Mathematics for Science

Time allowed: **2 hours**

Points: **60 marks**

Lecturer: Kahenya, N.P

Instructions:

Attempt all Questions. Read Questions keenly before responding to the questions. The items carry different weights/marks. Show your working clearly.

SECTION A (Compulsory)

a) Convert the rectangular coordinates (3, - 4) to polar coordinates. (2 points)

b) Distinguish between an ellipse and a circle (without a diagram). (2 points)

c) Convert the following polar equation into rectangular equation; $\mathrm{r} = \mathrm{Cos}\theta + 2\mathrm{Sin}\theta$, and hence identify the graphs of the polar equation and the resultant equation. (4 points)

d) Find the numerical value of k if the expression $x^3 + kx^2 + 7x - 6$ has a reminder of – 4 when divided by $(x + 2)$. (3 points)

e) Show that $(x - 2)$ is a factor of the expression $x^4 - 13x^2 + 36$. (2 points)

f) A school committee of 9 members is to be constituted from 8 parents, 6 teachers and the headteacher. In how many ways can the committee be formed to include the headteacher and 5 parents? (2 points)

g) Determine the general equation of a circle which has a diameter with endpoints at $(0,5)\text{and }(8,5)$ (4 points)

h) Solve the following equation for the values of x between 0 and 100 degrees. (4 points)

$$6 \sin x \cos x = 1$$

i) A parabolic signal satellite disk has the shape $x^2 = 9y$. At what point should the signal receiver be located? (3 points)

j) Simplify; $\frac{\cos^2 x - 1}{\sin 2x}$ and hence determine acute angle x if $\frac{\cos^2 x - 1}{\sin 2x} = -2$ (4 points)

SECTION B

Question 1 (compulsory) – 20 marks

a) Determine the vertex, focus, directrix, and axis for parabola $y^2 - 4x + 10y + 13 = 0$ (4 points)

b) Find the partial fraction decomposition of; $\frac{2x^2+10x-3}{(x+1)(x^2-9)}$ (4 points)

c) Determine the center and the radius of the circle equation; (3 points)

$$x^2 + y^2 - 10x + 14y + 25 = 0$$

d) Prove that; $\frac{\sin A}{\sin 2A} + \frac{\cos A}{1+\cos 2A} = \sec A$. (3 points)

e) Determine the numerical values of a and b if $(x - 1)$ and $(x + 2)$ are both factors of $x^3 + ax^2 + bx - 2$ (3 points)

f) A computer science student decided to generate some codes from the word DOUBLES.

(i) How many 3-letter code words can be formed from this word? (1 point)

(ii) How many of these 3-letter codes contain the letter D? (2 points)

Question 2 (Optional) – 10 marks

a) Identify the conic section given the equation below. (2 points)

$$2x^2 - y^2 - 7 = 0$$

b) Convert into rectangular equation the polar equation $r^2 = 4\sin 2\theta$. What name is given to the curve of the polar equation. (4 points)

c) Write the equation of the hyperbola below in standard form. (4 points)

$$25x^2 - 9y^2 - 100x - 72y - 269 = 0$$

Question 3 (Optional) – 10 marks

a) How many selections of 4 letters can be made from the 9 letters? (2 points)

b) Prove that; (4 points)

$$\frac{\sin^3\theta}{\cos^2\theta} + \sin\theta = \tan\theta\sec\theta$$

c) Find all the values of x where $0^o \leq x \leq 360^o$ that satisfy the equation; (4 points)

$$3\cos x - 5\sin 2x = 0$$

Question 4 (Optional) – 10 marks

a) Find the remainder when the expression $x^3 - 4x^2 + 3x + 5$ is divided by $(x - 3)$. (1 point)

b) Given that $(x + 2)$ and $(x - 3)$ are factors of $ax^3 + ax^2 + bx + 12$. Find the numerical values of a and b. (4 points)

c) Decompose into partial fractions; (5 points)

$$\frac{x^2-6x+2}{x^2(x-2)^2}$$