Automatic Control Systems

Lecture-11

Describe Types of System Compensators

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Session Objectives

By the end of this session, learners will be able to:

- Define system compensator
- Explain different types of compensators
- Find transfer function of each type of compensator

Introduction

- A feedback control system that provides an optimum performance without any necessary adjustment is ***rare***.
- In building a control system, we know that proper modification of the plant dynamics may be a simple way to meet the performance specifications.
- This, however, may not be possible in many practical situations because the plant may be fixed and not modifiable.
- Then we must adjust parameters other than those in the fixed plant.

Introduction (cont'd)

- It is then required to reconsider the structure of the system and redesign the system.
- The design problems, therefore, become those of improving system performance by insertion of a ***compensator***.
- ***Compensator***: is an additional component or circuit that is inserted into a control system to equalize or compensate for a deficient performance.

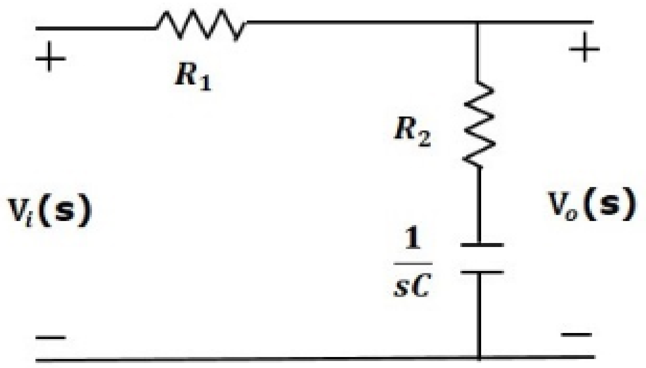
Commonly Used Compensators

- Among the many kinds of compensators, widely employed compensators are the
  - lead compensators
  - lag compensators
  - lag–lead compensators

Lag Compensator

The lag compensator is an Electrical Network which produces an sinusoidal output having the phase lag when a sinusoidal input is applied.

The lag compensator circuit in the s domain is shown in the following figure.



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Transfer function of Lag Compensator

Here the capacitor is in series with the resistor R2 and the output is measured across this combination.

$$V_i(s) = I(s)R_1 + I(s)R_2 + \frac{I(s)}{sC}$$

$$V_o(s) = \mathrm{I(s)}R_2 + \frac{I(s)}{sC}$$

The transfer function T.F=$\frac{V_0(s)}{V_i(s)}$

The transfer function T.F=$\frac{V_0(s)}{V_i(s)} = \frac{(R_2+\frac{1}{sC})I(s)}{(R_1+R_2+\frac{1}{sC})I(s)}$

$$= \frac{\boldsymbol{R_2Cs+1}}{\boldsymbol{R_1Cs+R_2Cs+1}}$$

$$= \frac{\boldsymbol{R_2Cs+1}}{\boldsymbol{(R_1+R_2)Cs+1}}$$

$$= \frac{\boldsymbol{R_2C}}{\boldsymbol{(R_1+R_2)C}}\left(\frac{\boldsymbol{s+\frac{1}{R_2C}}}{\boldsymbol{s+\frac{1}{(R_1+R_2)C}}}\right)$$

Assume

$$T = R_2C \; and \; \alpha = \frac{R_1 + R_2}{R_2}$$

$$\frac{V_0(s)}{V_i(s)} = \frac{1}{\alpha}\left(\frac{s + \frac{1}{T}}{s + \frac{1}{(R_1 + R_2)C}}\right)$$

$$= \frac{1}{\alpha}\left(\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}\right)$$

Here, $\alpha \; is$ always greater than 1

Features

- Doesn’t affect transient response
- Reduces stability margin
- Reduces bandwidth
- SNR increases
- Reduces steady state error

Effects of phase Lag Compensation

- Gain crossover frequency increases.
- Bandwidth decreases.
- Phase margin will be increase.
- Response will be slower before due to decreasing bandwidth, the rise time and the settling time become larger.

