Automatic Control Systems

Lecture-12

Describe Types of System Controllers

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Session Objectives

By the end of this session, learners will be able to:

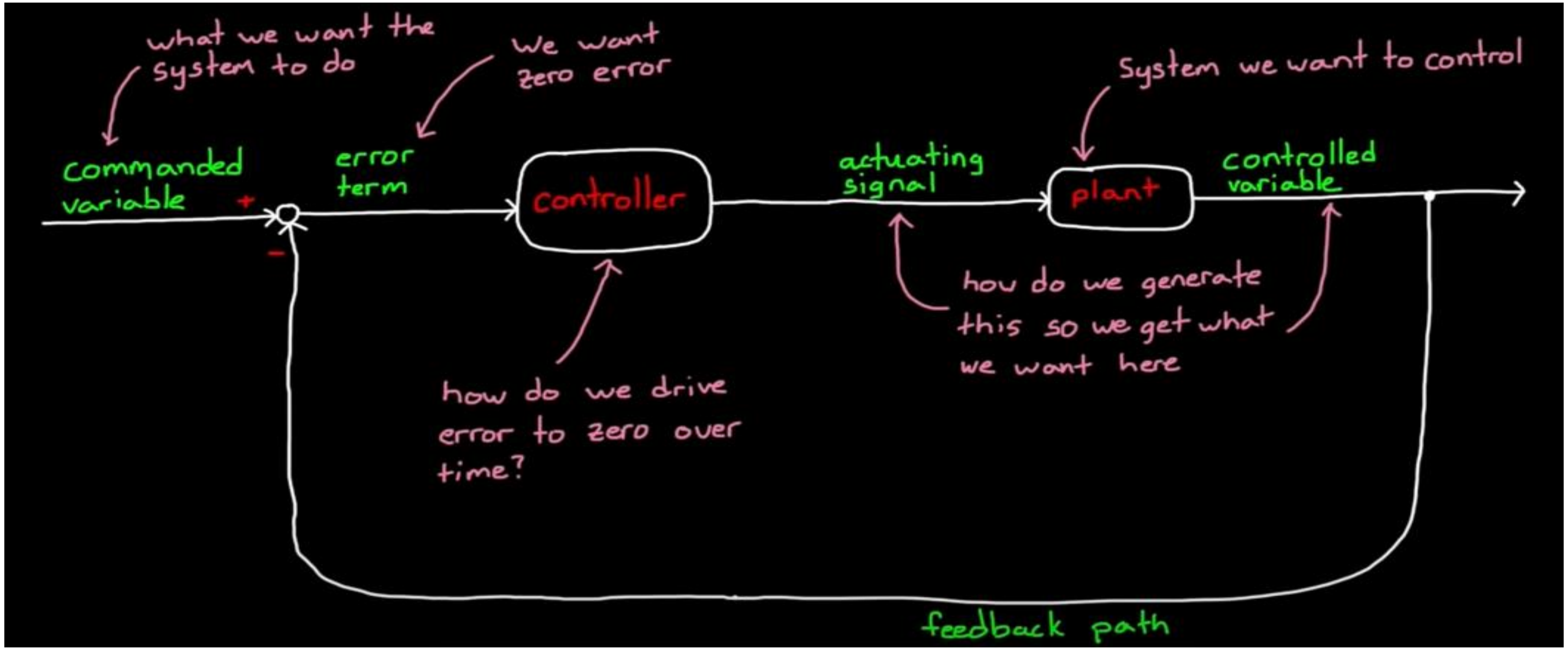
- Define a system controller
- Classify system controllers based on control action
- Explain advantages of each type of controller
- Find the transfer function of each type of controller

Definition of a System Controller

A controller compares controlled values with the desired values and has a function to correct the deviation produced.

Comparison of the actual value of plant output with the desired value is carried out by an automatic controller. It determines the deviation and generate a control signal to reduce the deviation to zero or a small value.

