resistances for each layer of the hollow cylinder or pipe are given as

$$R_{th,1} = \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi k_1 L} \quad , \quad R_{th,2} = \frac{ln\left(\frac{r_3}{r_2}\right)}{2\pi k_2 L} \quad \text{and} \quad R_{th,3} = \frac{ln\left(\frac{r_4}{r_3}\right)}{2\pi k_3 L}$$

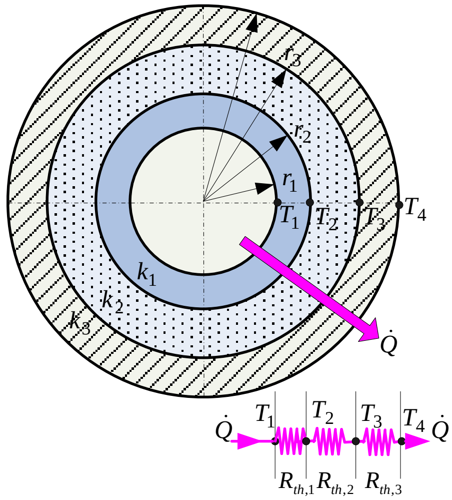

Figure 7.12: The thermal resistance circuit for heat transfer through a three-layered composite hollow cylinder in series arrangement

The resistances are in series in the thermal resistance circuit, and thus the total thermal resistance is simply the arithmetic sum of the individual thermal resistances in the path of heat transfer. Hence, the total thermal resistance is derived as

$$R_{total} = R_{th,1} + R_{th,2} += \frac{1}{2\pi L}\left(\frac{ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{ln\left(\frac{r_4}{r_3}\right)}{k_3}\right) \quad \text{.......(7.33)}$$

Then rate of overall heat transfer for a three - layered composite hollow cylinder or pipe in series arrangement is given by

$$\dot{Q} = \frac{\Delta T_{overall}}{R_{total}} = \frac{T_1 - T_4}{\frac{1}{2\pi L}\left(\frac{ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{ln\left(\frac{r_4}{r_3}\right)}{k_3}\right)}$$

$$\dot{Q} = \frac{2\pi L(T_1 - T_4)}{\left(\frac{ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{ln\left(\frac{r_4}{r_3}\right)}{k_3}\right)} \quad \text{..............(7.34)}$$

Equation (7.34) is identical to equation (7.19) obtained previously for a composite hollow cylinder in section 7.6.2.

7.4.5.3. Heat Transfer through Composite Structure in Combined Series - Parallel Arrangement

Thermal resistance circuit or electric analogy approach becomes more suitable when the composition or structure of the wall becomes more complex. This means the thermal resistance concept is appropriate to solve steady heat transfer problems that involve parallel layers or combined series-parallel arrangements. However, the result obtained is somewhat approximate, since the surfaces of the third layer after two parallel layers are probably not isothermal. Hence, two assumptions commonly used in solving complex multidimensional heat transfer problems by treating them as one-dimensional (e.g., in the *x*-direction) using the thermal resistance circuit are

(i) any plane wall normal to the *x*-axis is *isothermal* (i.e. to assume the temperature to vary in the *x*-direction only) and
(ii) any plane parallel to the *x*-axis is *adiabatic* (i.e. to assume heat transfer to occur in the *x*-direction only).

Let us consider the composite plane wall which consists of the combined series - parallel arrangement as shown in figure 7.13. The thermal resistance circuit which consists of two parallel resistances and two series resistances (for a conduction layer and a convective layer) can be represented as shown in the same figure.

In this arrangement different amount of heat flows through each parallel conduction layer (1 and 2) and same amount of heat flows through series layers namely a conduction layer 3 and a convective layer. Therefore, two thermal resistances ($R_{th,1}$, $R_{th,2}$) are placed in parallel and two thermal resistances ($R_{th,3}$, $R_{th,conv}$) are arranged in series. The total thermal resistance for the given circuit is then derived as

$$R_{total} = \frac{R_{th,1} \times R_{th,2}}{R_{th,1} + R_{th,2}} + R_{th,3} + R_{th,conv} \qquad \text{.........(7.35)}$$

where thermal resistances for each layer of the plane wall and for convective layer are given as

$$R_{th,1} = \frac{L_1}{A_1 k_1}, \quad R_{th,2} = \frac{L_2}{A_2 k_2}, \quad R_{th,3} = \frac{L_3}{A_3 k_3} \quad \text{and} \quad R_{th,conv} = \frac{1}{hA_3}$$

Then rate of overall heat transfer for a composite plane wall in the combined series-parallel arrangement is given by

$$\dot{Q} = \frac{\Delta T_{overall}}{R_{total}} = \frac{T_1 - T_\infty}{\frac{R_{th,1} \times R_{th,2}}{R_{th,1} + R_{th,2}} + R_{th,3} + R_{th,conv}} \qquad \text{...........(7.36)}$$

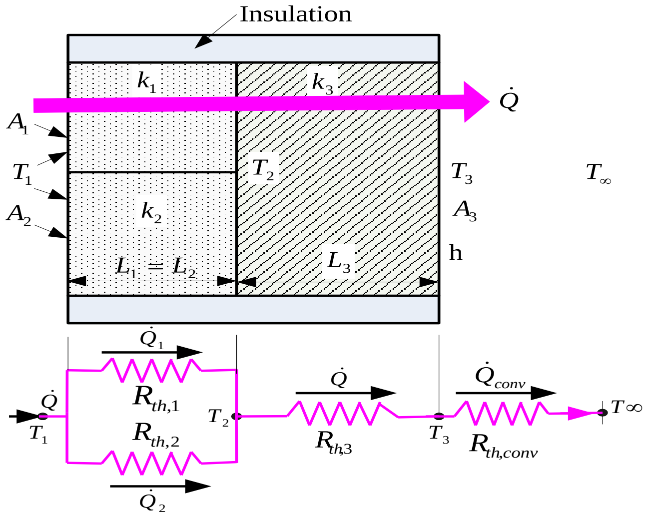

Figure7.13. Thermal resistance circuit for combined series-parallel arrangement [1]