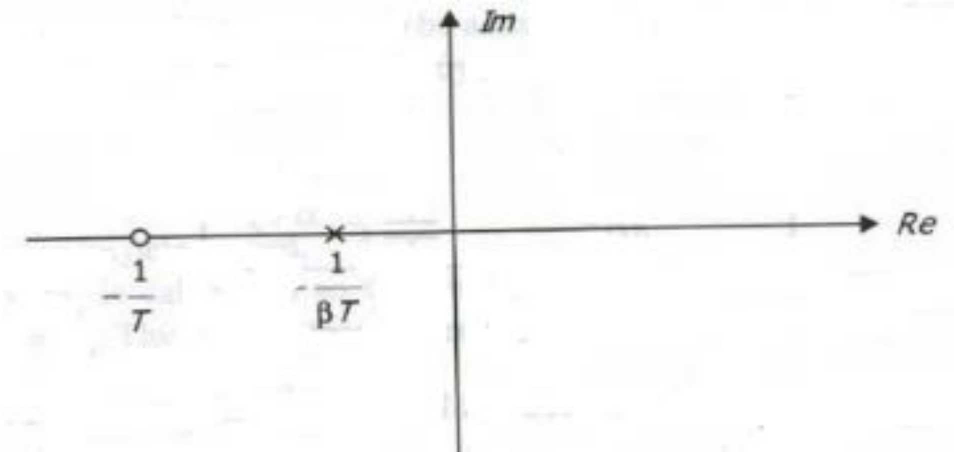
Advantages of Phase Lag Compensation

- Phase lag network allows low frequencies and high frequencies are attenuated.
- Due to the presence of phase lag compensation the steady state accuracy increases.

Disadvantages of Phase Lag Compensation

- In lag compensator, the attenuation offered by it shifts the gain crossover frequency to a lower point, thereby decreasing the bandwidth.
- Though the system response is longer due to decreased bandwidth; however, the response is quite slow.

The lag compensator has a zero at s=-1/T and pole at s=-1/αT. Since α>1, the pole is always located to the right of the zero. The figure below shows the pole-zero plot of the lag compensator.



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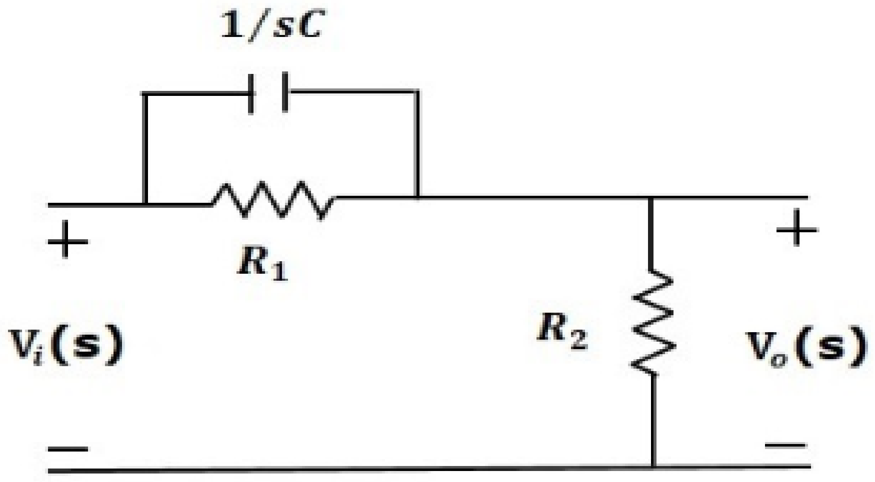

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Transfer function of Lead Compensator