The system block diagram with a controller



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Types of Controllers

Industrial controllers are classified based on the control action as follow:

- Proportional controllers [P]
- Integral Controllers [I]
- Derivative Controllers [D]
- Proportional plus Integral Controllers [PI]
- Proportional plus Derivative Controllers [PD]
- Proportional plus Integral plus derivative Controllers [PID]

Smarajit Gosh (2007), Control systems, Pearson Education, page 284

Use of Controllers

- Controllers improve steady state accuracy by reducing the steady state errors
- As the steady state accuracy improves, the stability also improves
- They also help in reducing the offsets in the system
- Maximum overshoot of the system can be controlled
- Also, they help in reducing the noise signals produced in the system
- Slow response can be made faster

Proportional Controllers

The actuating signal is proportional to the error signal.

The error signal e(t) is the difference between the reference input r(t) and the feedback signal.

Proportional controller is used where the error signal is weak and need some amplification. Offset can be tolerated and load changes are small.

Transfer function of Proportional Controller

Let us consider u(t)→actuating signal (output of the controller)

e(t)→ error signal (input to the controller)

$$u(t) \propto e(t)$$
$$u(t) = K_p e(t)$$

By applying Laplace transform on both sides, we get

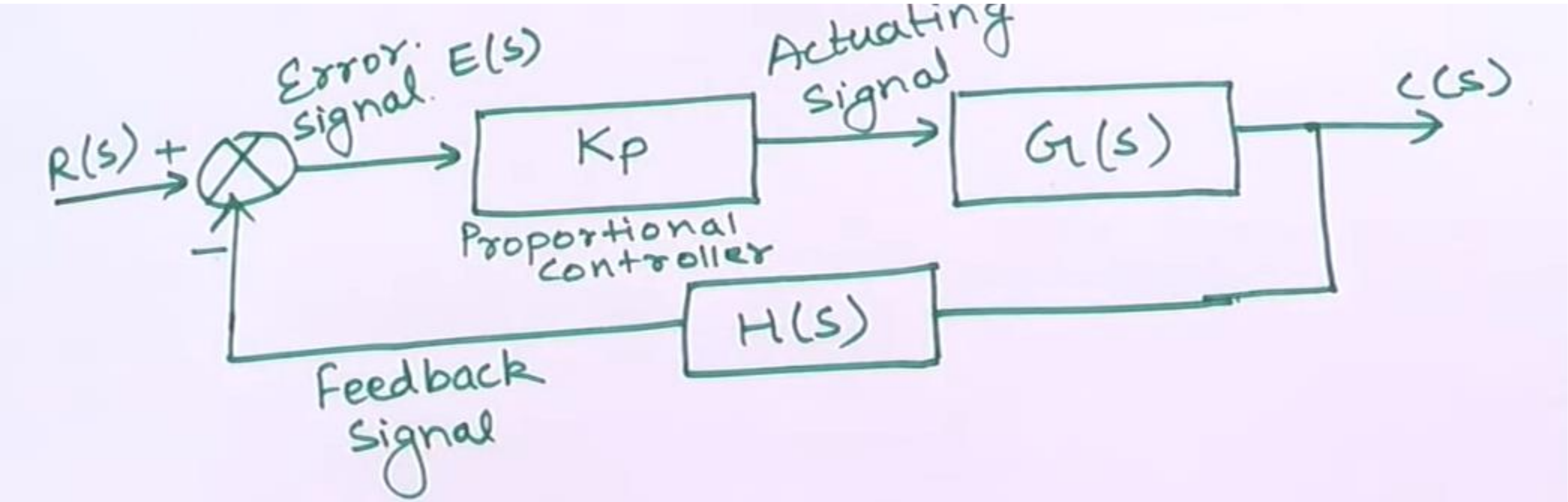
$$U(s) = K_p E(s) \ \therefore K_p \rightarrow Proportional\ Constant$$

The transfer function $\frac{U(s)}{E(s)} = K_p$

Advantages of Proportional Controller

- Reduce steady state error
- Makes the response of the system faster by increasing forward path gain
- Maximum overshoot can be reduced to some extent

A system block diagram with Proportional Controller



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Integral Controller

Integral controller produces an output which is integral of the error signal.