7.5. Combined Heat Transfer and Overall Heat Transfer Coefficient for a Plane Wall and Tube

In Engineering practice the heat transfer or heat exchange often occurs not only between solid surfaces but also between a solid surface and a fluid (liquid, gas) in contact with it. It may happen in some cases all modes of heat transfer (conduction, convection and radiation) are to be taken in account for the calculation of the rate of heat transfer. Among these modes, the problems involving the conduction and convection only will be discussed in this section.

When a stationary solid surface comes in contact with a moving fluid, a thin boundary layer develops adjacent to the wall and there is no relative velocity with respect to surface in this layer. This thin layer is known as *stagnant film* and heat flow in this film is covered by conduction and convection processes as well. As thermal conductivity of fluid is low, the heat transfer from the moving fluid to the wall is mainly due to convection. The resistance of the stagnant film is included in the convective heat transfer coefficient (h) and the rate of convective heat transfer is determined by using the Newton's law of cooling as mentioned earlier.

7.5.1. Plane Wall Subjected to Convective Medium on both Sides

Let us consider steady state one dimensional heat transfer through a plane wall of thickness L, area A, and thermal conductivity k that is exposed to convection on both sides as shown in figure 7.14. The left face of the plane wall is exposed to a hot fluid at temperature $T_{\infty 1}$ with heat transfer coefficient h_1 and the right face is exposed to a cold fluid at temperature $T_{\infty 2}$ with heat transfer coefficient h_2. As the temperature of the hot fluid $T_{\infty 1}$ is greater than that of the cold fluid $T_{\infty 2}$, the temperature varies linearly in the plane wall, and asymptotically approaches $T_{\infty 1}$ and $T_{\infty 2}$ in the fluids as we move away from the plane wall as shown in the figure.

For steady state one dimensional heat transfer, rate of the heat flow through each conductive and convective layer should be same. i.e.

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1)$$
$$\dot{Q} = \frac{kA}{L}(T_1 - T_2) \qquad \text{……….} (7.37)$$
$$\dot{Q} = h_2 A(T_2 - T_{\infty 2})$$

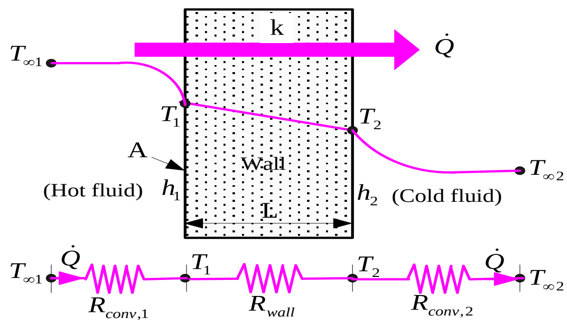

Figure 7.14: The heat transfer through a plane wall subjected to convection on both sides with equivalent thermal resistance circuit [1]

Rearranging the above equations for the temperature differences, we get

$$T_{\infty 1} - T_1 = \frac{\dot{Q}}{A}\frac{1}{h_1}$$
$$T_1 - T_2 = \frac{\dot{Q}}{A}\frac{L}{k} \qquad \text{……….} (7.38)$$
$$T_2 - T_{\infty 2} = \frac{\dot{Q}}{A}\frac{1}{h_2}$$

Adding above equations yields

$$T_{\infty 1} - T_{\infty 2} = \frac{\dot{Q}}{A}\left(\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}\right) \qquad \text{……….} (7.39)$$

Rearranging equation (7.39), an expression for rate of the overall heat transfer for combined conduction and convection heat transfer can be derived as,

$$\dot{Q} = \frac{A(T_{\infty 1} - T_{\infty 2})}{\left(\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}\right)} \quad \text{………. (7.40)}$$

It is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA(T_{\infty 1} - T_{\infty 2}) \quad \text{……. (7.41)}$$

A comparison of equations (7.40) and (7.41) reveals that

$$U = \frac{1}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}} \quad \text{………. (7.42)}$$

where U is called overall or combined heat transfer coefficient.

Application of Electric Analogy Approach

The above heat transfer problem can also be easily solved by using the electric analogy approach or thermal resistance circuit. The equivalent thermal resistance circuit for the heat transfer problem discussed above is shown in the same figure 7.14.

Since the thermal resistances are in series, the total thermal resistance is determined by simply adding the individual resistances, just like the electrical resistances connected in series. Hence, the total thermal resistance for the circuit is then given by

$$R_{total} = R_{conv,1} + R_{wall} + R_{conv,2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} = \frac{1}{A}\left(\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}\right) \quad \text{……..… (7.43)}$$

Then rate of the overall heat transfer for a plane wall subjected to convective medium on both sides is given by

$$\dot{Q} = \frac{\Delta T_{overall}}{R_{total}} = \frac{(T_{\infty 1} - T_{\infty 2})}{\frac{1}{A}\left(\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}\right)} = \frac{A(T_{\infty 1} - T_{\infty 2})}{\left(\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}\right)} \quad \text{………. (7.44)}$$

This equation (7.44) derived by using electric analogy approach is identical to equation (7.40). Thus, it is not required to know the surface temperatures of the wall in order to evaluate the rate of steady state heat transfer through it. All we need to know is the convection heat transfer coefficients and the fluid temperatures on both sides of the wall.

