Automatic Control Systems

Lecture-7

Represent a System Using Signal Flow Graph

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Session Objectives

By the end of this session, learners will be able to:

- Define a signal flow graph
- Explain terms used in a signal flow graph
- Explain the rules used in signal flow graphs
- Construct signal flow graph of a system
- Use Mason's Gain formula to find the system transfer function
- Convert a block diagram into a signal flow graph

Definition of Signal Flow Graph

A signal flow graph describes how a signal gets modified as it travels from input to output.

It is a graphical representation of the relationships between the variables of a system.

The overall transfer function can be obtained very easily by using Mason's gain formula.

Benjamin C. Kuo (1975), Automatic Control Systems, 3rd Edition, Prentice Hall, page 51.

Definition of Terms

- **Node:** A node is a point representing a variable of a signal.
- **Branch:** A branch is a directed line segment joining two nodes. The arrow on the branch indicates the direction of signal flow.
- **Transmittance**: The gain acquired by the signal when it travels from one node to another . Transmittance can be real or complex.
- **Input node:** It is a node that contains only outgoing branches
- **Output node:** It is a node that has only incoming branches.

Definition of Terms(cont’d)

- **Mixed node:** It is a node that has both outgoing and incoming branches.
- **Path:** A path is a traversal of connected branches in the direction of arrows.
- **Loop:** It is a path starting and ending on the same node.
- **Forward path:** It is a path from input node to output node**.**

Definition of Terms(cont'd)

- **Forward path gain:** The gain product of the branches in the forward path is called forward path gain
- **Loop gain**: Product of gains of a loop.
- **Individual loop (Non touching loop):** If the loop doesn't have common node, then they are said to be non touching.

Benjamin C. Kuo (1975), Automatic Control Systems, 3rd Edition, Prentice Hall, pages 67-69.

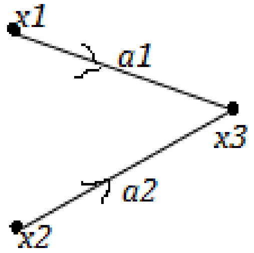
Rules for Signal flow graph

- **Rule 1:**

i)

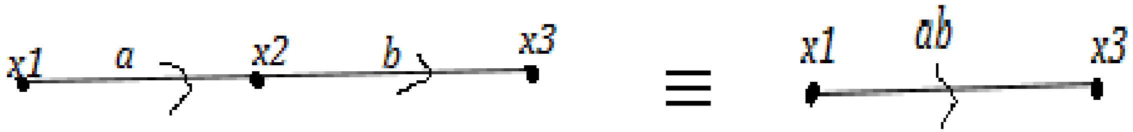
$$x_2 = ax_1$$

ii)