It decreases steady state error but stability is low.

Transfer function of Integral Controller

Let us consider u(t)→actuating signal (output of the controller)

e(t)→ error signal (input to the controller)

$$u(t) \propto \int e(t)$$

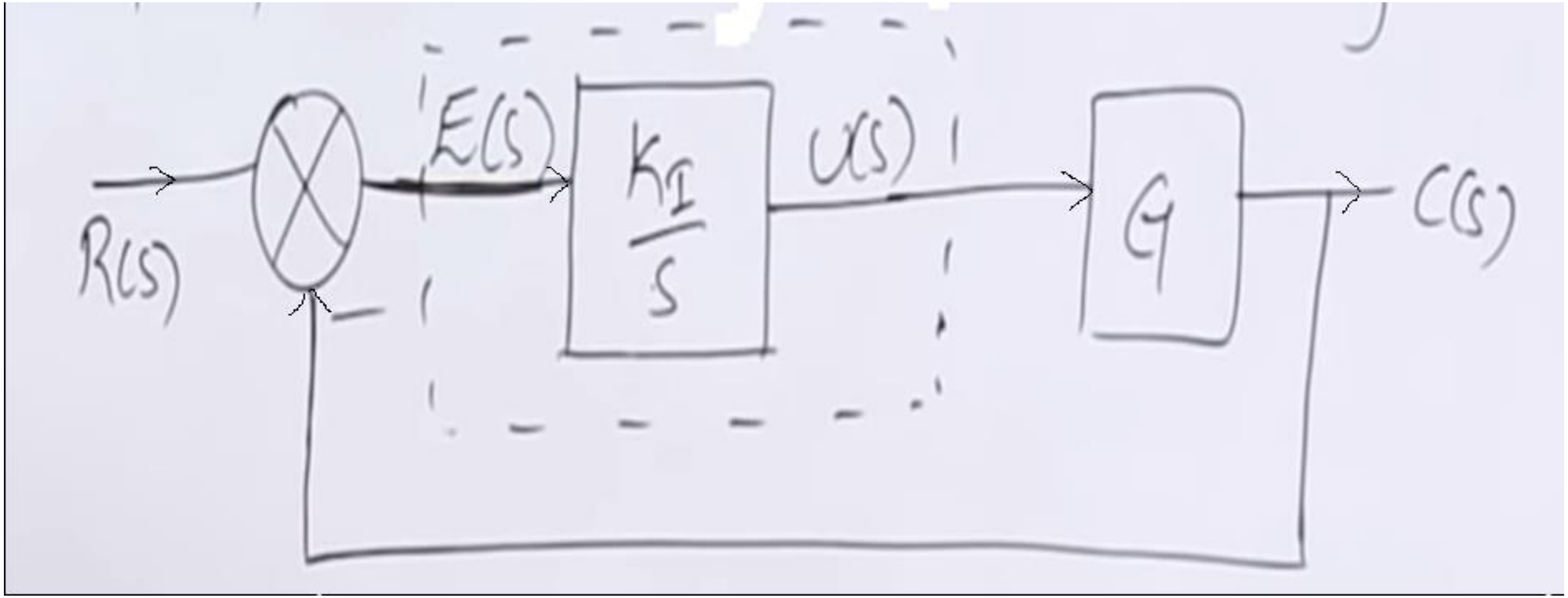
$$u(t) = K_i \int e(t)$$

By applying Laplace transform on both sides, we get

$$U(s) = K_i \frac{E(s)}{s} \therefore K_i \rightarrow Integral\ Constant$$

The transfer function $\frac{U(s)}{E(s)} = \frac{K_i}{s}$

A system block diagram with Integral Controller



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Derivative Controller

Derivative controller produces an output which is derivative of error signal.