7.5.2. Hollow Cylinder Subjected to Convective Medium on both Sides

Let us consider a hollow cylinder or pipe with inner and outer radii of r_1 and r_2, length L and thermal conductivity k as shown in figure 7.15. The inner surface of the cylinder is exposed to a hot fluid with temperature $T_{\infty 1}$ and convective heat transfer coefficient of h_1 whereas the outer surface of the cylinder is exposed to a cold fluid with temperature $T_{\infty 2}$ and convective heat transfer coefficient of h_2.

For steady state one dimensional heat transfer, rate of the heat flow through each layer should be same. i.e.

$$\dot{Q} = h_1 A_1 (T_{\infty 1} - T_1)$$

$$\dot{Q} = \frac{2\pi k L}{ln\left(\frac{r_2}{r_1}\right)} (T_1 - T_2)$$

$$\dot{Q} = h_2 A_2 (T_2 - T_{\infty 2}) \qquad \text{……….} (7.45)$$

where A_1 and A_2 are the inside and outside peripheral surface areas of the hollow cylinder or pipe respectively.

Rearranging the above equations for the temperature differences, we get

$$T_{\infty 1} - T_1 = \frac{\dot{Q}}{A_1} \frac{1}{h_1}$$

$$T_1 - T_2 = \dot{Q} \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi k L}$$

$$T_2 - T_{\infty 2} = \frac{\dot{Q}}{A_2} \frac{1}{h_2} \qquad \text{……….} (7.46)$$

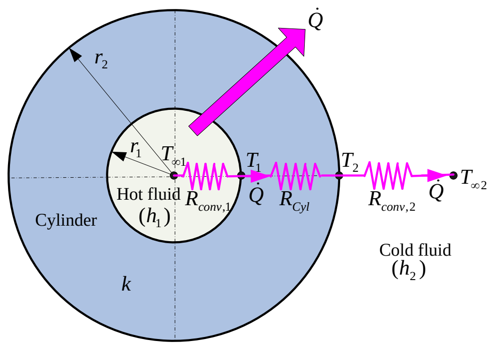

Figure 7.15. The heat transfer through a hollow cylinder subjected to convection from both the inner and the outer sides with thermal resistance circuit [1].

Adding above equations yields

$$T_{\infty 1} - T_{\infty 2} = \dot{Q}\left(\frac{1}{h_1 A_1} + \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi kL} + \frac{1}{h_2 A_2}\right) \quad \text{.......... (7.47)}$$

Rearranging equation (7.47), an expression for overall heat transfer for combined conduction and convection heat transfer can be derived as,

$$\dot{Q} = \frac{(T_{\infty 1} - T_{\infty 2})}{\left(\frac{1}{h_1 A_1} + \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi kL} + \frac{1}{h_2 A_2}\right)} \quad \text{....... (7.48)}$$

Equation (7.48) can also be expressed as

$$\dot{Q} = \frac{A_1(T_{\infty 1} - T_{\infty 2})}{\left(\frac{1}{h_1} + \frac{A_1}{2\pi kL} ln\left(\frac{r_2}{r_1}\right) + \frac{A_1}{A_2}\frac{1}{h_2}\right)} \quad \text{....... (7.49)}$$

Or, $$\dot{Q} = \frac{A_2(T_{\infty 1} - T_{\infty 2})}{\left(\frac{A_2}{A_1}\frac{1}{h_1} + \frac{A_2}{2\pi kL} ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_2}\right)} \quad \text{....... (7.50)}$$

Since it is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling, equations (7.49) and (7.50) can also be expressed in the simpler form as

$$\dot{Q} = U_1 A_1 (T_{\infty 1} - T_{\infty 2}) \quad \text{.......... (7.51)}$$

and $$\dot{Q} = U_2 A_2 (T_{\infty 1} - T_{\infty 2}) \quad \text{.......... (7.52)}$$

A comparison of equations (7.49) and (7.51) as well as equations (7.50) and (7.52) reveals that

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{A_1}{2\pi kL} ln\left(\frac{r_2}{r_1}\right) + \frac{A_1}{A_2}\frac{1}{h_2}} \quad \text{………. (7.53)}$$

and

$$U_2 = \frac{1}{\frac{A_2}{A_1}\frac{1}{h_1} + \frac{A_2}{2\pi kL} ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_2}} \quad \text{………. (7.54)}$$

where U_1 and U_2 are called inside overall heat transfer coefficient based on the internal peripheral area A_1 and outside overall heat transfer coefficient based on the external peripheral area A_2 of the hollow cylinder respectively.

Putting for the inner and outer peripheral areas of the hollow cylinder $A_1 = 2\pi r_1 L$ and $A_2 = 2\pi r_2 L$, equations (7.53) and (7.54) reduce to

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{r_1}{k} ln\left(\frac{r_2}{r_1}\right) + \frac{r_1}{r_2}\frac{1}{h_2}} \quad \text{………. (7.55)}$$

and

$$U_2 = \frac{1}{\frac{r_2}{r_1}\frac{1}{h_1} + \frac{r_2}{k} ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_2}} \quad \text{………..(7.56)}$$

Application of Electric Analogy Approach

The above heat transfer problem can also be easily solved by using the electric analogy approach or thermal resistance circuit. The equivalent thermal resistance circuit for the heat transfer problem discussed above is shown in the same figure 7.15.