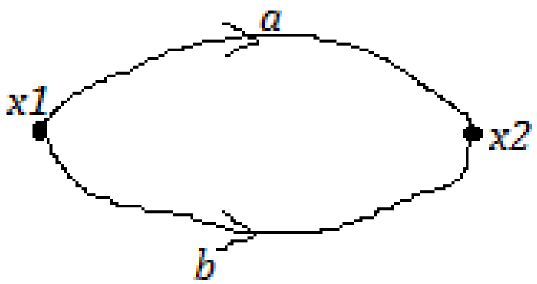
$$x_3 = a_1x_1 + a_2x_2$$

- **Rule 2:**



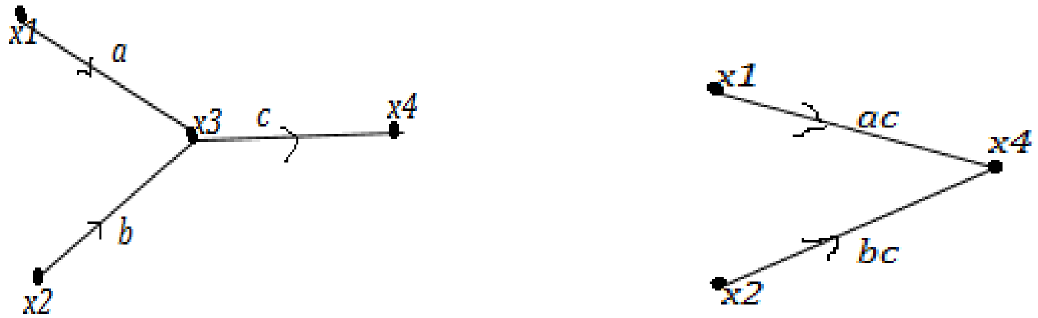
$$\boldsymbol{x_3 = abx_1}$$

- **Rule 3:**



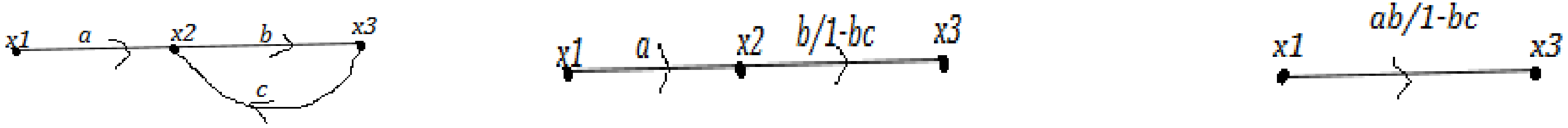
$$x_2 = (a + b)x_1$$

- **Rule 4:**



$$x_4 = acx_1 + bcx_2$$

- **Rule 5:**



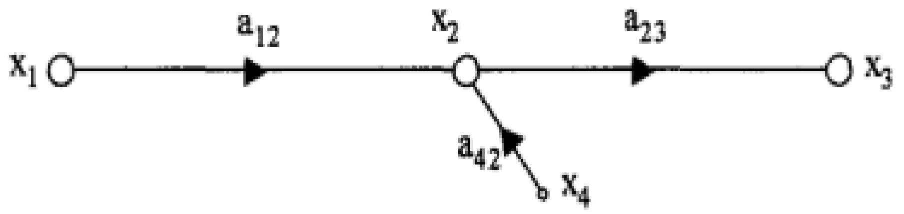
$$x_3 = \frac{}{1 - bc} x_1$$

Smarajit Gosh (2007), Control systems, Pearson Education, page 180.

Example of a signal flow graph

In the following example, there are four nodes representing variables x_1, x_2, x_3 and x_4.

The transmittance or gain of the branches are a_{12}, a_{23} and a_{42} .