Advantage

It increases the stability of the control system

Transfer function of Derivative Controller

Let us consider u(t)→actuating signal (output of the controller)

e(t)→ error signal (input to the controller)

$$u(t) \propto \frac{de(t)}{ds}$$

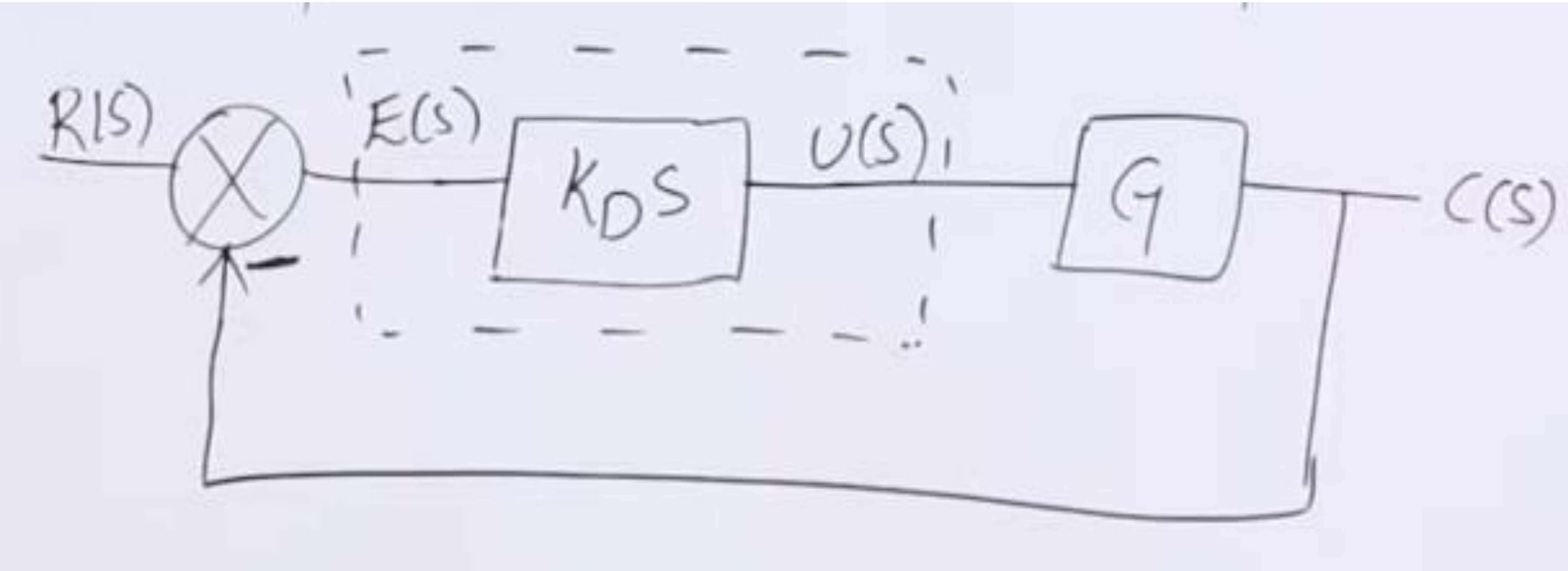
$$u(t) = K_d \frac{de(t)}{ds}$$

By applying Laplace transform on both sides, we get

$$U(s) = K_d s E(s) \therefore K_d \rightarrow Derivative\ Constant$$

The transfer function $\frac{U(s)}{E(s)} = K_d s$

A system block diagram with Derivative Controller



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Proportional plus Integral (PI) Controller

It produces an output which is the combination of outputs of proportional and integral controller