The transfer function T.F=$\frac{V_0(s)}{V_i(s)}$

$$V_i(s) = I(s)\left[\frac{R_1 * \frac{1}{sC}}{R_1 + \frac{1}{sC}}\right] + I(s)R_2$$

$$= I(s)\left[\frac{R_1}{R_1Cs+1}\right] + I(s)R_2$$

$$V_o(s) = R_2I(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 I(s)}{I(s)\left[\frac{R_1}{R_1Cs+1}\right]+I(s)R_2}$$

$$= \frac{R_2}{\left[\frac{R_1}{R_1Cs+1}\right]+R_2}$$

$$= \frac{R_2(R_1Cs+1)}{R_1+R_2(R_1Cs+1)}$$

$$= \frac{R_1R_2C(s+\frac{1}{R_1C}}{R_1+R_2+R_1R_2Cs}$$

$$= \frac{R_1 R_2 C\left(s+\frac{1}{R_1 C}\right)}{R_1 R_2\left(s+\frac{R_1+R_2}{R_1 R_2 C}\right)}$$

$$= \frac{s+\frac{1}{R_1 C}}{s+\frac{R_1+R_2}{R_1 R_2 C}}$$

Assume $T = R_1 C \quad and\ \beta = \frac{R_2}{R_1+R_2}$

The transfer function becomes

$$T.F = \frac{s+\frac{1}{T}}{s+\frac{1}{\beta T}}$$

$$= \beta\left(\frac{sT+1}{s\beta T+1}\right)$$

The lead compensator has a zero at s=-1/T and pole at s=-1/βT. Since 0<β<1, the zero is always located to the right of the pole as shown in the figure below

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Features

- Improves transient response and makes the system faster
- Increases stability margin
- Increases bandwidth
- SNR decreases
- Doesn’t affect steady state error much

Effect of Phase Lead Compensation

- The velocity constant K_v increases.
- The slope of the magnitude plot reduces at the gain crossover frequency so that relative stability improves and error decrease due to error is directly proportional to the slope.
- Phase margin increases.
- Response become faster.

Advantages of Phase Lead Compensation

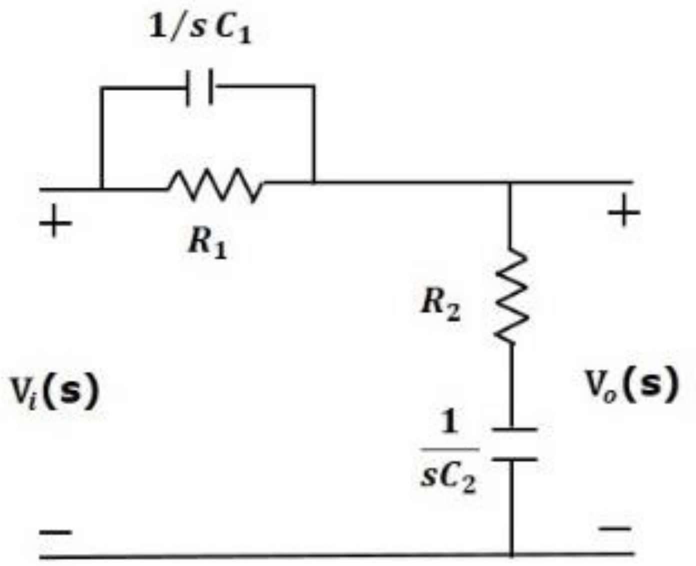
- Due to the presence of phase lead network the speed of the system increases because it shifts gain crossover frequency to a higher value.
- Due to the presence of phase lead compensation maximum overshoot of the system decreases.

Lag-Lead Compensator

Lag-Lead Compensator is an electrical network which produces phase lag at one frequency region and phase lead at other frequency region.

It is a combination of both the lag and the lead compensators.