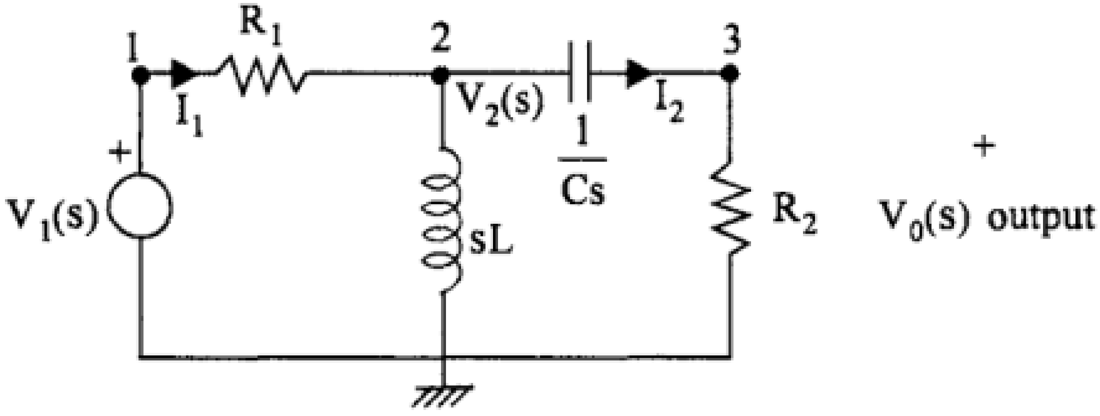


$$x_2 = a_{12}x_1 + a_{42}x_4$$

$$x_3 = a_{23}x_2$$

Construction of a signal flow graph of a system

Consider a network shown in the figure below



Draw the signal flow graph

Identifying the currents and voltages in the branches, we have

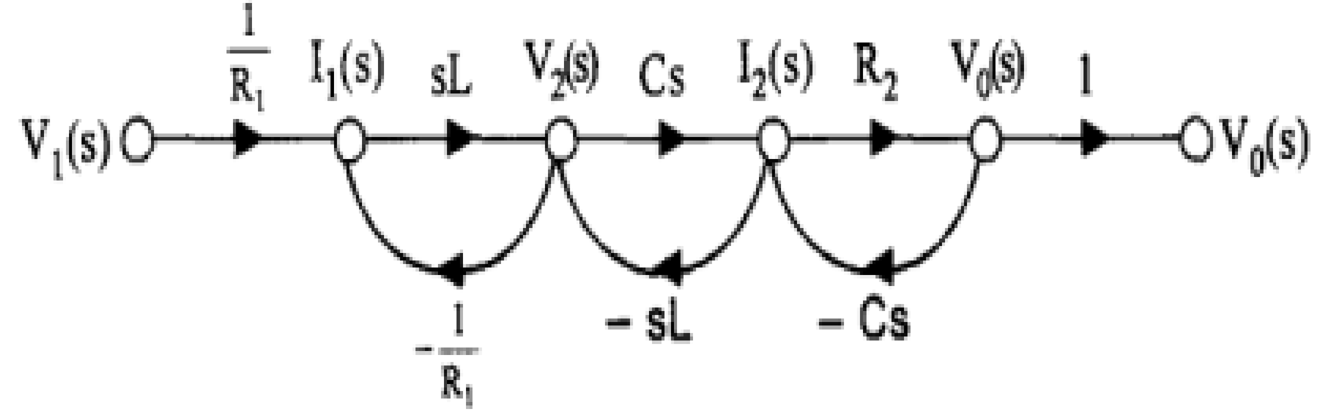
$$I_1(s) = \frac{V_1(s) - V_2(s)}{R_1}$$

$$V_2(s) = [I_1(s) - I_2(s)].\, sL$$

$$I_2(s) = [V_2(s) - V_0(s)].\, Cs$$

$$V_0(s) = I_2(s).\, R_2$$

The variables $I_1(s), I_2(s), V_1(s), V_2(s) and\ V_o(s)$ are represented by nodes and these nodes are interconnected to satisfy the relationships between them



Mason's Gain Formula

Let $R(s) \rightarrow Input\ of\ the\ system$

$C(s) \rightarrow Output\ of\ the\ system$

The transfer function of the system $T(s) = \frac{C(s)}{R(s)}$

$$T(s) = \frac{1}{\Delta}\sum_k P_k \Delta_k$$

Where $T = T(s) = Transfer\ function\ of\ the\ system$

$P_k = Forward\ path\ gain\ of\ k^{th}\ forward\ path.$

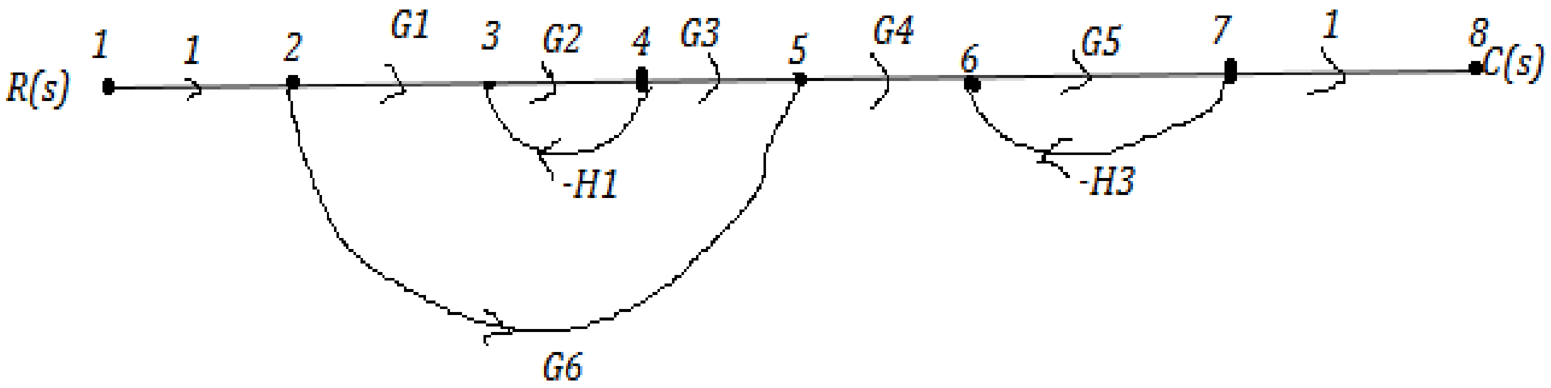
$\Delta = 1 - (sum\ of\ individual\ loop\ gains) + (sum\ of\ two\ non\ touching\ loops) + (sum\ of\ three\ non\ touching\ loops) + \cdots \ldots.$

$\Delta_k = 1 - loop\ gains\ which\ are\ not\ touching\ to\ k^{th}\ forward\ path.$

Smarajit Gosh (2007), Control systems, Pearson Education, page 181.