Advantage

To decrease steady state error without affecting the system stability

Transfer function of Proportional plus Integral Controller

Let us consider u(t)→actuating signal (output of the controller)

e(t)→ error signal (input to the controller)

$$u(t) \propto e(t) + \int e(t)$$

$$u(t) = K_p e(t) + K_i \int e(t)$$

By applying Laplace transform on both sides, we get

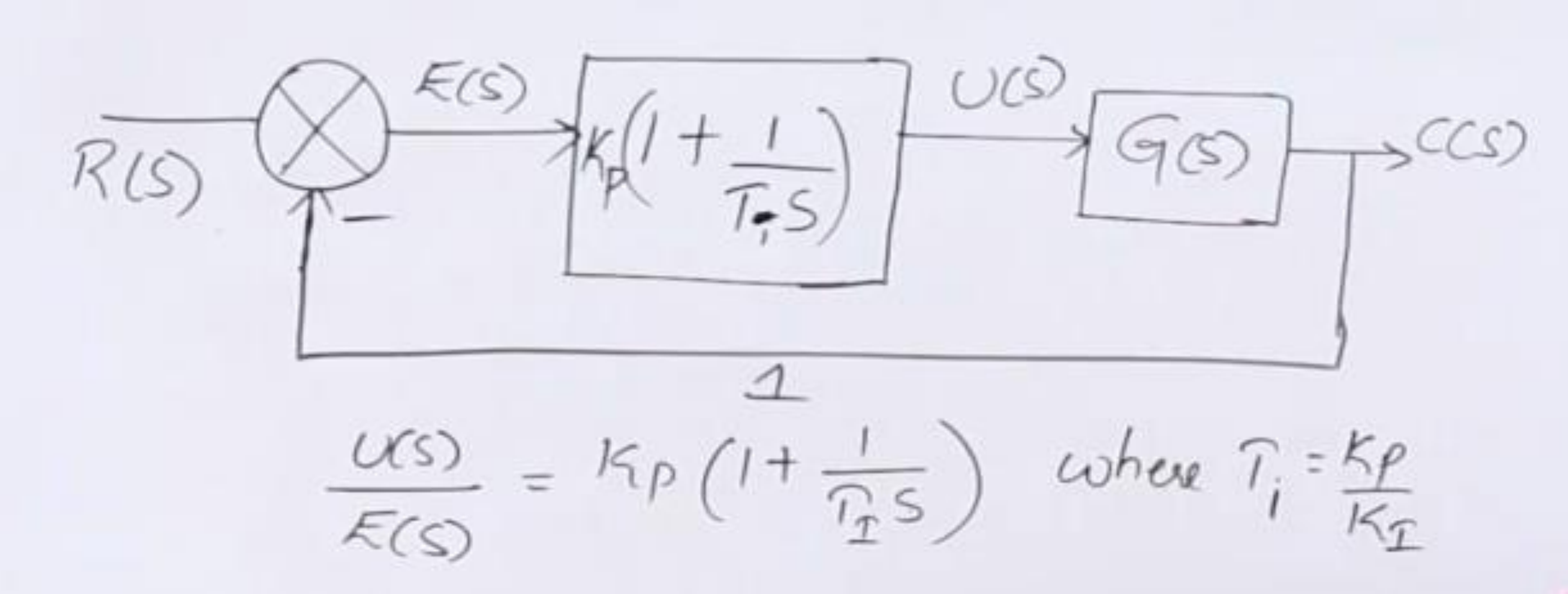
$$U(s) = K_p E(s) + K_i \frac{E(s)}{s}$$

The transfer function $\frac{U(s)}{E(s)} = \left(K_p + \frac{K_i}{s}\right)E(s)$

$$= K_p\left(1 + \frac{K_i}{K_p s}\right)E(s)$$

$$= K_p\left(1 + \frac{1}{T_i s}\right)E(s) \quad \therefore T_i = \frac{K_p}{K_i}$$

A system block diagram with Proportional plus Integral (PI) Controller



$$\frac{U(S)}{E(S)} = K_p\left(1+\frac{1}{T_I S}\right) \quad \text{where } T_i = \frac{K_p}{K_I}$$

Proportional plus Derivative (PD) Controller

It produces an output which is the combination of the outputs of proportional and derivative controllers

Advantage

To increase the stability of the system without affecting steady state error.