The lag-lead compensator circuit in the s domain is shown in the following figure



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Transfer function of Lag-Lead Compensator

The transfer function T.F=$\frac{V_0(s)}{V_i(s)}$

$$V_o(s) = R_2 I(s) + \frac{I(s)}{sC_2}$$

$$V_i(s) = \frac{\frac{R_1}{sC_1}}{\frac{1}{sC_1}+R_1} I(s) + R_2 I(s) + \frac{1}{sC_2} I(s)$$

$$\frac{V_o(s)}{V_i(s)} = \frac{R_2 I(s)+\frac{I(s)}{sC_2}}{\frac{\frac{R_1}{sC_1}}{\frac{1}{sC_1}+R_1} I(s)+R_2 I(s)+\frac{1}{sC_2} I(s)}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{R_2C_2s+1}{sC_2}}{\frac{R_1}{R_1C_1s+1}+R_2+\frac{1}{sC_2}}$$

$$= \frac{\frac{R_2C_2s+1}{sC_2}}{\frac{R_1C_2s+R_2C_2s[R_1C_1s+1]+R_1C_1s+1}{sC_2[R_1C_1s+1]}}$$

$$= \frac{R_2C_2s+1}{\frac{R_1C_2s+R_1C_1R_2C_2s^2+R_2C_2s+R_1C_1s+1}{R_1C_1s+1}}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{(R_1C_1s+1)(R_2C_2s+1)}{R_1C_1R_2C_2s^2 + R_1C_1s + R_2C_2s + R_1C_2s + 1}$$

$$= \frac{R_1C_1R_2C_2\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}{R_1C_1R_2C_2(s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{C_1R_2}\right)s+\frac{1}{R_1C_1R_2C_2}}$$

$$= \frac{\left(s+\frac{1}{T_1}\right)\left(s+\frac{1}{T_2}\right)}{\left(s+\frac{1}{\alpha T_1}\right)\left(s+\frac{1}{\beta T_2}\right)}$$

Where

$$T_1 = R_1 C_1$$
$$T_2 = R_2 C_2$$
$$\alpha\beta T_1 T_2 = R_1 C_1 R_2 C_2$$
$$\alpha\beta = 1$$
$$\frac{1}{\alpha T_1} + \frac{1}{\beta T_2} = \frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{C_1 R_2}$$

with α and β are time constants and attenuation constants respectively.

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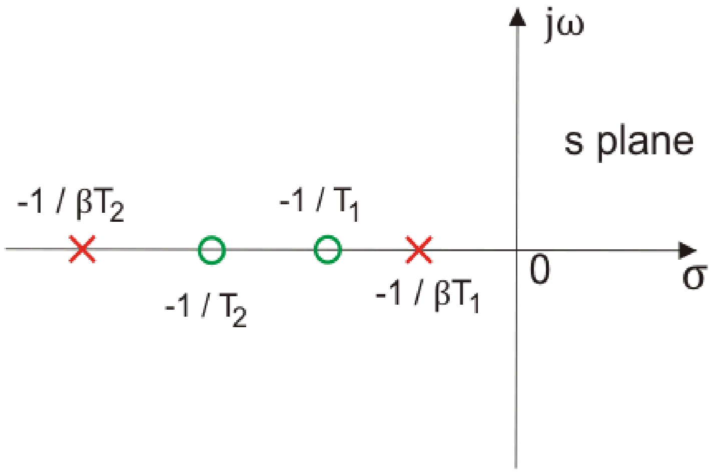
We have

Transfer function $\frac{V_o(s)}{V_i(s)} = \frac{(1+T_1 s)(1+T_2 s)}{(1+\alpha T_1 s)(1+\beta T_2 s)}$

Let us draw the pole zero plot for the above transfer function



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Clearly we have -1/T (which is a zero of the transfer function) is far to the origin than the -1/(βT)(which is the pole of the transfer function). Thus we can say in the **lag-lead compensation** pole is more dominating than the zero and because of this lag-lead network may introduces positive phase angle to the system when connected in series.

Advantages of Phase Lag Lead Compensation

- Due to the presence of phase lag-lead network the speed of the system increases because it shifts gain crossover frequency to a higher value.
- Due to the presence of phase lag-lead network accuracy is improved.

References

1. Katsuhiko Ogata (1997), Modern Control Engineering, Prentice Hall.
2. Benjamin C. Kuo (1975), Automatic Control Systems, 3rd Edition, Prentice Hall.
3. Smarajit Gosh (2007), Control systems, Pearson Education.

THANK YOU FOR YOUR ATTENTION!!