Solved Example

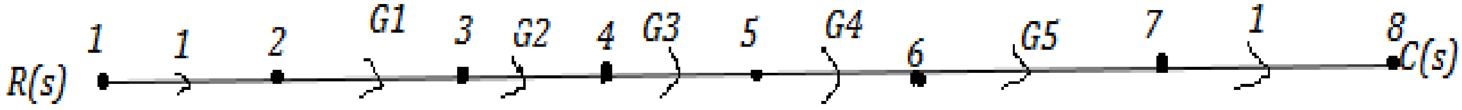
Find the overall transfer function of the system whose signal flow graph is shown in the figure



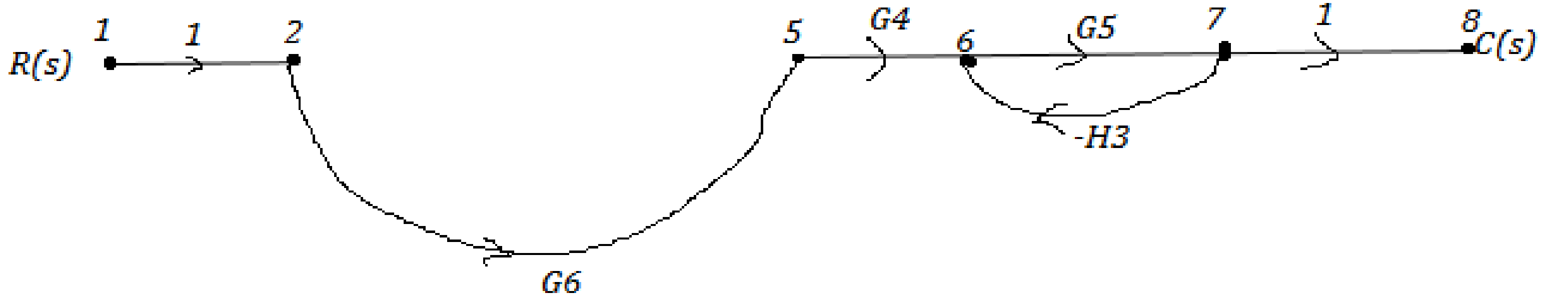
Solution:

Using Mason's Gain Formula $T = \frac{1}{\Delta} \sum_k P_k \Delta_k$

Forward paths



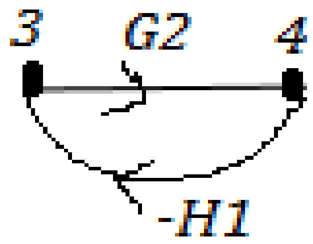
$$P_{1=G_1 G_2 G_3 G_4 G_5}$$



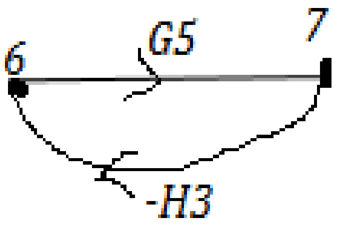
$$P_{2=G_4G_5G_6}$$

Δ
$= 1 - (sum\ of\ individual\ loops)$
$+ (sum\ of\ 2\ non\ touching\ loops)$

Individual loops:

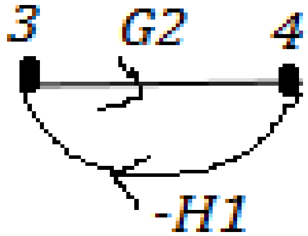


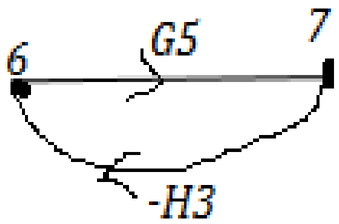
$$P_{11} = -G_2H_1$$



$$P_{12} = -G_5H_3$$

There are 2 Non Touching loops





$P_{21=G_2 G_5 H_1 H_3}$

Now $\Delta = 1 + G_2 H_1 + G_5 H_3 + G_2 G_5 H_1 H_3$

$\Delta_1 = 1 - (0) = 1$

$\Delta_2 = 1 - (-G_2 H_1) = 1 + G_2 H_1$

Thus, The system transfer function $T(s) = \frac{G_1 G_2 G_3 G_4 G_5 + G_4 G_5 G_6 (1 + G_2 H_1)}{1 + G_2 H_1 + G_5 H_3 + G_2 G_5 H_1 H_3}$