Transfer function of Proportional plus Integral Controller

Let us consider u(t)→actuating signal (output of the controller)

e(t)→ error signal (input to the controller)

$$u(t) \propto e(t) + \frac{de(t)}{ds}$$

$$u(t) = K_p e(t) + K_d \frac{de(t)}{ds}$$

By applying Laplace transform on both sides, we get

$$U(s) = K_p U(s) + K_d s U(s)$$

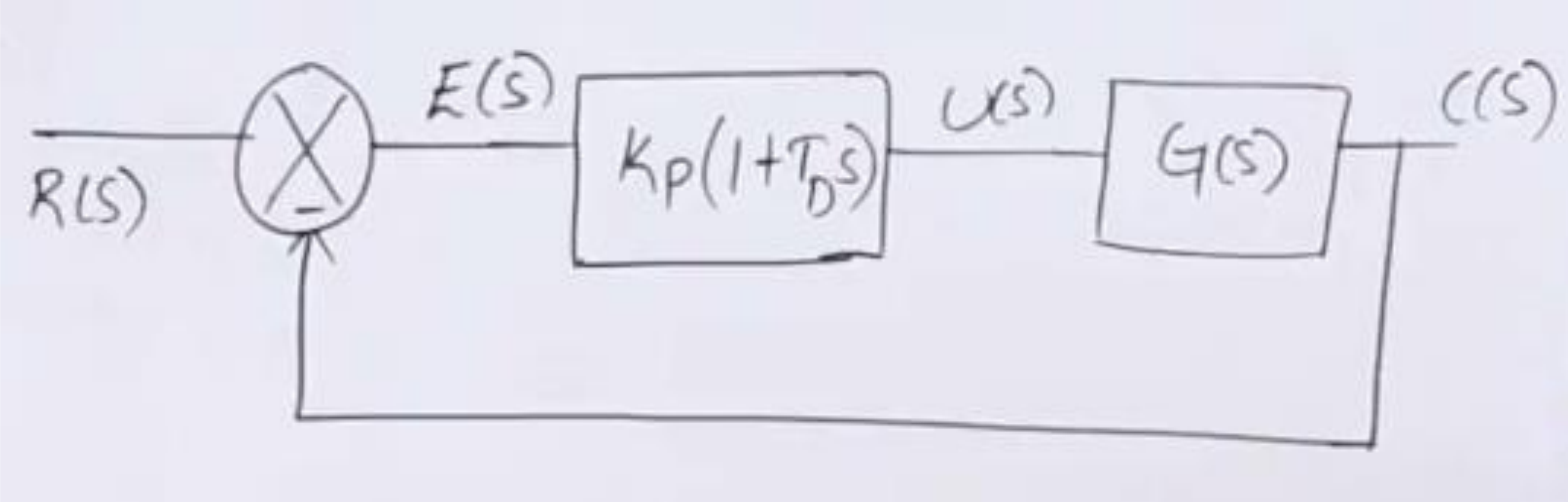
$$U(s) = \left[K_p + K_d s\right] U(s)$$

The transfer function $\frac{U(s)}{E(s)} = \left[K_p + K_d s\right]$

$$= K_p\left[1 + \frac{K_d}{K_p}\mathrm{s}\right]$$

$$= K_p[1 + T_d s] \;\therefore T_d = \frac{K_d}{K_p}$$

A system block diagram with Proportional plus Derivative (PD) Controller



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Proportional plus Integral plus Derivative (PID) Controller

It produces an output which is the combination of the outputs of proportional, integral and derivative controllers