Since the thermal resistances are in series, the total thermal resistance is determined by simply adding the individual resistances, just like the electrical resistances connected in series. Hence, the total thermal resistance for the circuit is then given by

$$R_{total} = R_{conv,1} + R_{cyl} + R_{conv,2} = \frac{1}{h_1 A_1} + \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi kL} + \frac{1}{h_2 A_2} \quad \text{………. (7.57)}$$

Then rate of the overall heat transfer for a composite hollow cylinder is given by

$$\dot{Q} = \frac{\Delta T_{overall}}{R_{total}} = \frac{(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1 A_1} + \frac{ln\left(\frac{r_2}{r_1}\right)}{2\pi kL} + \frac{1}{h_2 A_2}} \quad \text{……….} (7.58)$$

This equation (7.58) derived by using electric analogy approach is identical to equation (7.48).

7.6. Nature of Heat Convection: Free and Forced Convection

Convection is one of the major modes of heat transfer. Convective heat transfer occurs due to the random motion of molecules within fluid medium (diffusion) and it may happen between a solid surface and the fluid flowing over it as mentioned earlier. According to the mechanism of the fluid flow or fluid motion over the solid surface, convection heat transfer is classified into following two types: free convection and forced convection.

7.6.1. Free or Natural Convection

Convection heat transfer process in which the flow or motion of a fluid is caused by a density gradient due to the temperature gradient is called free convection or natural convection. During the free convection warmer or lighter (low density) molecules near the hot surface rise while cooler or heavier (high density) molecules will fall to fill their place which leads to a bulk movement of the fluid. Thus, the free convection can only occur in a gravitational field. During the free convection, the rate of mass flow of a fluid is usually low and the heat transfer takes place at very low rate. A common example of the free convection is the cooling of a room without a fan by natural circulation of air.

7.6.2. Forced Convection

Forced convection is a mechanism of heat transfer in which the flow or motion of a fluid is caused by some external sources or devices such as pump, fan, blower, etc. The motion or mass flow rate of the fluid in such cases will be relatively higher than that in free convection and therefore will result in increased rate of heat transfer which produces results more quickly than in free convection. However, in any forced convection situation, some amount of free convection is always present whenever there is gravitational force exist. When the free convection is not negligible, such flows are typically referred to as *mixed convection*. There are so many examples of the forced convection (e,g. heat exchangers, air conditioning, central heating etc.), but a common example of the forced convection is the cooling of a room by using a fan.

The Newton's law of cooling can be used to determine the rate of heat transfer for both the forced and free convections. However, this law is actually a definition of the convective heat transfer coefficient (*h*). Hence, the distinction between forced and natural convection is the determination of the average value of convective heat transfer coefficient (h) by different methods or empirical correlations in terms of dimensionless numbers like Nusselt (Nu), Prandtl (Pr), Reynolds (Re) numbers etc. which are out of the scope of this book.

7.7. Heat Radiation: Types of Bodies with Respect to Radiation; Radiation Surface Properties

7.7.1. Nature of Heat Radiation

Heat (thermal) radiation is the energy emitted by the surface of matter in the form of electromagnetic waves (or photons) as a result of the changes in the electronic configurations of the atoms or molecules because of their temperature. During the radiation heat transfer the thermal energy of the source gets converted into energy in the form of electromagnetic waves or photons. The waves or photons can travel through space and strike other receiving body resulting in absorption of part of energy carried by these photons and gets reconverted into the thermal energy of the receiving body.

Thermal or heat radiation is an electromagnetic phenomenon of varying wavelengths closely allied to the transmission of light and radio. Wavelength of the thermal radiation lies between 0.1 μm to 100 μm. It proceeds in straight lines at the speed of light, ($3\text{x}10^8$m/s). This speed is the product of the wavelength and frequency of the radiation

$$C = \lambda f \qquad \text{……….} \ (7.61)$$

where C is the speed of light, λ is wavelength and f is the frequency.

Unlike conduction and convection, the transfer of heat by radiation does not require the presence of an intervening medium. In fact, heat transfer by radiation is fastest in a vacuum. However, any material medium between the emitting and receiving surfaces could impede radiation transfer of energy. This is how the energy of the sun reaches the earth.

The rate of heat transfer by conduction and convection largely depends upon temperature difference rather than temperature level. But in heat radiation the temperature of the emitting surface controls the rate of heat transfer as it depends upon fourth power of the surface temperature.

Since the heat radiation is very much a surface phenomenon, its characteristics depends upon various properties of the emitting surface including its temperature, spectral absorptivity and emissive power. The heat radiation is not monochromatic, i.e., it does not consist of just a single wavelength like laser, but it comprises a wide wavelength band. A model radiator so called a black body has to be defined for characterization of certain radiation surface properties.

7.7.2. Concept of a Black Body

A black body is an idealized physical body that absorbs all incident heat radiation in the form of electromagnetic waves. Because of this perfect absorptivity at all wavelengths, a black body is also the best possible emitter of the heat radiation which it radiates in continuous spectrum that depends upon its surface temperature. The body appears black, since it does not reflect or emit any visible light. The concept of a black body is an idealization with which the radiation characteristics of real bodies can be conveniently compared. A black body has the following properties [2]:

(i) It absorbs all the incident radiation energy falling on it and does not transmit or reflect it regardless of wavelength and direction.
(ii) It has an emissivity value of unity and therefore emits maximum amount of heat radiation at all wavelengths at any specified surface temperature.
(iii) The radiation energy emitted by a black body is independent of direction (i.e., it is a diffuse emitter).