Procedures for converting Block diagram to Signal flow graph

1. Assume nodes at the input, output, at every summing point, at every branch point and in between cascaded blocks.
2. Draw the nodes separately as small circles and number the circles in the order 1,2,3,….
3. From the block diagram find the gain between each node in the main forward path and connect all the corresponding circles by straight lines and mark the gain on that nodes.

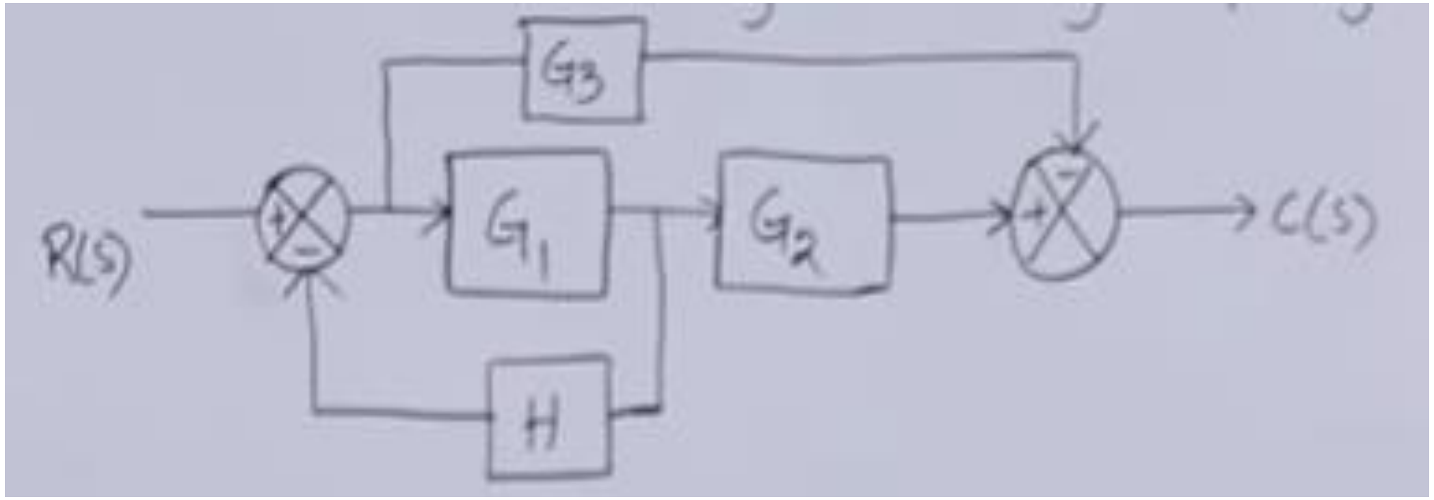
4. Draw the field forward paths between various nodes and mark the gain of field forward path along with sign.

5. Draw the feedback path between the various nodes and mark the gain of the field back paths along with sign.

K. Webb, MAE 4421-Control of Aerospace and Mechanical Systems, Lecture Notes, page 30, Available at https://web.engr.oregonstate.edu/~webbky/MAE4421_files/Section%202%20Block%20Diagrams%20&%20Signal%20Flow%20Graphs.pdf