Advantage

To decrease steady state error and increase the system stability

Transfer function of Proportional plus Integral plus Derivative Controller

Let us consider u(t)→actuating signal (output of the controller)

e(t)→ error signal (input to the controller)

$$u(t) \propto e(t) + \int e(t) + \frac{de(t)}{ds}$$

$$u(t) = K_p e(t) + K_i \int e(t) + K_d \frac{de(t)}{ds}$$

By applying Laplace transform on both sides, we get

$$U(s) = K_p E(s) + K_i \frac{E(s)}{s} + K_d sE(s)$$

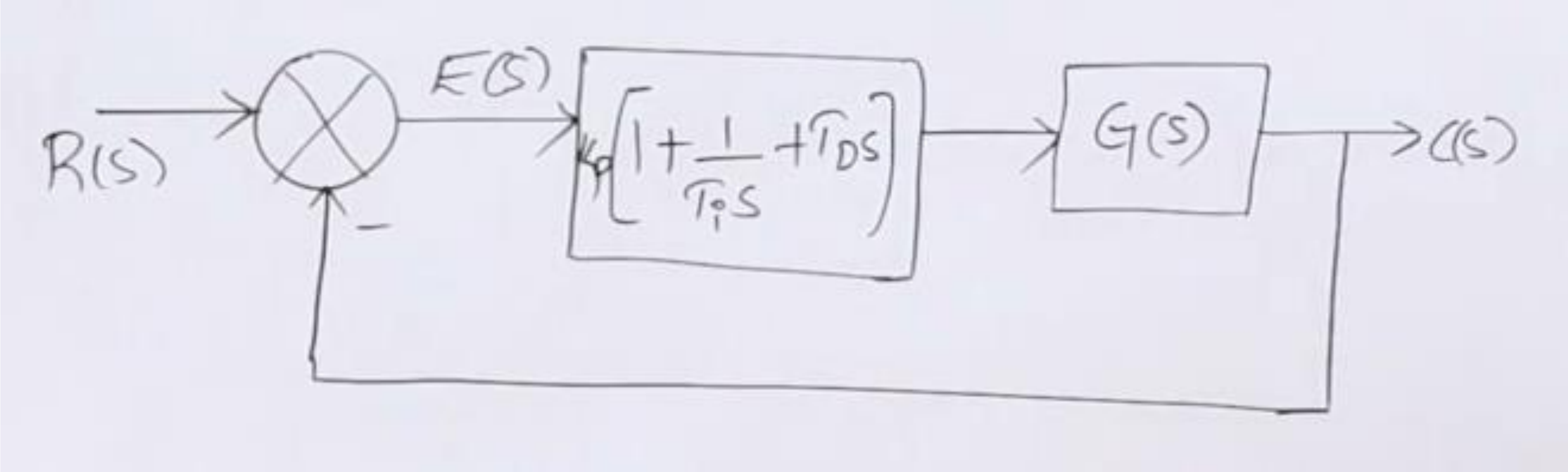
$$U(s) = \left[K_p + \frac{K_i}{s} + K_d s\right] E(s)$$

The transfer function $\frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s$

$$= K_p \left[1 + \frac{K_i}{K_p s} + \frac{K_d}{K_p} s\right]$$

$$= K_p \left[1 + \frac{1}{T_i s} + K_d s\right] \therefore T_i = \frac{K_p}{K_i} \; and \; T_d = \frac{K_d}{K_p}$$

A system block diagram with Proportional plus Integral plus Derivative (PID) Controller



https://www.youtube.com/watch?v=vzajydHWqrE&t=233s

References

1. Katsuhiko Ogata (1997), Modern Control Engineering, Prentice Hall.
2. Benjamin C. Kuo (1975), Automatic Control Systems, 3rd Edition, Prentice Hall.
3. Smarajit Gosh (2007), Control systems, Pearson Education.
4. https://www.youtube.com/watch?v=vzajydHWqrE&t=233s
5. https://www.youtube.com/watch?v=wkfEZmsQqiA

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