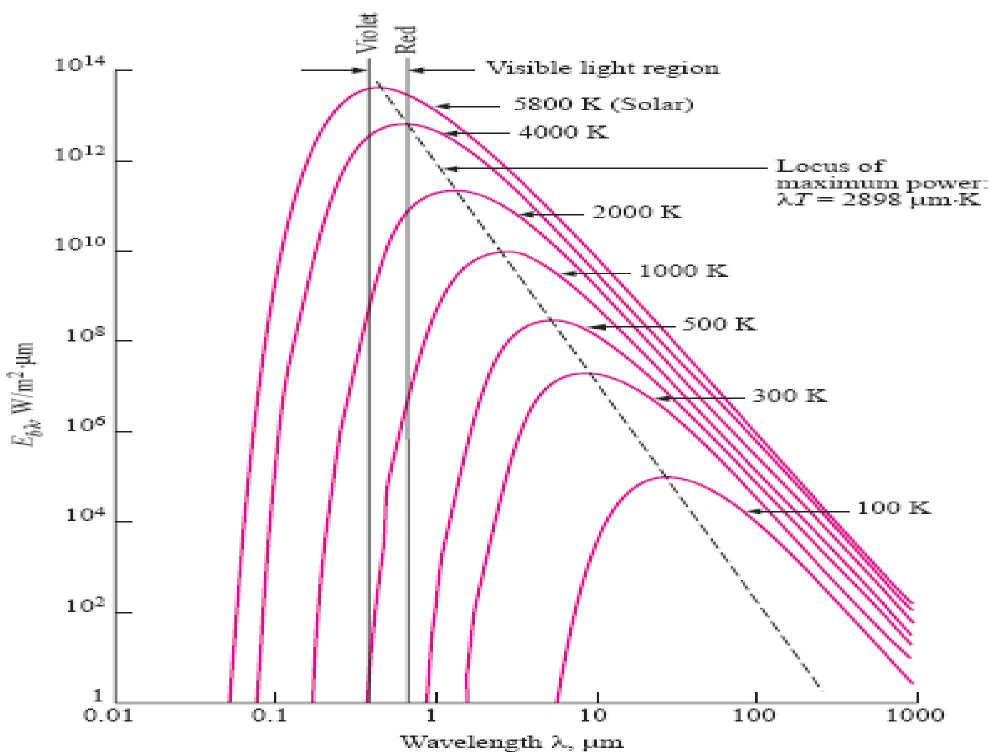

Figure 7.18. Distribution of spectral emissive power of a black body with wavelength at different surface temperatures [1]

The wavelength distribution for a black body as shown in figure 7.18 is given by *Planck's law* of black body radiation which indicates the general shape of the spectral emissive power ($E_{b,\lambda}$) against wavelength (λ) for a black body at different surface temperatures. According to Planck's law for black body radiation the emissive power per unit area per unit wavelength which is also called specific radiative intensity or spectral radiance is given by

$$E_{b,\lambda} = \frac{2\pi hc^2/\lambda^5}{e^{(hc/\lambda kT)}-1} \qquad \text{……..(7.62)}$$

where h = 6.626 x 10^{-34} Js and k = 1.38 x 10^{-23} J/K are called Planck constant and Boltzmann constant respectively as well as c and T are the speed of light in vacuum and surface temperature in Kelvin respectively.

Wien's law

Figure 7.18 shows that the wavelength at which the emissive power is maximum (peak) and the peak of the curve shifts towards shorter wavelength with increasing temperature. The locus of this maximum power is given by the *Wien's law* which relates the temperature T and wavelength λ at which the emissive power is maximum and expressed as,

$$\lambda\, T = 2898\, \mu m\, K \qquad \text{……….(7.63)}$$

A radiating surface at a high temperature, perhaps over 800^0C, will emit some wavelengths which are within the visible light spectrum, approximately 10^{-6} to 10^{-7} meters. At lower temperatures, less than 800^0C, the radiation will be in infrared range, approximately 10^{-2} to 10^{-6} meters.

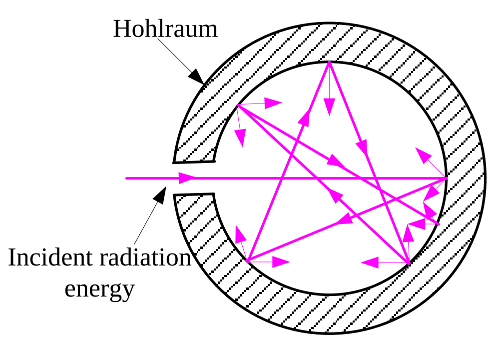

Figure 7.19. A hohlraum for an approximation of a theoretical black body [2]

A perfect black body does not exist in nature; however, graphite is a good approximation. Experimentally, black body radiation may be established in a rigid walled cavity with a small hole (Hohlraum) and is maintained at a constant temperature as shown in figure 7.19. Any incident radiation energy entering the hole would have to reflect off the walls of the cavity multiple times before it escaped in which process it is nearly completely to be absorbed so that very little of the original incident energy is reflected back out of the hole. Absorption occurs regardless of the wavelength of the entering radiation. The hole then is a close approximation of a theoretical black body.

7.7.3. Types of Bodies with respect to Radiation

The bodies are classified according to their radiation surface properties.

Black Body

A black body is one which neither reflects nor transmits any part of the incident radiation energy but absorbs all of it regardless of the wavelength. Hence, for the black body, $\alpha = \in = 1$ as shown in figure 7.20, $\rho = 0$ and $\tau = 0$. In practice, a perfect black body does not exist. However, graphite can approximately be taken as a black body.