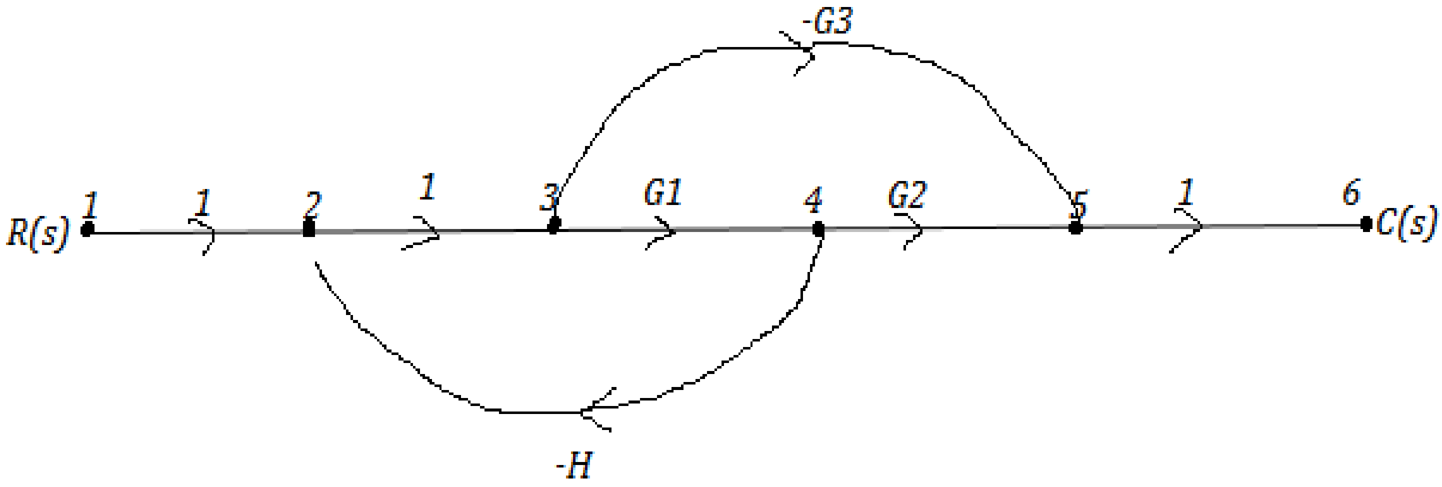
Example

Convert the given block diagram to signal flow graph and determine $\frac{C(s)}{R(s)}$



Solution:

Considering the nodes at the input, output, at the every summing point, branch point and cascaded blocks, we get



The system transfer function $\frac{C(s)}{R(s)}$ is obtained with the help of Mason's Gain Formula

$$T(s) = \frac{C(s)}{R(s)} = \frac{1}{\Delta}\sum_k P_k \Delta_k \text{ with}$$

Δ=1-(sum of all loop gains)+(sum of 2 non touching loops)+…

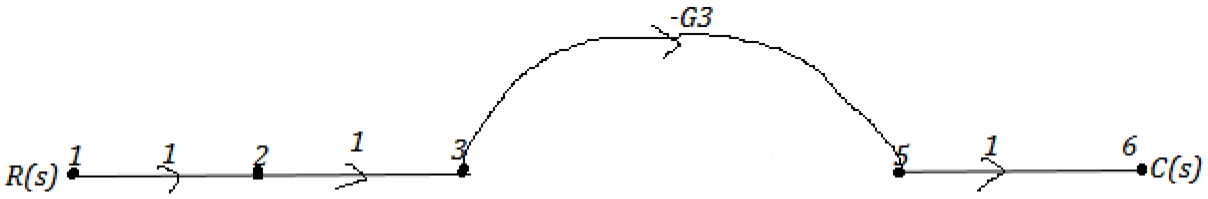
P_k=Gain of k^{th} forward path

Δ_k=1-gain of the loop non touching to k^{th} forward path.

We have 2 forward paths i.e k=1,2

First forward path

$$P_{.1} = G_1 G_2$$



$$P_2 = -G_3$$

$$\Delta_1 = 1 - (0) = 1$$

$$\Delta_2 = 1 - (0) = 1$$

$$\Delta = 1 - (-G_1 H) = 1 + G_1 H$$

Thus, the overall system transfer function is given by

$$T(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

By replacing parameters with their corresponding expressions, we get

$$T(s) = \frac{G_1G_2 - G_3}{1 + G_1H}$$

References

1. Smarajit Gosh (2007), Control systems, Pearson Education
2. Manirakiza and Kanyarwanda (2020), ELT 303Lecture Note, IPRC Gishari.
3. K. Ogata (1997), Modern Control Engineering, 3rd Edition, Prentice Hall.
4. Benjamin C. Kuo (1975), Automatic Control Systems, 3rd Edition, Prentice Hall
5. K. Webb, MAE 4421-Control of Aerospace and Mechanical Systems, Lecture Notes, Available at https://web.engr.oregonstate.edu/~webbky/MAE4421_files/Section%202%20Block%20Diagrams%20&%20Signal%20Flow%20Graphs.pdf

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