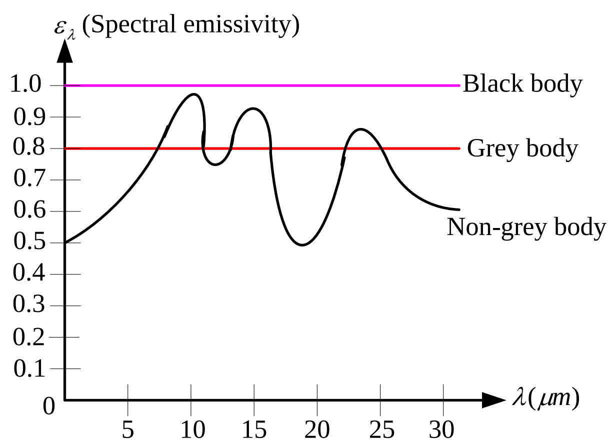

Figure 7.20. Variation of emissivity with wavelength of different bodies [3]

White Body

A white body is defined as a hypothetical body whose surface reflects the entire radiation energy incident on its surface. Hence, a white body surface does not absorb and transmit the radiation energy incident on its surface and has a reflectivity value of unity. For a white body, $\rho = 1$, $\alpha = 0$ and $\tau = 0$.

Opaque Body

A body is said to be an opaque body if no incident radiation energy is transmitted through it. For the opaque body, $\tau = 0$ and from equation (7.65), $\alpha + \rho = 1$. Solids generally do not transmit unless the material is very thin. Therefore, solids and liquids are usually considered as opaque.

Transparent Body

A body is said to be a transparent body if all the incident radiation energy is transmitted through it. For the transparent body, $\tau = 1$ and $\alpha = \rho = 0$. Gases are practically transparent to radiation, except that some gases are known to absorb radiation strongly at certain wavelengths. Ozone, for example, strongly absorbs ultraviolet radiation. Gases such as hydrogen, oxygen and nitrogen and their mixture (air) have a transmissivity value of unity and therefore considered as transparent.

Grey Body

If a body emits radiation energy across the black body wavelength spectrum but only a fraction of the power of the black body, then it is called a grey body. In other words, a grey body is defined as a hypothetical body with constant surface absorptivity (and then emissivity) over all wavelengths and temperatures like black body but its emissivity and therefore emissive power is less than that of a black body ($\propto = \propto_{\lambda}$= constant) as shown in figure 7.21. Whereas a *coloured body* is one whose surface absorptivity varies with the wavelength of radiation ($\propto \neq \propto_{\lambda}$).

The term black and grey do not necessarily refer to the color of the body; they merely describe its effectiveness as a radiator. A black body is a perfect radiator, but a grey body is not perfect radiator.

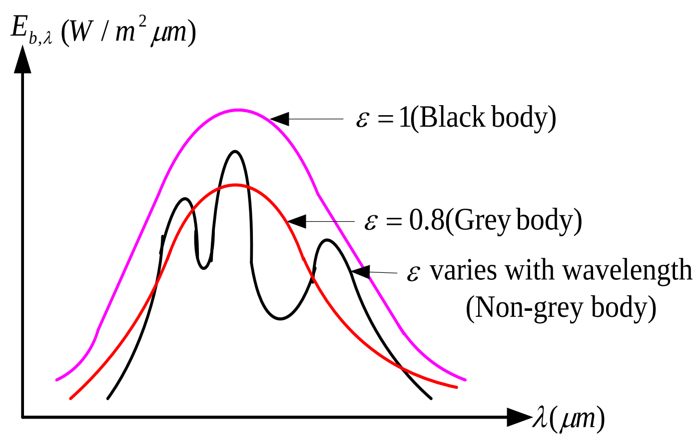

Figure 7.21. Variation of spectral emissive power with wavelength for different bodies. [4]

For a grey surface the emissivity ($\in_g$), which is a ratio of the grey surface emission (E_g) to the black surface emission(E_B), must therefore be of the form

$$\in_g = \frac{E_g}{E_B} < 1 \qquad \text{………. (7.64)}$$

7.7.3. Radiation Surface Properties

All surfaces are capable of emitting, absorbing, reflecting or transmitting radiation energy. In other words, when a surface received the incident radiation energy (Q_i), three things happen; a part of it is reflected back (Q_r), a part is transmitted through (Q_t), and the remaining part is absorbed (Q_a) depending upon the characteristics of the surface, as shown in figure 7.22.

By using the conservation of energy principle for any given surface,

$$Q_a + Q_r + Q_t = Q_i \qquad \text{……….(7.65)}$$

Dividing both sides of equation (7.65) by Q_i, we get

$$\frac{Q_a}{Q_i} + \frac{Q_r}{Q_i} + \frac{Q_t}{Q_i} = 1$$

$$\therefore \alpha + \rho + \tau = 1 \qquad \text{………. (7.66)}$$

where α, ρ and τ are the radiation surface properties known as absorptivity, reflectivity and transmissivity of a surface respectively and are defined below. Thus, the sum of absorptivity, reflectivity and transmissivity must be equal to unity for any surface.

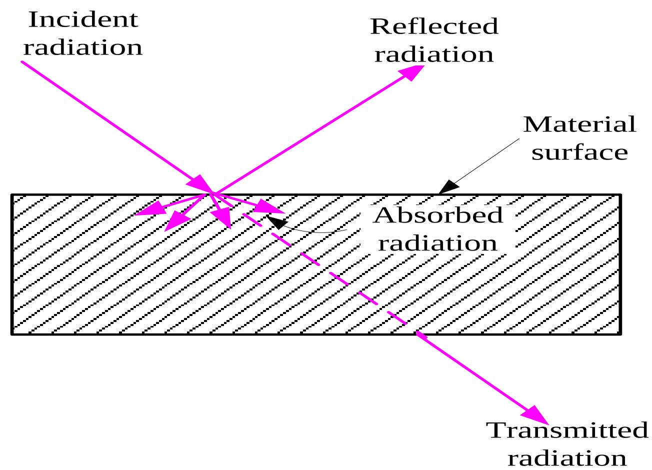

Figure 7.22. Absorption, reflection and transmission of incident radiation on a surface [2]

When the incident radiation energy is absorbed by a surface, it is converted into internal energy of the body. For a black body which absorbs all the radiation energy incident upon it, $\alpha = 1$, and hence $\rho = \tau = 0$. Similarly, for a white body which reflects all the radiation energy incident upon it, $\rho = 1$, and hence $\alpha = \tau = 0$.

Kirchhoff's Law

In general, both emissivity ($\in$)and absorptivity (α) of a surface depend on the temperature and the wavelength of the radiation. *Kirchhoff's law* of radiation states that the emissivity and the absorptivity of a surface at a given temperature and wavelength are equal. In many practical applications, the surface temperature and the temperature of the source of incident radiation are of the same order of magnitude, and the average absorptivity of a surface is taken to be equal to its average emissivity ($\in = \alpha$). Applying Kirchhoff's law, equation (7.66) can be expressed as

$$\in + \rho + \tau = 1 \qquad \text{……… (7.67)}$$

Thus, the sum of emissivity, reflectivity and transmissivity must also be equal to unity for any surface.

In the followings, the radiation surface properties and different types of bodies named as per their surface characteristics with respect to radiation are briefly described.

Emissivity

Emissivity ($\in$) of a surface is the ratio of the energy radiated by a surface to energy radiated by a black body surface at the same temperature. Its value is in the range $0 \leq \in \leq 1$. A blackbody is a perfect emitter, so it has emissivity value 1 ($\in = 1$). A real or grey body surface has the emissivity value less than 1 ($\in < 1$).

Absorptivity

Absorptivity (α) is an important radiation property of a surface. It is defined as the ratio of the radiation energy absorbed by a surface to the total radiation energy incident on the surface (Q_a/Q_i). Like emissivity, its value is in the range $0 \leq \alpha \leq 1$. A blackbody absorbs the entire radiation incident on it, i.e., a blackbody is a perfect absorber ($\alpha = 1$) as it is a perfect emitter ($\in = 1$). A real or grey body surface has the absorptivity value less than 1 ($\alpha < 1$).

Reflectivity

Reflectivity (ρ) of a surface is defined as the ratio of the radiation energy reflected by a surface to the total radiation energy incident on the surface (Q_r/Q_i). A black body surface has the reflectivity value zero ($\rho = 0$) and a real or grey body surface has reflectivity value greater than zero ($\rho > 0$).

Transmissivity

Transmissivity (τ) of a surface is defined as the ratio of the radiation energy traversing through the surface to the total radiation energy incident on the surface (Q_t/Q_i). A black body surface and opaque (nontransparent) body surface have the transmissivity value zero (τ = 0) and a real or grey body surface which is not opaque has the transmissivity value greater than zero (τ > 0).

Lecture Highlights

- According to the mechanism of the fluid flow, *convection heat transfer* is classified into two types: free or natural convection and forced convection.
- *Differences between free convection and forced convection are:*

Free convection	**Forced convection**
1. It is the heat transfer process in which flow of fluid is caused by density gradient.	1. It is the heat transfer process in which flow of fluid is caused by some external devices such as pump, fan, blower etc.
2. Mass flow rate of fluid is usually low during the free convection.	2. Mass flow rate of fluid is comparatively higher in forced convection.
3. Rate of heat transfer is usually very low.	3. Rate of heat transfer is relatively higher.
4. For example: Cooling of a room without a fan by natural circulation of air.	4. For example: Cooling of a room by a fan.

- *Thermal resistance*: It is the property of a substance by virtue of which it opposes the flow of heat.
- *Thermal resistance of a plane wall* is given by

$$R_{th,wall} = \frac{L}{kA}$$

- *Thermal resistance of a hollow cylinder* is given by $$R_{th,cyl} = \frac{\ln(r_2/r_1)}{2\pi kL}$$
- *Thermal resistance of a convective layer* is given by $$R_{th,conv} = \frac{1}{hA_s}$$
- *Thermal resistance of a radiation layer* is given by $$R_{th,rad} = \frac{1}{\in\sigma A_S(T_S+T_\infty)(T_S^2+T_\infty^2)}$$

- *Rate of heat flow through a plane wall* given by

$$\dot{\boldsymbol{Q}} = \frac{kA}{L}(T_1 - T_2) = \frac{\Delta T_{overall}}{R_{th,wall}}$$

where $R_{th,wall} = \dfrac{L}{kA}$

- *Rate of heat flow through a plane wall with three layers* is given by

$$\dot{Q} = \frac{A(T_1 - T_4)}{\left(\frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3}\right)} = \frac{\Delta T_{overall}}{R_{\boldsymbol{total}}}$$

where $R_{total} = \dfrac{1}{A}\left(\dfrac{L_1}{k_1} + \dfrac{L_2}{k_2} + \dfrac{L_3}{k_3}\right)$

- *Rate of heat flow through a hollow cylinder* is given by

$$\dot{Q} = \frac{2\pi kL(T_1 - T_2)}{ln\left(\frac{r_2}{r_1}\right)} = \frac{\Delta T_{overall}}{R_{th,cyl}}$$

where $R_{th,cyl} = \dfrac{\ln(r_2/r_1)}{2\pi kL}$

- *Rate of heat flow through a hollow cylinder with three layers* is given by

$$\dot{Q} = \frac{2\pi L(T_1 - T_4)}{\left(\frac{ln\left(\frac{r_2}{r_1}\right)}{k_1} + \frac{ln\left(\frac{r_3}{r_2}\right)}{k_2} + \frac{ln\left(\frac{r_4}{r_3}\right)}{k_3}\right)} = \frac{\Delta T_{overall}}{R_{total}}$$

where $R_{total} = \dfrac{1}{2\pi L}\left[\dfrac{\ln(r_2/r_1)}{k_1} + \dfrac{\ln(r_3/r_2)}{k_2} + \dfrac{\ln(r_4/r_3)}{k_3}\right]$

- *Rate of heat flow through a plane wall subjected to convective medium on both sides* is given by

$$\dot{Q} = \frac{A(T_{\infty 1} - T_{\infty 2})}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}} = UA(T_{\infty 1} - T_{\infty 2})$$

where $U = \dfrac{1}{\frac{1}{h_1} + \frac{L}{k} + \frac{1}{h_2}}$ is called overall or combined heat transfer coefficient.

- *Rate of heat flow through a hollow cylinder subjected to convective medium on both sides* is given by

$$\dot{Q} = \frac{A_1\left(T_{\infty 1} - T_{\infty 2}\right)}{\frac{1}{h_1} + \frac{A_1}{2\pi kL}\ln\left(r_2/r_1\right) + \frac{A_1}{A_2}\frac{1}{h_2}} = U_1 A_1\left(T_{\infty 1} - T_{\infty 2}\right)$$

Or, $$\dot{Q} = \frac{A_2\left(T_{\infty 1} - T_{\infty 2}\right)}{\frac{A_2}{A_1}\frac{1}{h_1} + \frac{A_2}{2\pi kL}\ln\left(r_2/r_1\right) + \frac{1}{h_2}} = U_2 A_2\left(T_{\infty 1} - T_{\infty 2}\right)$$

where

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{A_1}{2\pi kL}\ln\left(r_2/r_1\right) + \frac{A_1}{A_2}\frac{1}{h_2}}$$ and

$$U_2 = \frac{1}{\frac{A_2}{A_1}\frac{1}{h_1} + \frac{A_2}{2\pi kL}\ln\left(r_2/r_1\right) + \frac{1}{h_2}}$$

are called *inside overall or combined heat transfer coefficient* and *outside overall or combined heat transfer coefficient* respectively. $A_1 = 2\pi r_1 L$ and $A_2 = 2\pi r_2 L$ are inside curved surface area and outside curved surface area respectively.

- *The net rate of heat radiated for any real body can be derived as*

  $$\dot{Q}_{net} = \in \sigma\, A_s\, (T_1^4 - T_2^4)$$

  where T_1 and T_2 are surface temperatures of two bodies 1 and 2 which are exchanging radiation heat respectively.
- *Relationship between Radiation surface properties:* $\alpha + \rho + \tau = 1$

  As $\alpha = \in$ then $\in + \rho + \tau = 1$.

  where α is the *absorptivity* which is defined as the ratio of absorbed heat energy to the incident heat energy, ρ is the *reflectivity* which is defined as the ratio of reflected heat energy to the incident heat energy and τ is the *transmissivity* which is defined as the ratio of transmitted heat energy to the incident.
- *Grey body*: It is defined as the body whose surface emissivity does not vary with temperature and wavelength of the radiation as in the case of the black body radiation but the value of emissivity is less than 1. Hence, for a grey body, $0 < \in = \alpha < 1$.
- *Black body:* It is the body which absorbs all the incident radiation. Hence, for a black body $\alpha = 1$, $\rho = 0$ and $\tau = 0$. As black body is a good absorber as well as good emitter, $\alpha = \in = 1$.
- *White body:* If all the incident radiation falling on the body is reflected by its surface, then such a hypothetical body is called white body. Hence, for a white body, $\rho = 1$, $\alpha = \in = 0$ and $\tau = 0$.
- *Opaque body:* If no incident radiation is transmitted through a body, then such a body is called opaque body. Hence, for an opaque body, $\tau = 0$ and $\alpha + \rho = 1$.

- *Transparent body:* If all the incident radiation is transmitted through a body, then such a body is called transparent body. Hence, for a transparent body, $\tau = 1$ and $\alpha + \rho = 0$.
- *Differences between black body and grey body are:*

Black body	**Grey body**
1. It is the body which absorbs all the incident radiation. 2. It is a perfect absorber and emitter. 3. Its surface emissivity does not vary with temperature as well as wavelength of the radiation. 4. Its emissive power is higher than that of grey body. 5. The value of emissivity is 1. Hence, for a black body, $\in = \alpha = 1, \rho = 0, \tau = 0$. (for example, graphite).	1. It is the hypothetical body which cannot absorb all the incident radiation. 2. It is not a perfect absorber and emitter. 3. Its surface emissivity is assumed to be constant with temperature and wavelength of the radiation in contrary to the case of the non-grey body radiation. 4. Its emissive power is less than that of black body. 5. The value of emissivity is less than 1. Hence, for a grey body, $0 < \in = \alpha < 1, \rho \neq 0, \tau \neq 0$.

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